



FACULTY OF ENGINEERING
B.E. 2/4 (ECE/M/P/CSE/AE) II Semester (New) (Main)
Examination, May/June 2012
MATHEMATICS – IV

Time : 3 Hours]

[Max. Marks : 75

Note : Answer **all** questions from Part A. Answer **any five** questions from Part B.

PART – A

(25 Marks)

1. Give an example of multi valued and single valued complex functions. 2
2. From the following functions, which is harmonic function. 3
a) $r^2 \cos \theta$ b) $r^2 \cos 2\theta$ c) $r^2 \cos 3\theta$ d) None of above
3. Expand $\tan z$ in Taylor's series about the point $z = 0$. 2
4. Evaluate $\int_C \cot z \, dz$, where C is the circle $|z| = 2$. 3
5. Show that $E[aX + b] = aE[X] + b$, where a and b are constants and X is a random variable. 2
6. Moment generating function of a random variable X is given by :
$$M_X(t) = 1 + \frac{t}{1} \left(\mu + \frac{\sigma^2 t}{2} \right) + \frac{t^2}{2} \left(\mu + \frac{\sigma^2 t}{2} \right)^2 + \frac{t^3}{3} \left(\mu + \frac{\sigma^2 t}{2} \right)^3 + \dots$$
 find $E(X)$ and $\text{Var}[X]$. 3
7. The normal distribution is a limiting case of Poisson's when 2
a) $\lambda \rightarrow 0$ b) $\lambda \rightarrow \sigma$ c) $\lambda \rightarrow \infty$ d) $\lambda < \sigma$
8. Write the applications of chi-square test. 3
9. Write normal equations to fit the straight line $y = a + bx$ using method of least squares. 2

10. Match the following : 3

- | | |
|---|--|
| a) Line of regression of y on x | 1) $y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$ |
| b) Line of regression of x on y | 2) $x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$ |
| c) The value of coefficient of correlation lies between | 3) -1 and 0 |
| | 4) -1 and 1 |
| | 5) 0 and 1 |

PART - B (50 Marks)

11. a) If $f(z)$ is regular function of z , then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$. 5

b) State Cauchy's integral formula and using the Cauchy's integral formula, evaluate $\int_C \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $\frac{3}{2} \pm i, -\frac{3}{2} \pm i$. 5

12. a) Define conformal mapping. 5

Show that under the mapping $w = \frac{z-i}{z+i}$ real axis in the z -plane is mapped into the circle $|w| = 1$.

b) State and prove the Residue theorem. 5

13. a) Probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{2} \sin x, & \text{for } 0 \leq x \leq \pi \\ 0, & \text{otherwise} \end{cases} \text{ then find mean and variance of } X. \quad 5$$

b) For the following probability distribution. 5

X	-3	-2	-1	0	1	2	3
P(x)	0.001	0.01	0.1	K	0.1	0.01	0.001

Find K and $E(X^2 + 2X + 5)$



14. a) Find mean and variance of Gamma distribution. 5

b) The heights of 10 females of a given location are found to be 66, 63, 58, 64, 57, 64, 66, 60, 60, 62 inches. Is it reasonable to believe that the average height is greater than 60 inches ? Test at 5% significance level. 5

15. a) Fit the exponential curve $y = ae^{bx}$ to the following data. 5

x	2	4	6	8
y	25	38	56	84

b) Find the correlation coefficient between x and y for the following data. Find also two regression lines. 5

x	55	56	58	59	60	60	62
y	35	38	38	39	44	43	45

16. a) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ 5

b) Find mean and variance of normal distribution. 5

17. a) Find the Laurent's expansion of $f(z) = \frac{7z - 2}{(z + 1)z(z - 2)}$ in the region 5

$$1 < |z + 1| < 3.$$

b) If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}(\frac{3}{8})$, show that $\sigma_x = \frac{1}{2}\sigma_y$. 5