

- N.B. (1) Question No. 1 is compulsory.
 (2) Attempt any four questions from remaining six questions.
 (3) Figures to the right indicate full marks.
 (4) Assume suitable data if necessary.

1. (a) Prove by Mathematical Induction — 8

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- (b) Explain the terms :— 6
 (i) Poset
 (ii) Normal Subgroup
 (iii) Lattice.

- (c) In a survey of 60 people, it was found that 25 reads Newsweek Magazine, 26 reads Times and 26 reads Fortune. Also 9 reads both Newsweek and Fortune, 11 reads both Newsweek and Times, 8 reads Time and Fortune and 8 reads no magazine at all. 6
 (i) Find the number of people who read all three magazines.
 (ii) Determine number of people who read exactly one magazine.

2. (a) Define injective, surjective and bijective functions. 8

if $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined by the formulas —

$$f(x) = x + 2 \text{ and } g(x) = x^2$$

Find (i) f.g.f. (ii) g.f.g.

- (b) Define equivalence relation on a set. Let R be a relation on the set of integers defined by aRb iff $a-b$ is a multiple of 5. Prove that R is a equivalence relation. 6

- (c) State the converse, inverse and contrapositive of the following :— 6
 (i) If it is cold, then he wears a hat.
 (ii) If an integer is a multiple of 2, then it is even.

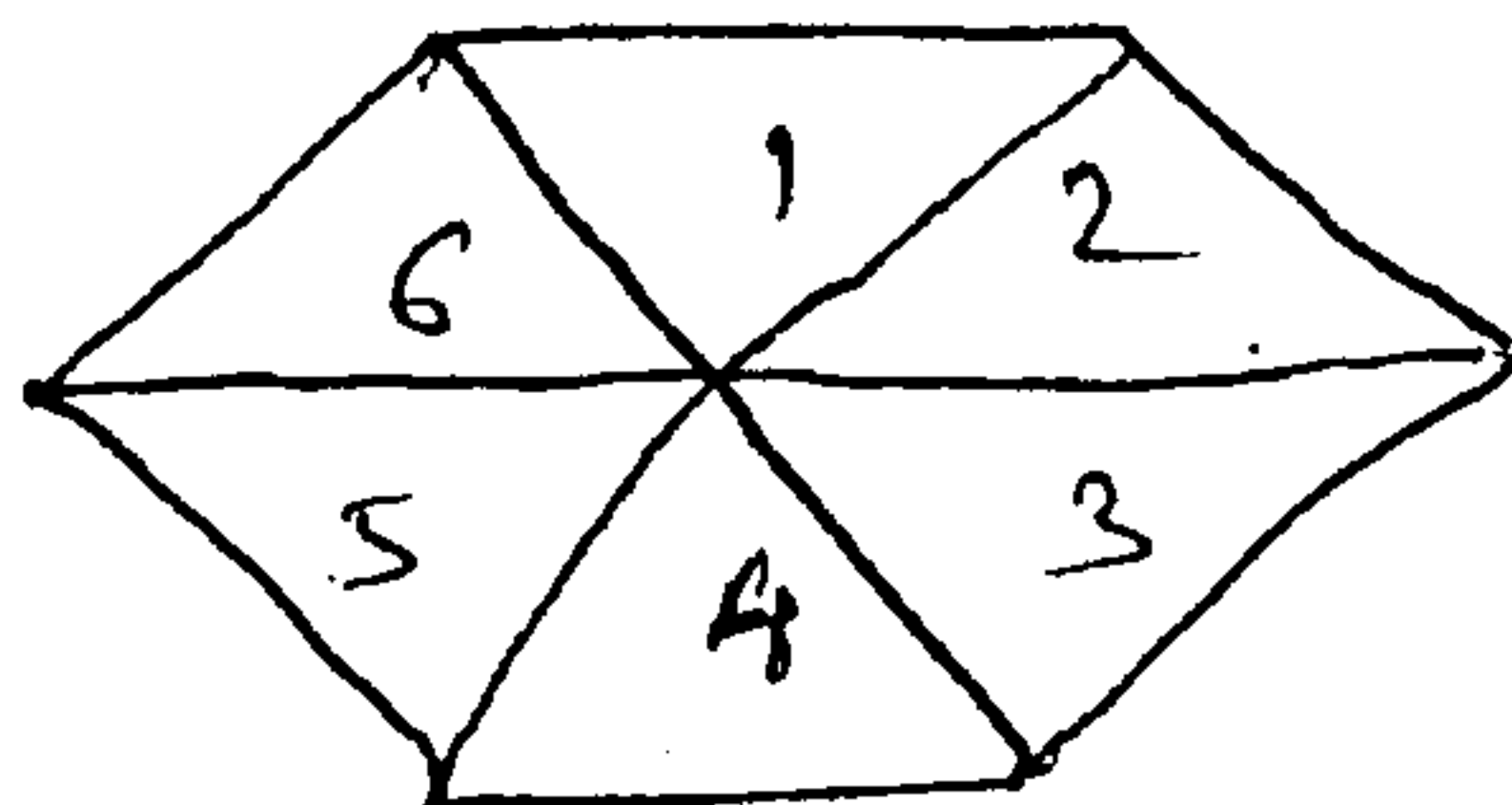
3. (a) Explain Hasse diagram. Draw the Hasse diagram of the relation given by :— 8

(i) $R_1 = \{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5) \}$

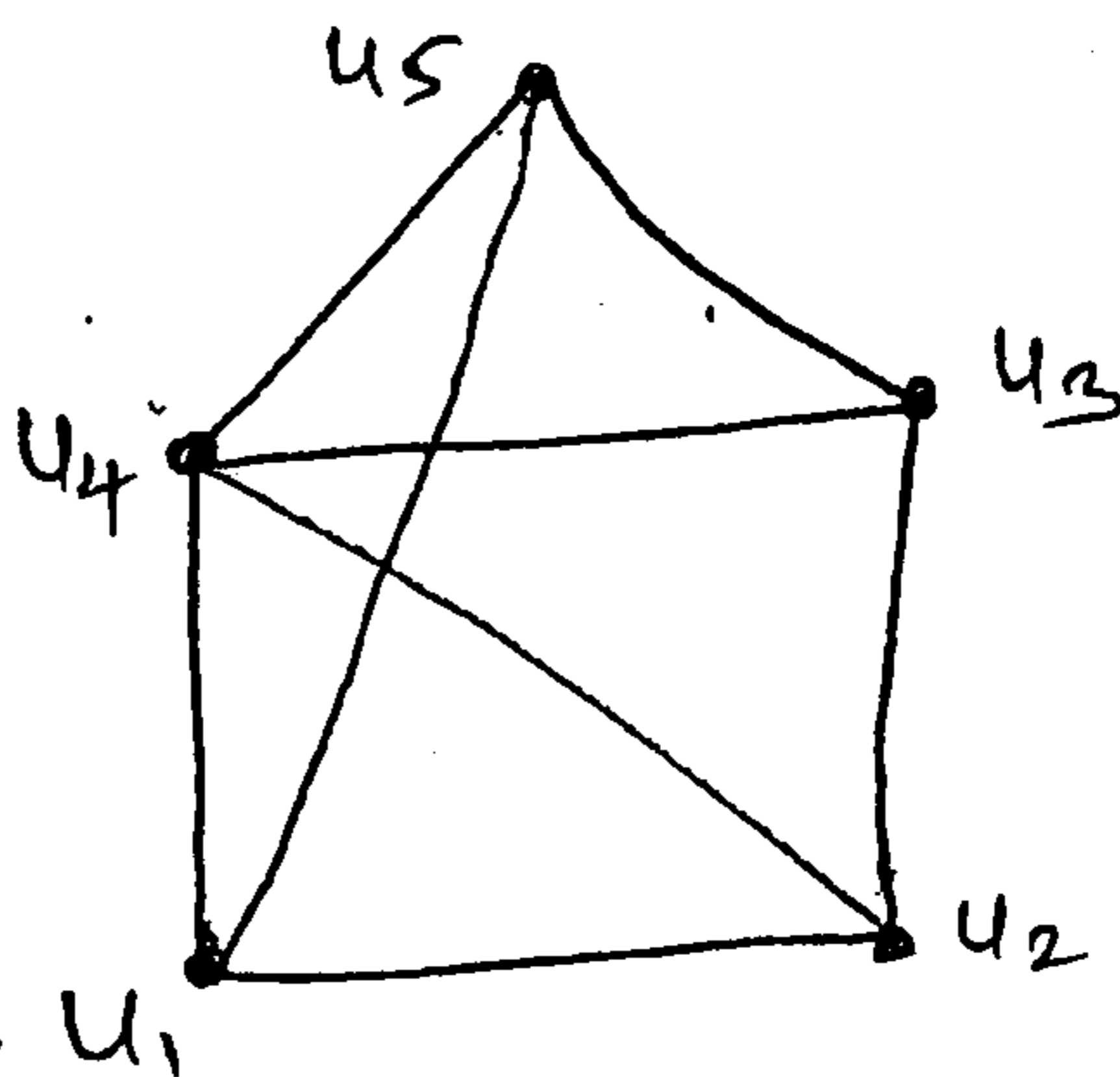
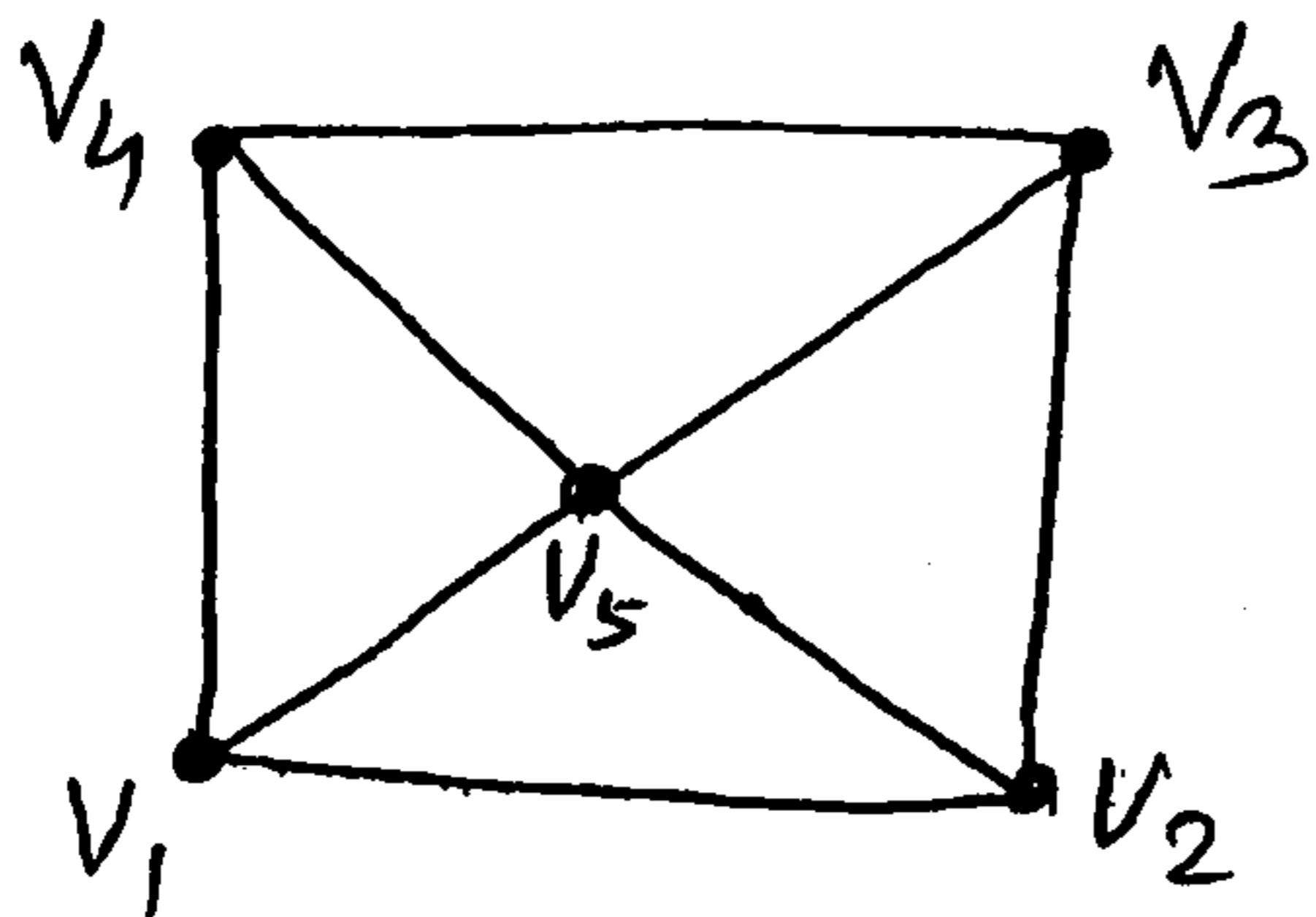
(ii) $R_2 = \{ (1, 1), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (5, 5) \}$

- (b) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (3, 4), (2, 1)\}$. Find the Transitive closure of R using Warshall's Algorithm. 6

- (c) Consider the region shown below. It is bounded by a regular hexagon whose sides are the length 1 units. Show that if any seven points are chosen in this region then two of them must be no further apart than 1 unit. 6



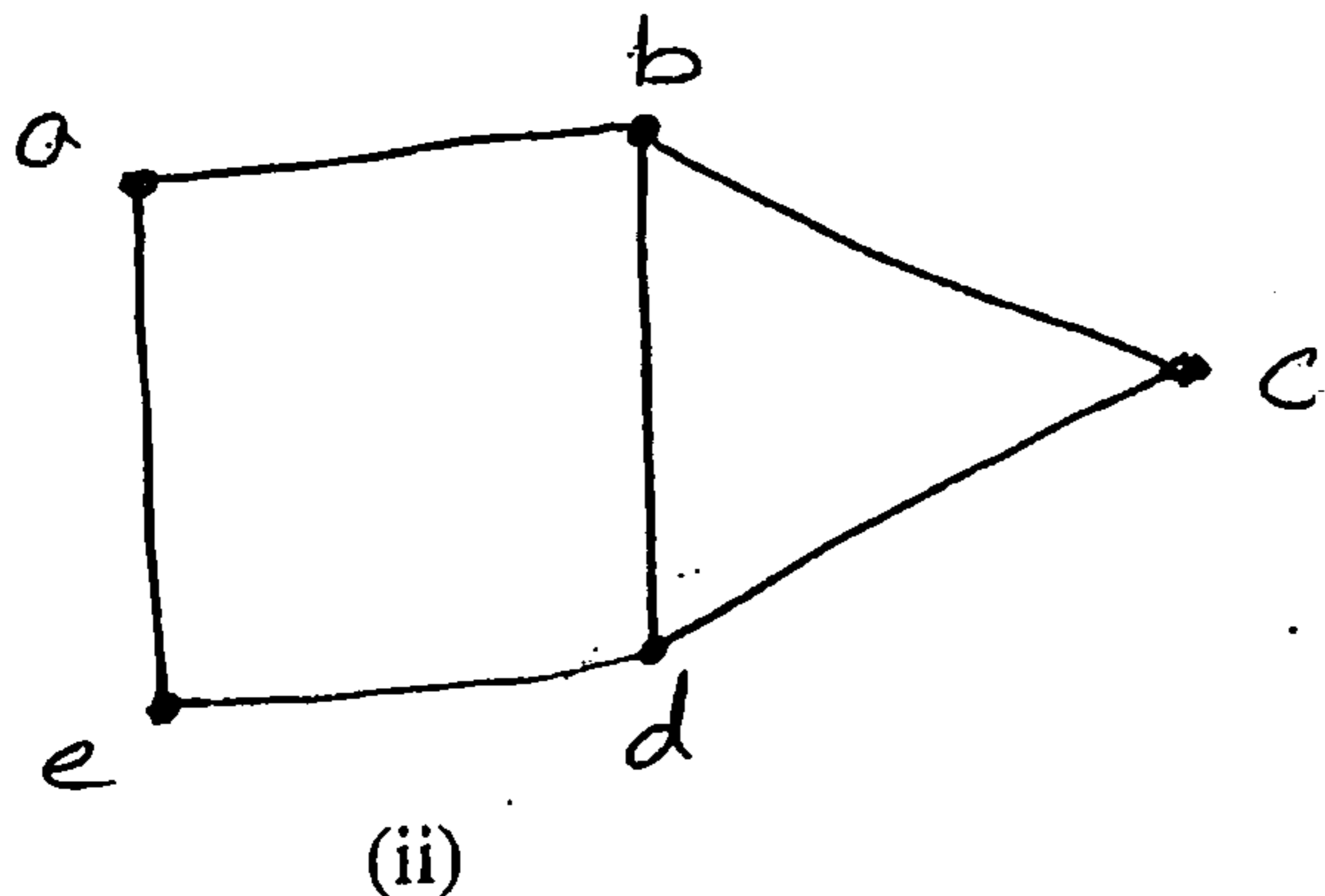
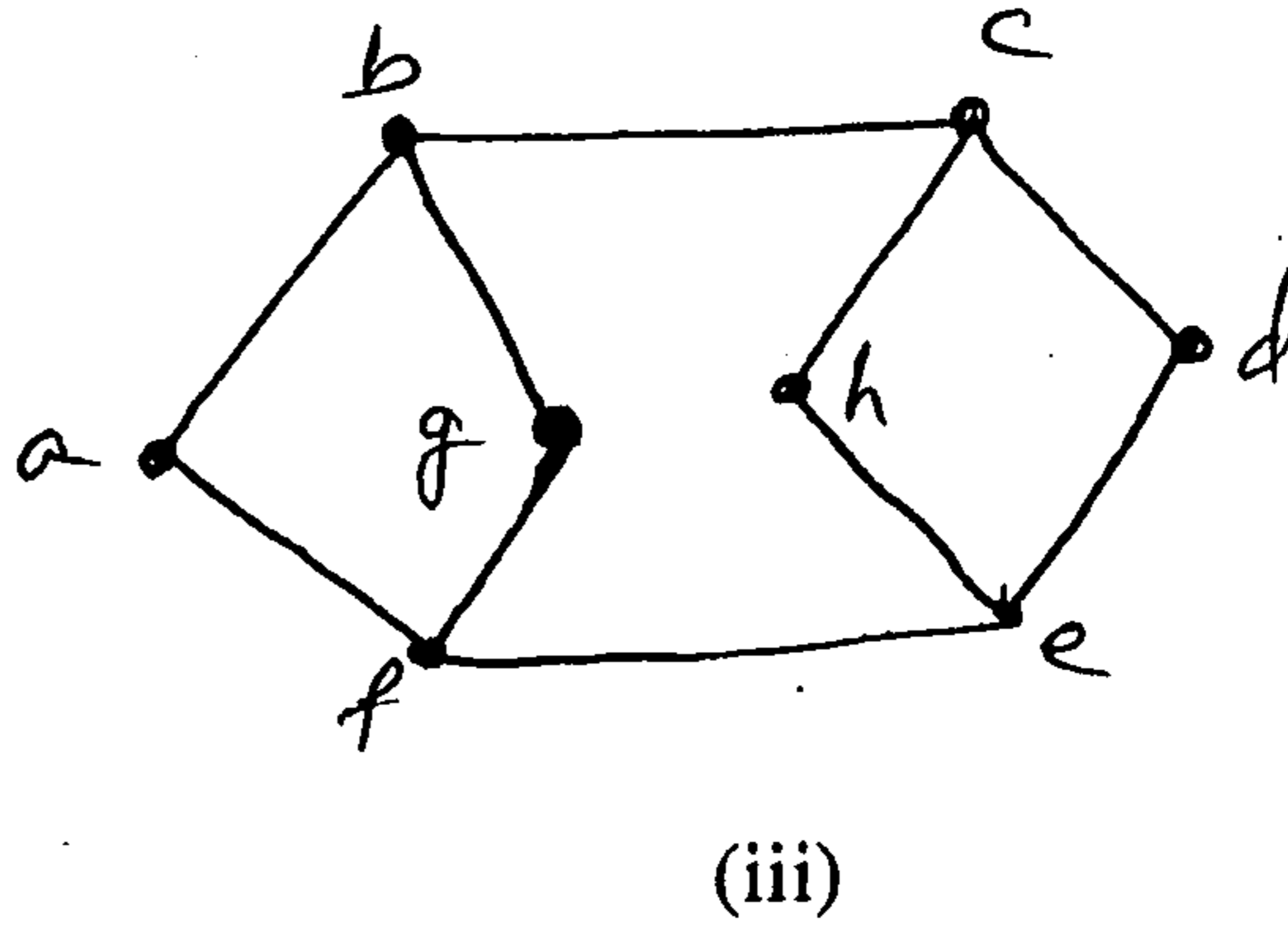
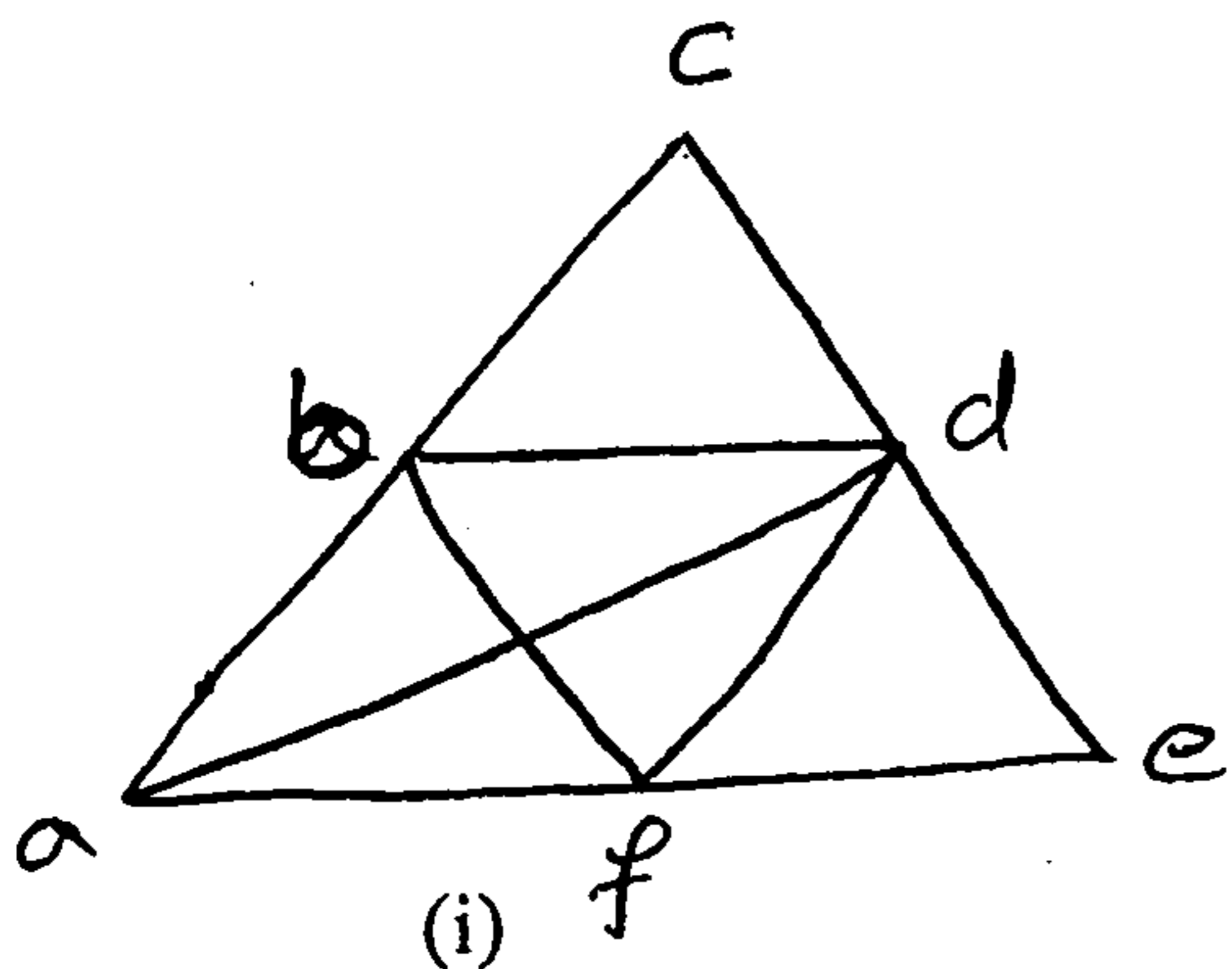
4. (a) Show that the following graphs are isomorphic. 8



(b) Let $R = \{(1, 2), (4, 3), (2, 2), (2, 1), (3, 1)\}$ be a relation on $s = \{1, 2, 3, 4\}$. Find the symmetric closure of R . 6

(c) Define : 6
 (i) Integral domain
 (ii) Field
 (iii) Normal Subgroup.

5. (a) What is a minimum spanning tree? Explain any one technique with example. 8
 (b) Define Cyclic Group. Prove that the set $A = \{0, 1, 2, 3, 4, 5\}$ is a finite abelian under addition modulo 6. 6
 (c) Determine whether the given graph has a Hamilton circuit or Eulerian circuit. If it does, find such a circuit. 6



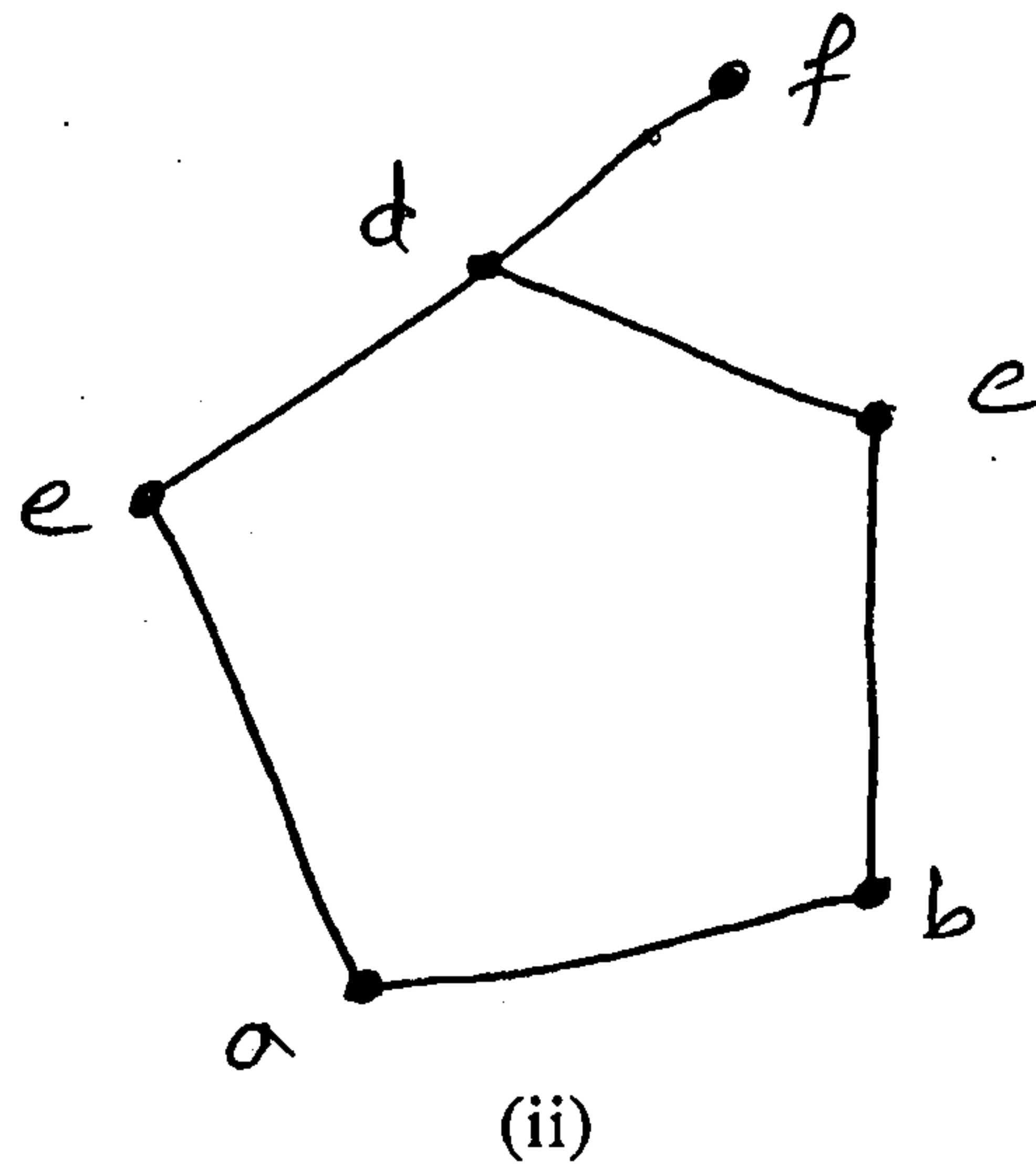
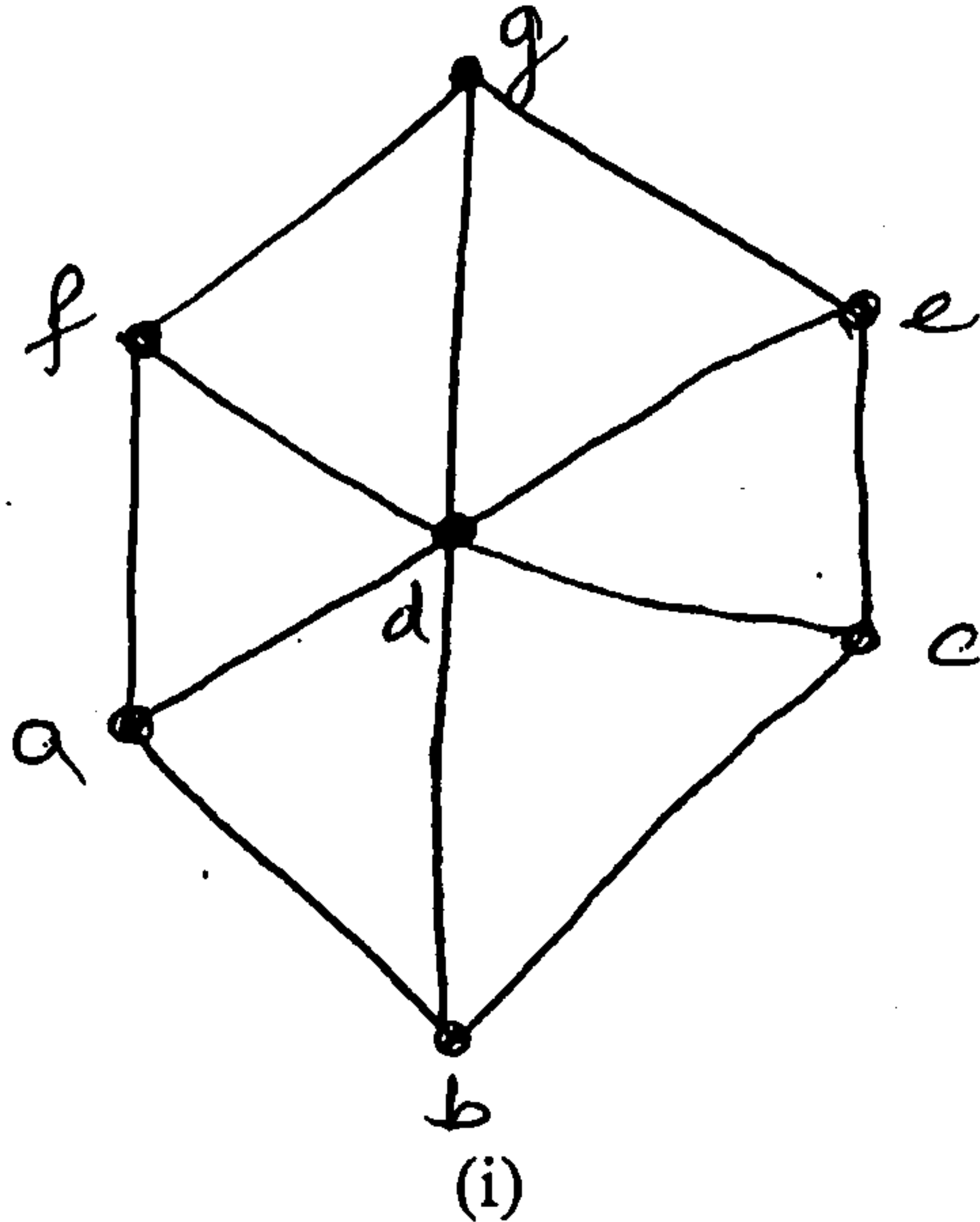
6. (a) Consider the (3, 5) group encoding function $E : B^3 \rightarrow B^5$ is defined by — 8

$E(000) = 00000$	$E(100) = 10011$
$E(001) = 00110$	$E(101) = 10101$
$E(010) = 01001$	$E(110) = 11010$
$E(011) = 01111$	$E(111) = 11100$

Decode the following words relative to a maximum decoding function :—

- (i) 11001 (ii) 01010 (iii) 00111 (iv) 11100

(b) Which of the following diagram in the figure represents a lattice? Justify. 6



(c) Define with example :— 6

- (i) Planer graph
- (ii) Semigroup
- (iii) Quantifiers.

7. (a) Find the solution to the recurrence relation :— 6

$$a_n = a_{n-1} + 2 \quad n \geq 2$$

subject to initial condition $a_1 = 3$.

(b) Find the complement of each element in D_{30} . 6

(c) Find the generating function for each of the following sequence :— 8

- (i) $\{0, 1, 2, 3, 4, \dots\}$
- (ii) $\{1, 2, 3, 4, \dots\}$
- (iii) $\{2, 2, 2, 2, \dots\}$
- (iv) $\{0, 0, 1, 1, 1, \dots\}$