USN

06MAT11

First Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the Answer Booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. i) The n^{th} derivative of $\frac{1}{x^p}$ is

A) $\frac{(-1)^{n+1}(p+n)!}{(p-1)!\,x^{p+n}}$ B) $\frac{(-1)^{n+1}(p+n-1)!}{(p-1)!\,x^{p+n}}$ C) $\frac{(-1)^{n}(p+n-1)!}{(p-1)!\,x^{p+n}}$ D) $\frac{(-1)^{n}(p+n-1)!}{p!\,x^{p}}$

ii) The n^{th} derivative of e^{ax} is

A) $a^n e^{ax}$ B) ae^x C) a^2e^x D) e^x

iii) The angle between radius vector and tangent is

A) $\tan\phi = r\frac{d\theta}{dr}$ B) $\tan\phi = r^2\frac{d\theta}{dr}$ C) $\tan\phi = \frac{1}{r}\frac{d\theta}{dr}$ D) $\tan\phi = \frac{dr}{d\theta}$

iv) The curve $r = \frac{a}{1+\cos\theta}$ intersect orthogonally with the following curve:

A) $r = \frac{b}{1-\cos\theta}$ B) $r = \frac{b}{1+\sin\theta}$ C) $r = \frac{b}{1+\sin^2\theta}$ D) $r = \frac{b}{1+\cos^2\theta}$ **(04 Marks)**

b. Find the n^{th} derivation of $y = \sinh 2x \sin 4x$. **(04 Marks)**

c. If $y = \sinh\left(m\log(x+\sqrt{x^2+1})\right)$, prove that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$. **(06 Marks)**

d. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

2 a. i) If $u = \log\left(\frac{x^2}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

A) 2u B) 3u C) u D) 1

ii) If $u = x^3 + y^3$, then $\frac{\partial^3 u}{\partial x^2 \partial y}$ is equal to

A) -3 B) 3 C) 0 D) $3x+3y$

iii) If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to

A) 1 B) r C) $\frac{1}{r}$ D) 0

iv) If an error of 1% is made in measuring its length and breadth, the percentage error in the area of a rectangle is

A) 0.2% B) 0.02% C) 2% D) 1% **(04 Marks)**

b. If $z = e^{ax+by} + (ax-by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(04 Marks)**

2 c. If $w = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, show that $\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$. **(06 Marks)**

d. If u, v are functions of r, s and r, s are functions of x, y, prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$. **(06 Marks)**

3 a. i) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$ is

A) $\frac{16}{25}\pi$ B) $\frac{25}{16}\pi$ C) $\frac{16\pi^2}{25}$ D) $\frac{25}{16}\pi^2$

ii) The curve $y^2(a - x) = x^2(a + x)$ is symmetrical about the ______ axis.

A) x B) y C) both x and y D) none of these

iii) The value of $\int_0^1 x^{3/2}(1 - x)^{3/2} dx$ is

A) $\frac{\pi}{32}$ B) $\frac{-\pi}{32}$ C) $\frac{3\pi}{128}$ D) $\frac{-3\pi}{128}$

iv) If $f(r, \theta) = f(-r, \theta)$ then the curve is symmetrical about the ______

A) initial line B) pole C) origin D) tangential line **(04 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1 + x^2)^{7/2}} dx$. **(04 Marks)**

c. Obtain the reduction formula for $\int_0^{\pi/4} \sec^n x \, dx$. **(06 Marks)**

d. Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. **(06 Marks)**

4 a. i) If $y = f(x)$ be the equation of the Cartesian curve then $\frac{ds}{dx}$ is equal to

A) $\sqrt{1 + y_1^2}$ B) $\sqrt{1 + y_1}$ C) $-\sqrt{1 + y_1^2}$ D) $-\sqrt{1 + y^2}$

ii) The area of the cardioid $r = a(1 + \cos\theta)$ is

A) $\frac{3}{2}\pi a$ B) $\frac{2}{3}\pi a$ C) $\frac{3}{2}\pi a^2$ D) $\frac{2}{3}\pi a^2$

iii) The surface area of the solid got by revolving the circle $r = 2a\cos\theta$ about the initial line is

A) $4\pi^2 a$ B) $4\pi a^3$ C) $4\pi a^2$ D) $8\pi a$

iv) The volume generated by the revolution of the curve $y = \frac{a^3}{a^2 + x^2}$ about its asymptote is

A) $\frac{\pi^2 a^3}{2}$ B) $\frac{\pi a^3}{2}$ C) $\frac{\pi a^2}{2}$ D) $\frac{\pi a}{2}$ **(04 Marks)**

b. Find the length of the arc of the curve $y = \log \sec x$ between the points for which $x = 0$ and $x = \pi/3$. **(04 Marks)**

4 c. Find the surface area of the solid got by revolving the arch of the cycloid $x = a(t + \sin t)$, $y = a(t + \cos t)$ about the base. **(06 Marks)**

d. Evaluate $\int_0^{\infty} \frac{\tan^{-1} \alpha x}{x(1+x^2)} dx$ where $\alpha \geq 0$ using the rule of differentiation under the integral sign. **(06 Marks)**

PART – B

5 a. i) The order of the differential equation $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 4y = 0$ is

A) 2 B) 0 C) 3 D) 1

ii) The integrating factor of the differential equation $\frac{dy}{dx} + y\cos x = \frac{\sin 2x}{2}$ is

A) $e^{\sin^2 x}$ B) $e^{\sin^3 x}$ C) $e^{\sin x}$ D) $\sin x$

iii) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\frac{y}{x}$ is

A) $\cos\left(\frac{y}{x}\right) - \log x = c$ B) $\cos\left(\frac{y}{x}\right) + \log x = c$

C) $\cos^2\left(\frac{y}{x}\right) + \log x = c$ D) $\cos^2\left(\frac{y}{x}\right) - \log x = c$

iv) By replacing $\frac{dr}{d\theta}$ by $-r^2\frac{dr}{d\theta}$ in the differential equation $f\left(r, \theta, -r^2\frac{dr}{d\theta}\right) = 0$, we get the differential equation of ________.

A) Orthogonal trajectory B) Polar trajectory
C) Parametric trajectory D) None of these. **(04 Marks)**

b. Solve : $(1-x^2)\frac{dy}{dx} - xy = 1$. **(04 Marks)**

c. Solve : $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$. **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $r = 2a(\cos\theta + \sin\theta)$ where a is a parameter. **(06 Marks)**

6 a. i) The series $\frac{1}{1^P} + \frac{1}{2^P} + \frac{1}{3^P} + \ldots\ldots\ldots$ converges if

A) $p > 0$ B) $p < 1$ C) $p > 1$ D) $p \leq 1$

ii) $\sum \sin\left(\frac{1}{n}\right)$ is

A) convergent B) divergent C) oscillatory D) none of these

iii) The convergence of the series $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \ldots\ldots\ldots$

A) Leibnitz test B) Raabe's test C) Ratio test D) Cauchy's root test

iv) If a series $\sum y_n$ is such that S_n does not tend to unique limit as $n \to \infty$, we say that the series $\sum y_n$ is

A) convergent B) divergent C) oscillatory D) none of these **(04 Marks)**

b. Determine the nature of the series $\sum\left(\sqrt{n^2+1} - n\right)$. **(04 Marks)**

6 c. Test the convergence of the series $\frac{2}{3.4}+\frac{2.4}{3.5.6}+\frac{2.4.6}{3.5.7.8}+\dots\dots\dots$ **(06 Marks)**

d. Find the nature of the series $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+\dots\dots$ **(06 Marks)**

7 a. i) A line makes angles α, β, γ with coordinate axes, then $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$ is equal to

A) 1 B) 2 C) −1 D) −2

ii) Find the angle between the planes $x-y+2z-9=0$ and $2x+y+z=7$ is

A) 30° B) 90° C) 60° D) 120°

iii) Two straight lines which lie in the same plane are called

A) parallel B) perpendicular C) coplanar D) non-coplanar

iv) The normal form of plane equation is

A) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ B) $l^2+m^2+n^2=1$

C) $l_1l_2+m_1m_2+n_1n_2=0$ D) $lx+my+nz=p$ **(04 Marks)**

b. Prove that the sum of the squares of the direction cosines of a line is equal to unity. **(04 Marks)**

c. Find the image of the point (1, 2, 3) in the plane x + y + x = 9. **(06 Marks)**

d. Find the shortest and the equation of the line of shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and the y – axis. **(06 Marks)**

8 a. i) The acceleration of the moving particle along the curve $x=\cos 3t$, $y=\sin 3t$, $z=-t$ is

A) $-3\sin t\,\hat{i}+3\cos 3t\,\hat{j}-\hat{k}$ B) $\cos t\,\hat{i}+\sin 3t\,\hat{j}-\hat{k}$

C) $-9\cos 3t\,\hat{i}-9\sin 3t\,\hat{j}$ D) $-12\cos 3t\,\hat{i}-12\sin 3t\,\hat{j}$

ii) The directional derivative of x^2yz+xz^2 at (-1, 2, 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$ is

A) $-\frac{7}{3}$ B) $\frac{7}{3}$ C) $\frac{3}{7}$ D) $-\frac{3}{7}$

iii) If a particle moves along a curve $\vec{R}(t)=x(t)i+y(t)j+z(t)k$ then $\frac{dR}{dt}$ is

A) Radial vector B) Tangential vector C) Normal vector D) Unit vector

iv) Curl (grad ϕ) is equal to

A) unity B) $\hat{i}+\hat{j}+\hat{k}$ C) zero D) none of these **(04 Marks)**

b. Find the angle between the tangents $\vec{r}=t^2\,\hat{i}+2t\,\hat{j}-t^3\,\hat{k}$ at the points $t=\pm 1$. **(04 Marks)**

c. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $|\vec{r}|=r$, find grad $\left(\text{div}\frac{\vec{r}}{r}\right)$. **(06 Marks)**

d. If $\vec{a}$ is a constant vector and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, show that $\frac{1}{2}\text{curl}(\vec{a}\times\vec{r})=\vec{a}$. **(06 Marks)**

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