## 3E1704

**B. Tech. (Sem. III) (Main) Examination, January - 2013**
**Petroleum Engg.**
**3PE1 Mathematics - III (Common with 3EC1, EIC, BM, AI, CR)**

Time : **3** Hours]

[Total Marks : **80**
[Min. Passing Marks : **24**

*Attempt any **five** questions, selecting **one** question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly. Figures on the right hand side indicate full marks.*

Use of following supporting material is permitted during examination.

*(Mentioned in form No. 205)*

1.   **Non-programmable Scientific Calculator**      2.    **Nil**

## UNIT - I

1   (a)   The error function is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \, du$$

Find the Laplace transform of the error function and hence prove that

$$L\left[erfc \, \sqrt{t}\right] = \frac{1}{\sqrt{s+1}\left[\sqrt{s+1}+1\right]}$$

8

(b)   Solve the ordinary linear differential equation

$$\frac{d^2 y}{dt^2} + m^2 y = a \cos nt, t > 0$$

give that $y(0) = y_0$ and $y'(0) = y_1$. State the condition when the solution so obtained does not exist.

8

**OR**

1   (a)   Find the inverse Laplace transform of the following :

(i)   $\dfrac{1}{(8s-27)^3}$   (ii)   $\dfrac{s e^{-as}}{s^2 - \omega^2}$

3+3

(b)   Give the statement of convolution theorem on inverse Laplace transform. Justify it with a suitable example.

2

(c)   The equation

$$Y(x, s) = c_1 e^{xs} + c_2 e^{-xs} - \dfrac{x}{s^4}$$   denotes the general solution

of a PDE obtained by Laplace transform method. If the solution satisfies the initial conditions $y(x, 0) = y_t(x, 0) = 0$ and $y$ remains finite for $x \to \infty, t \to \infty$ followed by $y(0, t) = 0$ then find the ultimate solution of the PDE.

## UNIT-II

2   (a)   Find the Fourier series for the function $f(x)$ defined by

$$f(x) = \begin{cases} \operatorname{sgn}(x), & x \neq 0 \ (\text{lies between} -\pi \text{ and } \pi ) \\ 0 & , x = 0 \end{cases}$$

where $\operatorname{sgn}(x)$ denotes the signum function.   8

(b)   Find the z-transform of the Dirac's delta function given by

$$\delta_n = \begin{bmatrix} 1 & , n = 0 \\ 0 & , n \neq 0 \end{bmatrix}$$   4

(c)   Find the inverse z-transform of $\log\left(\dfrac{z}{z+1}\right)$ using power series

method.   4

### OR

2   (a)   Compute approximately, the Fourier coefficients $a_0, a_1, a_2$ and $b_1, b_2$ in the Fourier series expansion of function tabulated as follows :

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 |
|------|---|---|---|---|---|---|
| $y$: | 9 | 18 | 24 | 28 | 26 | 20 |

Find the amplitude of the first harmonic.

10

(b)   Find the inverse z-transform of $\dfrac{3z^2 + 2z}{z^2 - 3z + 2}, 1 < |z| < 2$.

**6**

## UNIT-III

2.   (a)   Find the Fourier transform of the function $f(x) = |x| e^{-|x|}$.

**6**

(b)   Use the method of Fourier transform to obtain the displacement $u(x, t)$ of an infinite string, given that the string is initially at rest and the initial displacement is $f(x), -\infty < x < \infty$. Show that the solution can be represented

by $u(x, y) = \dfrac{1}{2} \left[ f(x + ct) + f(x - ct) \right]$.

**10**

### OR

3.   (a)   Use inverse cosine transform to find $f(x)$ in

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$$

**6**

(b)   Find the temperature distribution in a bar of length $L$, with its both ends and lateral surface insulated when the initial temperature in the bar is $f(x)$. Deduce the expression for temperature if $f(x) = x^2$ and $L = 10$.

**10**

## UNIT-IV

4.   (a)   Determine where the function $f(z) = \begin{cases} \dfrac{z^2 + 3iz - 2}{z + i} & , z \neq -i \\ 5 & , z = i \end{cases}$ is

continuous ? Can the function be refined to make it continuous at $z = -i$ ?

**8**

(b)   Evaluate $\oint_C \dfrac{e^{3z} dz}{(z - \ln 2)^4}$ where $C$ is the square with vertices at $\pm 1, \pm i$.

**4**

(c) Find the analytic function $w = u + iv$, if the real part is given by $u = e^x \cos y$. **4**

## OR

4 (a) Prove that the cross-ratio of four points is invariant under a bilinear transformation. **6**

(b) The ML-inequality of an analytic function $f(z)$ is

$$\left| \int_C f(z)\, dz \right| \le ML$$

where $|f(z)| \le M$ everywhere on $C$ and $L$ is the length of the curve. Use this inequality to find an upper bound for the absolute value of the integral $\int_c \left( e^z - \bar{z} \right) dz$ where $c$ is the boundary of the triangle with vertices at $Z = 0, -4, 3i$. **10**

## UNIT - V

5 (a) Expand $f(z) = \sin z$ in a Taylor's series about $z = \dfrac{\pi}{4}$. **6**

(b) Evaluate $I = \displaystyle\int_0^\pi \dfrac{a\,d\theta}{a^2 + \sin^2 \theta}, a > 0$. **10**

## OR

5 (a) Show, by contour integration, that

$$\int_0^\infty \frac{\sin x}{x\left(a^2 + x^2\right)}\, dx = \frac{\pi}{2a^2}\left(1 - e^{-a}\right). \ a > 0.$$ **8**

(b) Find the Laurent series of $f(z) = \dfrac{7z^2 + 9z - 18}{z^3 - 9z}$ in the regions :

(i) $0 < |z + 3| < 3$

(ii) $3 < |z + 3| < 6$

(iii) $|z + 3| > 6$. **8**