(CBGS)

SE Comp Applied Mathematics - III 31 May 2014
SE IT - III (CBGS)

QP Code : NP-18619

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No.1 is **compulsory.**
(2) Attempt any **three** questions from Question No.2 to Question No.6.
(3) Non-programmable **calculator** is **allowed.**

1. (a) Find $L^{-1}\left[\frac{Se^{-\pi s}}{S^2+2S+2}\right]$ 5

(b) State true or false with proper justification "There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$". 5

(c) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \frac{(3x^2-1)}{2}$ are orthogonal over $(-1, 1)$. 5

(d) Using Green's theorem in the plane, evaluate $\int_c (x^2-y)dx+(2y^2+x)dy$ around the boundary of the region defined by $y = x^2$ and $y = 4$. 5

2. (a) Find the fourier cosine integral representation of the function $f(x)=e^{-ax}, x>0$ and hence show that $\int_0^\infty \frac{\cos ws}{1+w^2}dw = \frac{\pi}{2}e^{-x}, x \geq 0$. 6

(b) Verify laplaces equation for $U=\left(r+\frac{a^2}{r}\right)\cos\theta$ Also find V and f(z). 6

(c) Solve the following eqn. by using laplace transform $\frac{dy}{dt}+2y+\int_0^t ydt = \sin t$ given that $y(o) = 1$. 8

3. (a) Expland $f(x)=\begin{cases}\pi x, & 0<x<1 \\ 0, & 1<x<2\end{cases}$ with period 2 into a fourier series. 6

(b) A vector field is given by $\bar{F}=\left(x^2+xy^2\right)i+\left(y^2+x^2y\right)j$ show that $\bar{F}$ is irrotational and find its scalar potential. 6

(c) Find the inverse z - transform of - 8

$$f(z)=\frac{z+2}{z^2-2z+1}, |z|>1$$

4. (a) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at (1, -1, 2) 6

(b) Given $L\left(\operatorname{erf}\sqrt{t}\right)=\frac{1}{S\sqrt{S+1}}$, evaluate $\int_0^\infty t.e^{-t}\operatorname{erf}\left(\sqrt{t}\right)dt$ 6

(c) Obtain the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half-range cosine series. 8

Hence show that - (i) $\sum_1^\infty \frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}$

(ii) $\sum_1^\infty \frac{1}{n^4}=\frac{\pi^4}{90}$

5. (a) If the imaginary part of the analytic function W=f(z) is $V=x^2-y^2+\frac{x}{x^2+y^2}$ find the real part U. 6

(b) If $f(k) = 4^k U(K)$ and $g(k) = 5^k U(K)$, then find the z- transform of $f(k)\cdot g(k)$ 6

(c) Use Gauss's Divergence theorem to evaluate $\iint_S \bar{N}\cdot\bar{F}\,ds$ where $\bar{F}=4xi+3yj-2z\hat{k}$ and S is the surface bounded by x = o, y = 0, z = 0 and 2x + 2y + z = 4. 8

6. (a) Obtain complex form of Fourier series for f(x) = con h 3x + sin h 3x in (–3, 3). 6

(b) Find the inverse Laplace transform of $\frac{(S-1)^2}{\left(S^2-2S+5\right)^2}$ 6

(c) Find the bilinear transformation under which 1, i, -1 from the z-plane are mapped onto 0, 1, ∞ of w-plane. Also show that under this transformation the unit circle in the w-plane is mapped onto a straight line in the z-plane. Write the name of this line. 8