SE/III/MECH/App. maths - III / Nov. 2012

24/11/12

KONKAN GYANPEETH COLLEGE OF ENGINEERING
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V-A4-II-Hf-Ex-12-D-1

Con. 7338-12. **KR-3050**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **four** from the remaining **six**.

1. (a) Discuss the analyticity of the function $f(z) = \frac{1}{z-1}$. **5**

(b) Obtain Laurent's expansion for the function $f(z) = \frac{1}{z^2 \sinh z}$. **5**

(c) Prove that $L\left\{2\sqrt{\frac{t}{p}}\right\} = \frac{1}{s^{2/3}}$ and hence show that $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{s}}$. **5**

(d) The matrix A is given by $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. **5**

Find the eigen values of $3A^3 + 5A^2 - 6A + 2I - 6SA^{-1}$.

2. (a) Evaluate $\int_0^{4+2i} \bar{z}\, dz$ along the path of the line from 0 to i and then to 4 + 2i. **6**

(b) Evaluate the integral $\int_{t=0}^{\infty} \int_{u=0}^{t} \frac{e^{-t} \sin u}{u}\, du\, dt$. **6**

(c) Discuss for all values of k, the following system of equations possesses trivial and non-trivial solutions :— **8**

$2x + 3ky + (3k + 4)z = 0$
$x + (k + 4)y + (4k + 2)z = 0$
$x + 2(k + 1)y + (3k + 4)z = 0$.

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3. (a) Show that $L^{-1}\left\{\frac{1}{s}\cdot\cos\left(\frac{1}{s}\right)\right\}=1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-\frac{t^6}{(6!)^2}+\ldots$ 6

(b) If $A(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$ prove that $\left[A(\alpha)\right]^{-1}=A(-\alpha)$. 6

(c) Evaluate $\int_C \frac{\cos\pi z}{z^2-1}dz$ where c is 8

(i) a rectangle with vertices at $2 \pm i$ and $-2 \pm i$.
(ii) a square with vertices at $\pm i$ and $2 \pm i$.

4. (a) Prove that the sum of the residues of the function $f(z)=\frac{e^z}{z^2+a^2}$ is $\frac{\sin a}{a}$. 6

(b) Prove that is circle $|z| = 1$ in the z-plane mapped onto the cardiode in the w-plane under the transformation $w = z^2 + 2z$. 6

(c) Obtain the Laplace transform of $\left\{t\cdot \operatorname{erf}\left(3\sqrt{t}\right)\right\}$. 8

5. (a) Find the orthogonal trajectories of u = constant where 6

$$u = x^2 - y^2 + 5x + y - \frac{y}{x^2+y^2}.$$

(b) Examine the linear dependence of the vectors [1 0 2 1], [3 1 2 1], [4 6 2 −4] and [−6 0 −3 −4] and find the relation between them if possible. 6

(c) Evaluate $\int_0^{2\pi}\frac{d\theta}{3-2\cos\theta+\sin\theta}$. 8

6. (a) Find the bilinear transformation which maps the points z = 2, 1, 0 onto w = 1, 0, i. 6

(b) If $f(t)=\begin{cases}3t & 0<t<2\\ 6 & 2<t<4\end{cases}$ where f(t) has period 4. 6

(i) Draw graph of f(t).
(ii) Find L { f(t) }.

(c) Show that $u=\left(\gamma+\frac{a^2}{\gamma}\right)\cos\theta$ is harmonic, find $v(\gamma, \theta)$ so that u + iv is analytic. 8

7. (a) Obtain :— 6

(i) $L^{-1}\left\{\frac{3s-8}{s^2+4}-\frac{4s-24}{s^2-16}\right\}$.

(ii) $L^{-1}\left\{\frac{3s-2}{s^{5/2}}-\frac{7}{3s+2}\right\}$.

(b) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$, 6

find $f(z)$ subject to the condition $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$.

(c) Show that the matrix $A = \begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$ is diagonalizable. Find the transforming 8

matrix and the diagonal form.

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