SE I CMPN | III (CR) 30/5/13
DSDT

mk.72-1st hlf 13-K

Con. 6572-13. **GS-6525**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.
(4) Assume suitable **data** if **necessary.**

1. (a) Prove by Mathematical Induction — **8**

$$1^2 + 2^2 + 3^2 + \ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(b) Explain the terms :— **6**
(i) Poset
(ii) Normal Subgroup
(iii) Lattice.

(c) In a survey of 60 people, it was found that 25 reads Newsweek Magazine, 26 reads Times and 26 reads Fortune. Also 9 reads both Newsweek and Fortune, 11 reads both Newsweek and Times, 8 reads Time and Fortune and 8 reads no magazine at all. **6**
(i) Find the number of people who read all three magazines.
(ii) Determine number of people who read exactly one magazine.

2. (a) Define injective, surjective and bijective functions. **8**
if $f : R \to R$ and $g : R \to R$ are defined by the formulas —
$f(x) = x + 2$ and $g(x) = x^2$
Find (i) f.g.f. (ii) g.f.g.

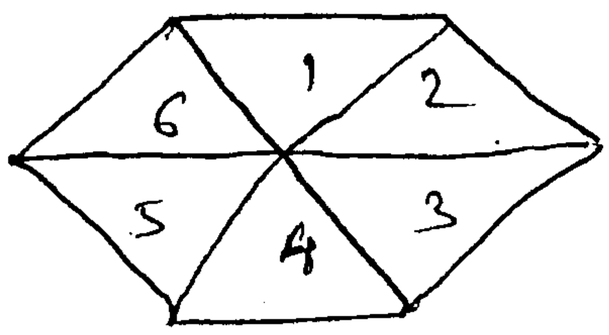
(b) Define equivalence relation on a set. Let R be a relation on the set of integers defined by aRb iff a–b is a multiple of 5. Prove that R is a equivalence relation. **6**

(c) State the converse, inverse and contrapositive of the following :— **6**
(i) If it is cold, then he wears a hat.
(ii) If an integer is a multiple of 2, then it is even.

3. (a) Explain Hasse diagram. Draw the Hasse diagram of the relation given by :— **8**
(i) R_1 = { (1, 1), (1, 2), (1, 3) (1, 4) (1, 5), (2, 2), (2, 3) (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5) }
(ii) R_2 = { (1, 1), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (5, 5) }

(b) Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (3, 4), (2, 1)}. Find the Transitive closure of R using Warshall's Algorithm. **6**

(c) Consider the region shown below. It is bounded by a regular hexagon whose sides are the length 1 units. Show that if any seven points are chosen in this region then two of them must be no further apart than 1 unit. **6**



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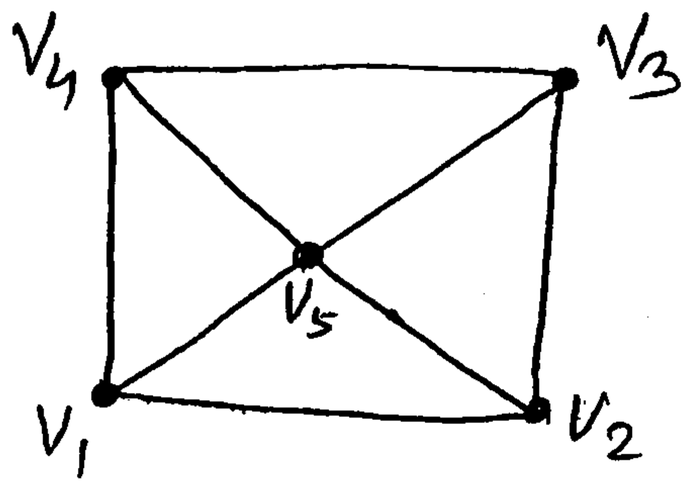
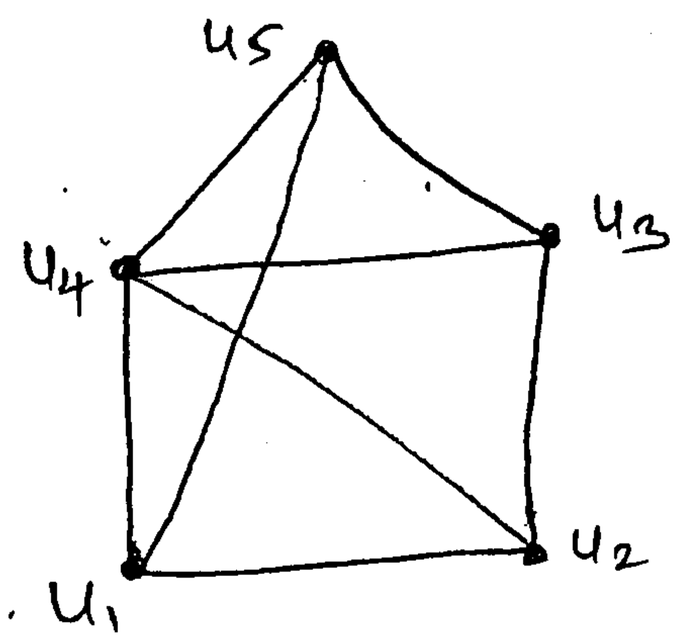
4. (a) Show that the following graphs are isomorphic. **8**





(b) Let R = {(1, 2), (4, 3), (2, 2), (2, 1), (3, 1)} be a relation on s = {1, 2, 3, 4}. Find the symmetric closure of R. **6**

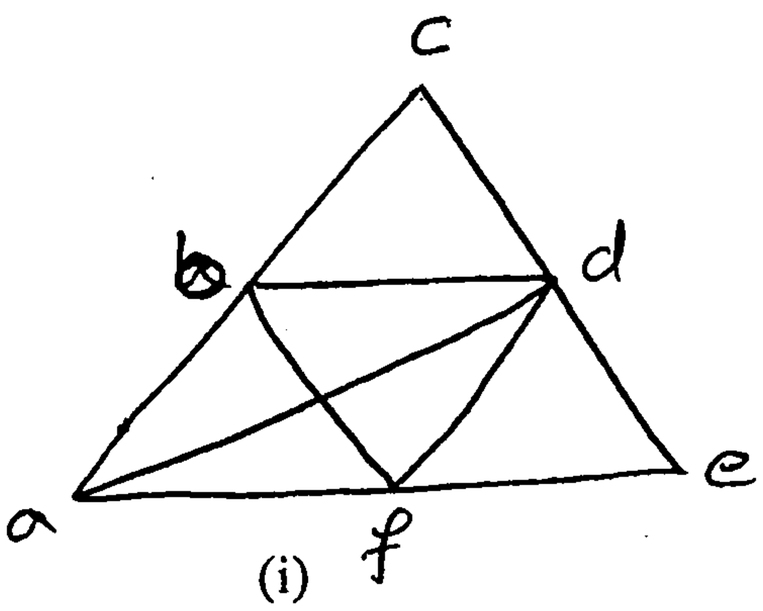
(c) Define : **6**

(i) Integral domain
(ii) Field
(iii) Normal Subgroup.

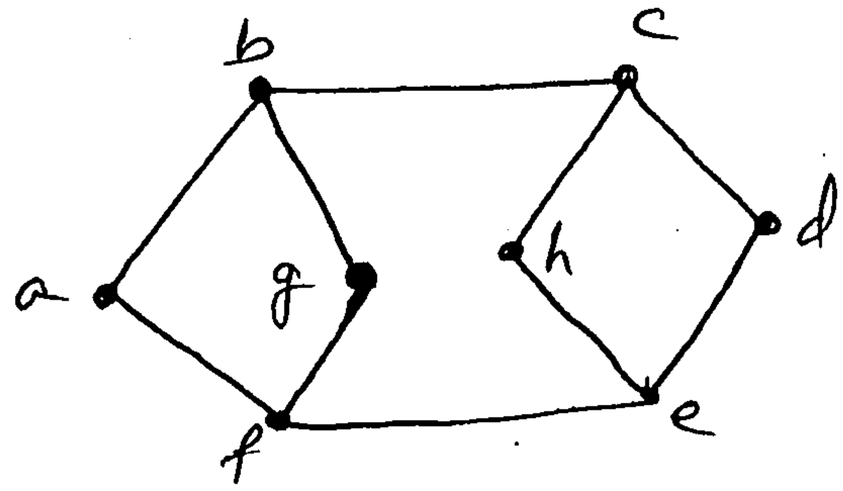
5. (a) What is a minimum spanning tree ? Explain any one technique with example. **8**

(b) Define Cyclic Group. Prove that the set A = {0, 1, 2, 3, 4, 5} is a finite abelian under addition modulo 6. **6**

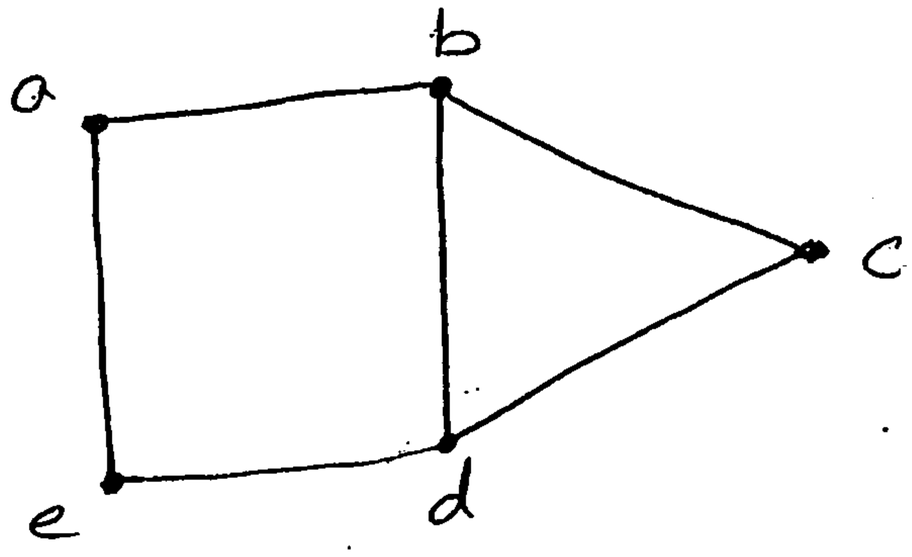
(c) Determine whether the given graph has a Hamilton circuit or Eulerian circuit. If it does, find such a circuit. **6**



(i)



(iii)



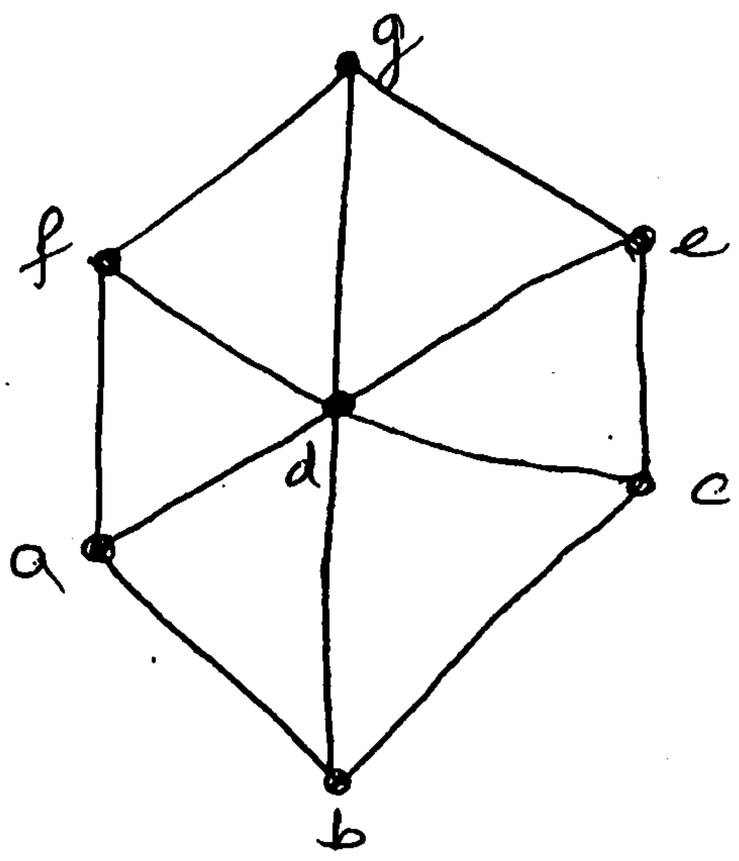
(ii)

6. (a) Consider the (3, 5) group encoding function $E : B^3 \rightarrow B^5$ is defined by — **8**

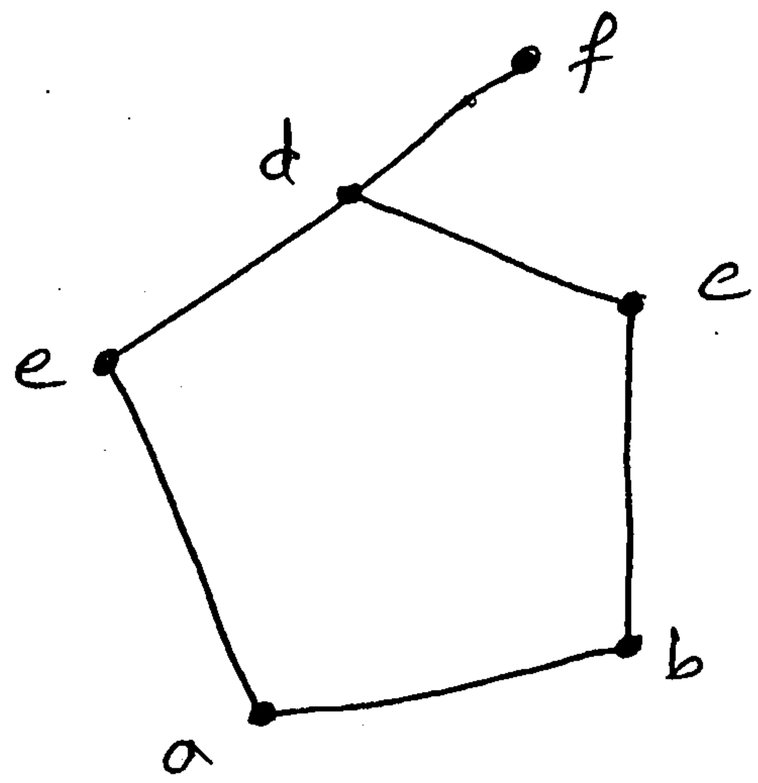
$E(000) = 00000$ $E(100) = 10011$
$E(001) = 00110$ $E(101) = 10101$
$E(010) = 01001$ $E(110) = 11010$
$E(011) = 01111$ $E(111) = 11100$

Decode the following words relative to a maximum decoding function :—
(i) 11001 (ii) 01010 (iii) 00111 (iv) 11100

(b) Which of the following diagram in the **figure** represents a lattice ? Justify. **6**



(i)



(ii)

(c) Define with example :— **6**
(i) Planer graph
(ii) Semigroup
(iii) Quantifiers.

7. (a) Find the solution to the recurrence relation :— **6**

$a_n = a_{n-1} + 2 \qquad n \geq 2$

subject to initial condition $a_1 = 3$.

(b) Find the complement of each element in D_{30}. **6**

(c) Find the generating function for each of the following sequence :— **8**
(i) {0, 1, 2, 3, 4, ...}
(ii) {1, 2, 3, 4, ...}
(iii) {2, 2, 2, 2, ...}
(iv) {0, 0, 1, 1, 1, ...}