# MAGNETIC CIRCUIT

A **magnetic circuit** is made up of one or more closed loop paths containing a magnetic flux. The flux is usually generated by permanent magnets or electromagnets and confined to the path by magnetic cores consisting of ferromagnetic materials like iron, although there may be air gaps or other materials in the path. Magnetic circuits are employed to efficiently channel magnetic fields in many devices such as electric motors, generators, transformers, relays, lifting electromagnets, SQUIDs, galvanometers, and magnetic recording heads.

***Definitions concerning Magnetic Circuit-***

**Magnetic flux- The** magnetic lines of force passing through a magnetic circuit is known as magnetic flux. It is denoted by a symbol ϕ and given by a formula $\phi = BA$, where B is the magnetic flux density and A is the area of the cross-section in $m^2$. The unit of magnetic flux is weber.

**Magnetic flux density-**It is the amount of magnetic flux per unit area at right angles to the flux. The unit of magnetic flux density is weber/$m^2$ and denoted by B. The formula is given by,

$$B = \frac{\phi}{a}$$

**Magnetomotive force** (MMF) 'drives' magnetic flux through magnetic circuits. The term 'magnetomotive force', though, is a misnomer since it is not a force nor is anything moving. It is perhaps better to call it simply MMF. In analogy to the definition of EMF. It is given by a formula, $MMF = NI$ The unit of magneto motive force is the ampere-turn (At),

**Magnetic field intensity-**Magnetizing force or Magnetic field intensity or magnetic field strength is the MMF required to magnetize a unit length of the magnetic flux path. The unit of magnetic field intensity is AT/m and is denoted by H.

$$H = \frac{NI}{l}$$

**Reluctance-**It is the opposition that the magnetic circuit offers for the flow of magnetic flux. We can also define the reluctance as the ratio of magneto-motive force to the magnetic flux. It is denoted by S and its unit is ampere-turns per weber.

$$Reluctance(S) = \frac{MMF}{flux} \qquad = \frac{l}{\mu A}.$$

where

- $l$ is the length of the element,
- $\mu = \mu_r \mu_0$ is the permeability of the material ($\mu_r$ is the relative permeability of the material (dimensionless), and $\mu_0$ is the permeability of free space), and
- $A$ is the cross-sectional area of the circuit.

Permeance- **Permeance is the reciprocal of reluctance. The ease with which the flux can pass through the material is known as permeance. Weber/AT is the unit of permeance.**

$$Permeance = \frac{1}{reluctance}$$

**Permeability-**It is the measure of the resistance of a material against the formation of a magnetic field. In simple words, the permeability of material means its conductivity for magnetic flux. The reciprocal of magnetic permeability is magnetic reluctivity. Greater permeability, greater is its conductivity.

Magnetic permeability is represented by a greek letter μ. It is given by a formula,

$$\mu = \frac{B}{H}$$

**Relative permeability**

It is the ratio of flux density of a magnetic material to the flux density produced in air by the same magnetizing force. The formula for relative permeability is,

$$\mu_r = \frac{\mu}{\mu_0}$$

where $\mu_r$ – relative permeability of the magnetic material.

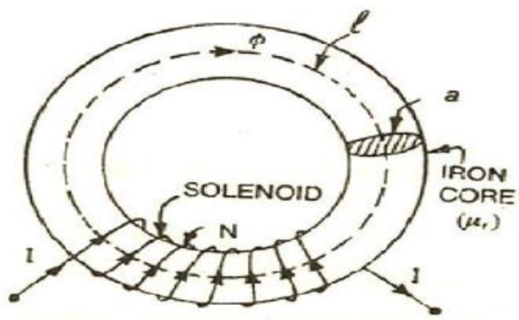
$\mu_o$ – absolute permeability of air or vacuum.

μ – absolute permeability of the magnetic material.

**Reluctivity**

The tendency of a magnetic circuit to conduct magnetic flux, equal to the reciprocal of the permeability of the circuit.

**Example Of Magnetic Circuit-**Consider a solenoid having N turns wound on an iron core. The magnetic flux of ø Weber sets up in the core when the current of I ampere is passed through a solenoid.




Let, l = mean length of the magnetic circuit

A = cross-sectional area of the core

μr = relative permeability of the core

Now the flux density in the core material

$$B = \frac{\varphi}{a} \quad (Weber/m^2)$$

Magnetising force in the core

$$H = B/\mu_0\mu_r$$

$$H = {}^{\varphi}/_{a}\,\mu_0\mu_r \text{ AT/m (Ampere turns/meter)}$$

According to work law, the work done in moving a unit pole once round the magnetic circuit is equal to the ampere-turns enclosed by the magnetic circuit.

$$Hl = NI$$

$$H = \frac{\varphi}{a\mu_0\mu_r} x\, l$$

$$H = NI$$

$$\varphi = \frac{NI}{{}^{l}/_{a\mu_0\mu_r}}$$

The above equation explains the following points:

1. Directly proportional to the number of turns (N) and current (I).
   It shows that the flux increase if the number of turns or current increases and decreases when either of the two quantity decreases. NI is the magneto motive force (MMF).
2. Inversely proportional to $l/a\ \mu_0\mu_r$, where $(l/a\ \mu_0\mu_r)$ is known as reluctance. The lower the reluctance, the higher will be the flux and vice- versa.

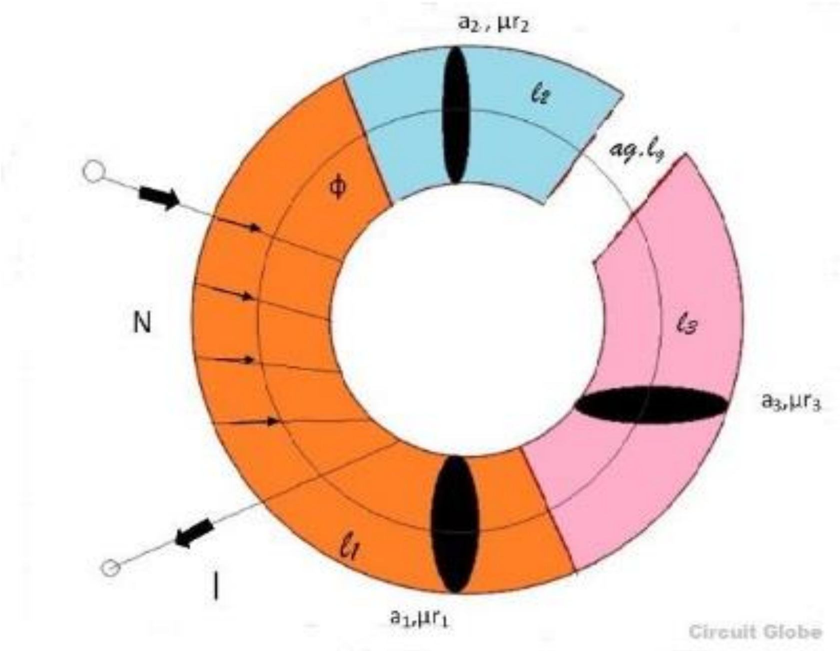
# Analogy between Magnetic circuit and Electric Circuit

| Magnetic Circuit | Electric Circuit |
|---|---|
| A closed path for a magnetic flux forms a magnetic circuit. | A closed path for an electric current form an electric circuit. |
| Magnetic flux does not flow in a magnetic circuit. | Electric current always flows in an electric circuit. |
| MMF is the cause for producing flux. | EMF is the cause for producing current. |
| Weber is the unit of flux. | Ampere is the unit of current. |
| $Flux = \frac{mmf}{reluctance}$ | $Current = \frac{emf}{resistance}$ |
| Reluctance opposes the flow of flux. | Resistance opposes the flow of current. |
| $Reluctance = \frac{l}{\mu_0\mu_r a}$ | $Resistance = \frac{\rho l}{a}$ |
| $Permeance = \frac{1}{reluctance}$ | $Conductance = \frac{1}{resistance}$ |
| Flux density, $B = \frac{\phi}{a}$ | Current density, $J = \frac{I}{a}$ |
| Magnetic field intensity, $H = \frac{NI}{l}$ | Electric field intensity, $E = \frac{V}{d}$ |
| Magnetic flux lines flow from the North pole to the South pole. | Electric current flows from the positive to negative terminal. |

**Difference-**

| Sl. No. | Electric Circuit | Magnetic Circuit |
|---|---|---|
| 1. | In an electric circuit, there is a real flow of current when emf is applied | In a magnetic circuit, there is no real flux but it is only a concept to understand a magnetic circuit. |
| 2. | Energy must be supplied to the electric circuit to maintain a constant flow of current | In a magnetic circuit, once established, the flux can be maintained without an energy source (e.g. permanent magnets) |
| 3. | In an electric circuit, it is possible to precisely point out where the emf is acting. | In a magnetic circuit, it is difficult to find out exactly where the mmf is acting. |

**Series Magnetic Circuit- The** Series Magnetic Circuit **is defined as the magnetic circuit having a number of parts of different dimensions and materials carrying the same magnetic field. Consider a circular coil or solenoid having different dimensions as shown in the figure below:**




Current I is passed through the solenoid having N number of turns wound on the one section of the circular coil. Φ is the flux, set up in the core of the coil.

$a_1$, $a_2$, $a_3$ are the cross-sectional area of the solenoid. $l_1$, $l_2$, $l_3$ *are* the length of the three different coils having different dimension joined together in series. $\mu_{r1}$, $\mu_{r2}$, $\mu_{r3}$ are the relative permeability of the material of the circular coil. $a_g$ and are the area and the length of the air gap.

The total reluctance (S) of the magnetic circuit is

$$S = S_1 + S_2 + S_3 + S_g$$

$$S = \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}$$

Total MMF = $\varphi \times S$ ............ (1)

Putting the value of S in equation (1) we get,

$$\text{Total mmf} = \varphi \; x \; \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0} \;\ldots\ldots\ldots (2)$$

(As $B = \varphi/a$) putting the valve of B in the equation (2) we obtain the following equation for the total MMF

$$\text{Total mmf} = \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

(As $H = B/\mu_0\mu_r$)

$$\text{Total mmf} = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

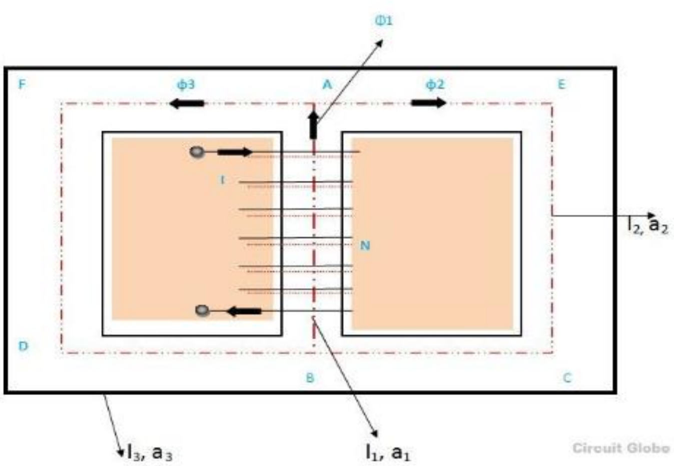
Procedure for the Calculation of the total MMF for the Series Magnetic Circuit

1. The magnetic circuit is divided into a different section or parts.
2. Now determine the value of the flux density (B) of the different sections. As we know $B = \varphi/a$ where $\varphi$ is the flux in Weber, and a is the area of cross-section in $m^2$
3. Determine the value of the magnetising force (H) as we know that $H = B/\mu_0\mu_r$ where B is the flux density in Weber/ $m^2$ and $\mu_0$ is absolute permeability and its value is $\mathbf{4\pi x10^{-7}}$, and $\mu_r$ is the relative permeability of the material, and its value will be given. The value of magnetising force (H) as $H_1$, $H_2$, $H_3$, Hg will be individually multiplied by the length of the different sections that is, $l_1$, $l_2$, $l_3$ and *lg* respectively.
4. Finally, add all the values of Hx ***l*** and therefore, the total MMF will be

$$\sum H \, x \, l = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

The value of H for the air gap part will always $\mu_g = B/\mu_0$.

**Parallel Magnetic Circuit-** A magnetic circuit having two or more than two paths for the magnetic flux is called a **parallel magnetic circuit**. Its behaviour can be compared to the parallel electric circuit. The parallel magnetic circuit contains different dimensional areas and materials having various numbers of paths.



The above figure shows a parallel magnetic circuit. In this circuit, a current-carrying coil is wound on the central limb AB. This coil sets up the magnetic flux $\varphi_1$ in the central limb of the circuit. The flux $\varphi_1$ which is in the upward direction is further divided into two paths namely ADCB and AFEB. The path ADCB carries flux $\varphi_2$, and the path AFEB carries flux $\varphi_3$. It is clearly seen fro the above circuit that

$$\varphi_1 = \varphi_2 + \varphi_3$$

The two magnetic paths ADCB and AFEB form the parallel magnetic circuit, thus, the ampere-turns (ATs) required for this parallel circuit are equal to the ampere-turns (ATs) required for any one of the paths.

As we know, reluctance is

$$S = \frac{l}{a_1 \mu_0 \mu_{r1}}$$

If $S_1$ = reluctance of path BA will be

$$S_1 = \frac{l_1}{a_1 \mu_0 \mu_{r1}}$$

$S_2$ =reluctance of path ADCB will be

$$S_2 = \frac{l_2}{a_2 \mu_0 \mu_{r2}}$$

$S_3$ = reluctance of the path AFEB will be

$$S_3 = \frac{l_3}{a_3 \mu_0 \mu_{r3}}$$

Therefore, the total MMF or the total Ampere turns required in the parallel magnetic circuit will be the sum of all the individual parallel paths.
Total mmf required = mmf required for the path BA +mmf required for the path ADCB + mmf required for the path AFEB

$$\text{Total mmf or Ampere turns} = \varphi_1 S_1 + \varphi_2 S_2 + \varphi_3 S_3$$

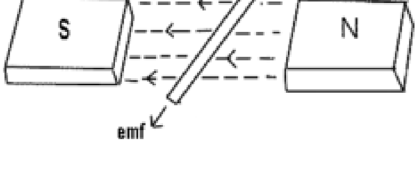
Where $\varphi_1$. $\Phi_2$, $\varphi_3$ is the flux and $S_1$, $S_2$, $S_3$ are the reluctances of the parallel path BA, ADCB and AFEB respectively.

**BASIC LAWS OF ELECTROMAGNETIC INDUCTION-**

The induction of an electromotive force by the motion of a conductor across a magnetic field or by a change in magnetic flux in a magnetic field is called **'Electromagnetic Induction'.**

This either happens when a conductor is set in a moving magnetic field (when utilizing AC power source) – *Dynamically Induced E.M.F* or when a conductor is always moving in a stationary magnetic field- *Statically Induced E.M.F.*

Faraday's law of Electromagnetic Induction-



- First law: Whenever a conductor is placed in a varying magnetic field, EMF induces and this emf is called an induced emf and if the conductor is a closed circuit than the induced current flows through it.
- Second law: The magnitude of the induced EMF is equal to the rate of change of flux linkages.

Based on his experiments we now have Faraday's law of electromagnetic induction according to which the amount of voltage induced in a coil is proportional to the number of turns and the changing magnetic field of the coil.

So now, the induced voltage is as follows:

**$e = N \times d\Phi/dt$**

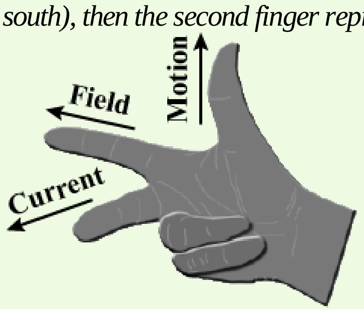
where, e is the induced voltage N is the number of turns in the coil
$\Phi$ is the magnetic flux t is the time

**Lenz's law of Electromagnetic Induction-Lenz law of electromagnetic induction states that, when an emf induces according to Faraday's law, the polarity (direction) of that induced emf is such that it opposes the cause of its production. The direction of the induced current may also be found by this law.**
According to Lenz's law

**$E = -N\ (d\Phi/\ dt)$ (volts)**

**Direction of induced e.m.f and currents-**

*The direction of induced emf is determined by right hand rule. As per the the rule, if the thumb is pointed in the direction of motion of the conductor and the first finger is pointed in the direction of the magnetic field (north to south), then the second finger represents the direction of the induced current.*




**EXAMPLE-**

**Example 6.13.** *A laminated soft iron ring of relative permeability 1000 has a mean circumference of 800 mm and a cross-sectional area 500 mm². A radial air-gap of 1 mm width is cut in the ring which is wound with 1000 turns. Calculate the current required to produce an air-gap flux of 0.5 mWb if leakage factor is 1.2 and stacking factor 0.9. Neglect fringing.*

**Solution.** Total AT reqd. $= \Phi_g S_g + \Phi_i S_i = \dfrac{\Phi_g l_g}{\mu_0 A_g} + \dfrac{\Phi_i l_i}{\mu_0 \mu_r A_i B}$

Now, air-gap flux $\Phi_g = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb, $l_g = 1$ mm $= 1 \times 10^{-3}$ m; $A_g = 500$ mm² $= 500 \times 10^{-6}$ m²

Flux in the iron ring, $\Phi_i = 1.2 \times 0.5 \times 10^{-3}$ Wb

Net cross-sectional area $= A_i \times$ stacking factor $= 500 \times 10^{-6} \times 0.9$ m²

$$\therefore \text{ total AT reqd.} = \frac{0.5\times10^{-3}\times1\times10^{-3}}{4\pi\times10^{-7}\times500\times10^{-6}} + \frac{1.2\times0.5\times10^{-3}\times800\times10^{-3}}{4\pi\times10^{-7}\times1000\times(0.9\times500\times10^{-6})} = 1644$$

$\therefore$ $I = 1644/1000 =$ **1.64 A**

**Example 6.14.** *A ring has a diameter of 21 cm and a cross-sectional area of 10 cm$^2$. The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of $8 \times 10^{-4}$ Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.*

**(Elect. Circuits, South Gujarat Univ.)**

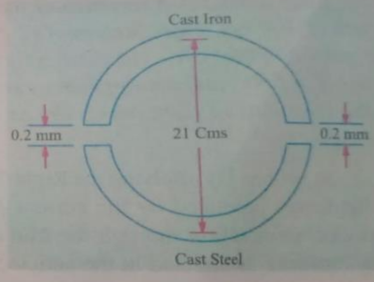


Fig. 6.34

**Solution.** $\Phi = 8 \times 10^{-4}$ Wb ; $A = 10$ cm$^2 = 10^{-3}$ m$^2$; $B = 8 \times 10^{-4}/10^{-3} = 0.8$ Wb/m$^2$

**Air gap**

$H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 6.366 \times 10^5$ AT/m

Total air-gap length $= 2 \times 0.2 = 0.4$ mm

$= 4 \times 10^{-4}$ m

$\therefore$ AT required $= H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$

**Cast Steel Path** (Fig. 6.34)

$H = B/\mu_0\,\mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796$ AT/m

path $= \pi D/2 = 21\ \pi/2 = 33$ cm $= 0.33$ m

AT required $= H \times l = 796 \times 0.33 = 263$

**Cast Iron Path**

$H = 0.8/\pi \times 10^{-7} \times 166 = 3{,}835$ AT/m ; path $= 0.33$ m

AT required $= 3{,}835 \times 0.33 = 1265$

Total AT required $= 255 + 263 + 1265 =$ **1783.**

**Example 6.15.** *A mild steel ring of 30 cm mean circumference has a cross-sectional area of 6 cm$^2$ and has a winding of 500 turns on it. The ring is cut through at a point so as to provide an air-gap of 1 mm in the magnetic circuit. It is found that a current of 4 A in the winding, produces a flux density of 1 T in the air-gap. Find (i) the relative permeability of the mild steel and (ii) inductance of the winding.*

**(F.E. Engg. Pune Univ.)**

**Solution.** *(a)* **Steel ring**

$$H = B/\mu_0\,\mu_r = 1/4\pi \times 10^{-7} \times \mu_r \text{ AT/m} = 0.7957 \times 10^7/\mu_r \text{ AT/m}$$

$$\text{m.m.f.} = H \times l = (0.7957 \times 10^7/\mu_r) \times 29.9 \times 10^{-2} = 0.2379 \times 10^6/\mu_r \text{ AT}$$

*(b)* **Air-gap**

$$H = B/\mu_0 = 1/4\pi \times 10^{-7} = 0.7957 \times 10^6 \text{ AT/m}$$

$$\text{m.m.f. reqd.} = H \times l = 0.7957 \times 10^6 \times (1 \times 10^{-3}) = 795.7 \text{ AT}$$

$$\text{Total m.m.f.} = (0.2379 \times 10^6/\mu_r) + 795.7$$

$$\text{Total m.m.f. available} = NI = 500 \times 4 = 2000 \text{ AT}$$

*(i)* $\therefore$ $2000 = (0.2379 \times 10^6/\mu_r) + 795.7 \quad \therefore \quad \mu_r =$ **197.5**

*(ii)* Inductance of the winding $= \dfrac{N\Phi}{I} = \dfrac{NBA}{I} = \dfrac{500 \times 1 \times 6 \times 10^{-4}}{4} =$ **0.075 H**

**Example 6.17.** *A rectangular iron core is shown in Fig. 6.35. It has a mean length of magnetic path of 100 cm, cross-section of (2 cm × 2 cm), relative permeability of 1400 and an air-gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$ ; and the respective currents are 1.6 A, 4 A and 3 A. The directions of the currents are as shown. Find the flux in the air-gap.*

**(F.Y. Engg. Pune Univ. )**

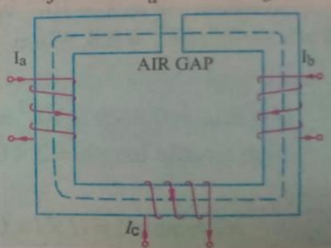


Fig. 6.35

**Solution.** By applying the Right-Hand Thumb rule, it is found that fluxes produced by the current $I_a$ and $I_b$ are directed in the clockwise direction through the iron core whereas that produced by current $I_c$ is directed in the anticlockwise direction through the core.

$\therefore$ total m.m.f. $= N_a I_a + N_b I_b - N_c I_c = 335 \times 1.6 + 600 \times 4 - 600 \times 3 = 1136$ AT

Reluctance of the air-gap $= \dfrac{l}{\mu_0 A} = \dfrac{5 \times 10^3}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.946 \times 10^6$ AT/Wb

Reluctance of the iron path $= \dfrac{l}{\mu_0 \mu_r A} = \dfrac{100 - (0.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 1400 \times 4 \times 10^{-4}} = 1.414 \times 10^6$ AT/Wb

Total reluctance $= (9.946 + 1.414) \times 10^6 = 11.36 \times 10^6$ AT/Wb

The flux in the air-gap is the same as in the iron core.

Air-gap flux $= \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{1136}{11.36 \times 10^6} = 100 \times 10^{-6}$ Wb $=$ **100 μWb**

**Example 6.21.** *A cast steel magnetic structure made for a bar of section 8 cm × 2 cm is shown in Fig. 6.35. Determine the current that the 500 turn-magnetising coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take $\mu_r$ = 600 and neglect leakage.*

(Elect. Technology Allahabad Univ. 1993)

**Solution.** Since path $C$ and $D$ are in parallel with each other w.r.t. path $E$ (Fig. 6.38), the m.m.f. across the two is the same.

$$\Phi_1 S_1 = \Phi_2 S_2$$

$$\therefore \quad \Phi_1 \times \frac{15}{\mu A} = 2 \times \frac{25}{\mu A}$$

$$\therefore \quad \Phi_1 = 10/3 \text{ mWb}$$

$$\therefore \quad \Phi = \Phi_1 + \Phi_2 = 16/3 \text{ mWb}$$

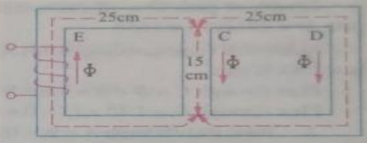



Fig. 6.38

Total AT required for the whole circuit is equal to the sum of

(*i*) that required for path $E$ and (*ii*) that required for either of the two paths $C$ or $D$.

$$\text{Flux density in path } E = \frac{16 \times 10^{-3}}{3 \times 4 \times 10^{-4}} = \frac{40}{3} \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{40 \times 0.25}{3 \times 4\pi \times 10^{-7} \times 600} = 4,420$$

$$\text{Flux density in path } D = \frac{2 \times 10^{-3}}{4 \times 10^{-4}} = 5 \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{5}{4\pi \times 10^{-7} \times 600} \times 0.25 = 1658$$

Total AT = 4,420 + 1,658 = 6,078 ;

Current needed = 6078/500 = **12.16 A**