Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.Tech (IT-NEW) / SEM-6 / IT-605A / 2013

2013

DISCRETE MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) A disjuctive normal form of $P \rightarrow Q$ is

      a) $\sim P \vee Q$

      b) $P \vee \sim Q$

      c) $(\sim P \wedge Q) \vee (P \wedge \sim Q)$

      d) $(P \wedge Q) \vee (P \wedge \sim Q)$.

   ii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv$

      a) $\sim p$ b) p

      c) $\sim q$ d) q.

iii) The minimal functionally complete set of connectives is

a) $\{ \sim , \equiv , \vee \}$ b) $\{ \sim , \vee \}$

c) $\{ \sim , \equiv \}$ d) none of these.

iv) If 6^{16} is divided by 17, the remainder will be

a) 15 b) 10

c) 1 d) none of these.

v) Let $(L , \vee , \equiv)$ be a lattice. Then for $a, b, c \in L$, which of the following does not hold in L ?

a) $a \vee (b \vee c) = (a \vee b) \vee c$

b) $a \in (a \vee b) = a$

c) $a \in b = b \in a$

d) $a \in (b \vee c) = (a \in b) \vee (a \in c)$.

vi) The number of binary relations on a set having 3 elements is

a) 3^3 b) 3^9

c) $3!$ d) none of these.

vii) The number of non-negative integral solutions of the equation $x + y + z = 17$, $x, y, z \geq 0$, is

a) 170 b) 171

c) 172 d) none of these.

viii) The generating function for the sequence $\{ 1, -2, 3, -4, 5, -6, \ldots\ldots \}$ is

a) $\frac{1}{1 - x^2}$ b) $\frac{1}{(1 + x)^2}$

c) $\frac{1}{(1 - x)^2}$ d) $\frac{1}{1 + x^2}$.

ix) In how many ways can 7 women and 3 men be arranged in a row if 3 men must always stand next to each other ?

a) $7! \times 3!$ b) $7! + 3!$

c) $3! \times 8!$ d) $7! \times 8!$.

x) A graph consisting of simply one circuit, with $n \geq 3$ and n odd, is

a) 2-chromatic b) 3-chromatic

c) 4-chromatic d) none of these.

xi) A complete bipartite graph $K_{m,\,n}$ is a tree when

a) $m = 1,\ n = 2$ b) $m = 2,\ n = 2$

c) $m = 2,\ n = 3$ d) none of these.

xii) For a perfect matching the corresponding graph in a matching problem should be

a) a tree

b) a bipartite graph

c) a cycle with even number of vertices

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that the following pair of propositions are logically equivalent :

a) $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q)$ and p.

b) $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$. 3 + 2

3. If m is a positive integer and $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, show that

a) $a + c \equiv b + d \pmod{m}$

b) $ac \equiv bd \pmod{m}$ 2 + 3

4. Find the closed form of the generating function for the sequence $\{ 2, 5, 8, 11, 14, \ldots\ldots \}$.

5. Prove that every tree with 2 or more vertices is 2-chromatic.

6. Show that the set of all positive divisors of 42 forms a poset under the relation $\leq$ defined as $a \leq b$ if a is a divisor of b. Draw its Hasse diagram.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Obtain the CNF of $\neg\ (p \rightarrow (q \rightarrow r))$. 5

   b) Prove the Pascal's identity : 5

   $c (n, r) = c (n - 1, r) + c (n - 1, r - 1)$, where the notation carries usual meaning.

   c) Prove that every tree has at most one perfect matching. 5

8. a) Prove that $[(p \vee q) \rightarrow (p \rightarrow r) \rightarrow (q \rightarrow r) \rightarrow r$ is a tautology. 5

   b) State Pigeon hole principle. Using that prove that if any five numbers from 1 to 8 are selected, then two of them will add to 9. 1 + 4

   c) What is perfect matching ? State Hall's perfect matching condition. Differentiate between maximum matching and perfect matching. 1 + 2 + 2

9. a) Check the validity of the following argument :

"If the band could not play rock music or the refreshments were not delivered on time, then the New Year's party would have been cancelled and Alice would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made." 5

b) Show with example that

i) any subset of L which is a lattice not be a sub-lattice

ii) the union of two sub-lattices may not be a sub-lattice. 2 + 3

c) Find the remainder when the sum

$1^5 + 2^5 + 3^5 + \ldots\ldots + 100^5$ is divided by 5. 5

10. a) Solve the following recurrence relation using generating function :

$a_n = 4\left(a_{n-1} - a_{n-2}\right) + 2^n \ (n \geq 2),\ a_0 = 1,\ a_1 = 4.$ 5

b) In the distributive lattice $(L, \vee, \geq)$, prove that for $a, b, c \in L$, $(a \vee b = a \vee c)$ and $(a \in b = a \in c)$ implies $b = c$. 5

c) Prove that, for a graph G with n vertices,

$\frac{n}{\beta(G)} \leq \chi(G) \leq n - \beta(G) + 1$, where $\chi(G)$ and $\beta(G)$ denote the chromatic number and independence numbers of G, respectively. 5

11. a) A box contains 10 balls each of white and red colour. What is the least number of balls one must pull out from the box to be sure to get a matched pair ? 5

b) Using Pigeon hole principle, find the total number of natural numbers that must be chosen to be sure of getting at least two of them whose difference is divisible by 11. 5

c) Prove that the chromatic polynomial for the complete graph K_n having n vertices is

$P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \ldots\ldots (\lambda - n + 1)$ and conversely. 5