Name : ....

Roll No. : ....

Invigilator's Signature : ....

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The degree and order of the differential equation $y = \frac{x\,d^2y}{dx^2} + r\sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}$ are

a) 2, 2 b) 2, 1

c) 3, 2 d) none of these.

ii) The geometric series $1 + r + r^2 + \ldots$ is convergent if

a) $-1 < r < 1$ b) $r > 1$

c) $r = 1$ d) none of these.

iii) The series 1 + 1 + 1 + ... is

a) convergent b) divergent

c) oscillatory d) none of these.

iv) An absolutely convergent series is

a) necessarily convergent

b) not necessarily convergent

c) conditionally convergent

d) none of these.

v) Leibnitz's test is applied to

a) a constant series

b) an alternating series

c) series of positive terms only

d) none of these.

vi) If W_1 and W_2 be two subspaces of a vector space $V(F)$ then $W_1 \cap W_2$ is

a) necessarily a subspace

b) not a subspace

c) is a subspace only when one is contained within another

d) none of these.

vii) In the vector space R^3 over the field R the vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) are

a) linearly independent

b) linearly dependent

c) none of these.

viii) Integrating factor of differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$ is

a) $\log x$ b) $\log(\log x)$

c) e^x d) none of these.

ix) The upper bound of the sequence $\{(-3)^n\}$ is

a) 4 b) 0

c) – 3 d) none of these.

x) If T is a linear mapping for V to V and $\alpha, \beta \in V$ and a, b are scalers, then

a) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$

b) $T(a\alpha + b\beta) = aT(\alpha) - bT(\beta)$

c) $T(a\alpha + b\beta) = aT(\alpha)$

d) $T(a\alpha + b\beta) = bT(\beta)$

xi) The roots of the auxiliary equation of the given differential equation $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y = 0$ are

a) 2, 4 b) 2, 2

c) 1, 1 d) none of these.

xii) Let $T: R^2 \to R^2$ be a linear transformation defined by $T(x, y) = (2x - y, x + y)$. Then kernel of T is

a) $\{(1, 2)\}$ b) $\{(0, 0)\}$

c) $\{(1, 2), (1, -1)\}$ d) none of these.

xiii) If $T : V_2 \to V_3$ be defined by

$T(v_2) = \{ (x,0,0) : x$ is a real number $\}$ then rank of T is

a) 3 b) 2

c) 1 d) 0.

xiv) If T is a linear transformation from vector space V into W, then

a) rank (T) + nullity (T) = dim (V)

b) rank (T) + nullity (T) = dim (W)

c) rank (T) – nullity (T) = dim (V)

d) none of these.

Group – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3. Show that the mapping defined by $T : R^2 \to R^3$

$T(x_1, x_2) = (x_1 + x_2,\ x_1 - x_2,\ x_2)$ is linear. Find the value of $T(1, 2)$.

4. Solve :

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x .$$

5. The linear transformation $T : R^3 \to R^3$ is defined by $T(x, y, z) = (x - y,\ x + 2y,\ y + 3z)$. Show that T is invertible and determine T^{-1}.

6. Prove that the set of vectors { (1, –2, 3), (2, 3, 1), (–1, 3, 2) } is linearly independent. Also verify whether this set forms a basis of V_3 or not.

7. Let $S = \{ (x, y, z) : (x, y, z) \in R^3, x + y + z = 0 \}$. Prove that S is a subspace. Find the dimension of S.

Group – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \left(y + \left(\frac{dy}{dx} \right)^2 \right)^{1/4}$$

b) Verify whether the differential equation $e^y dx + \left(xe^y + 2y \right) dy = 0$ is exact. If so, then solve it.

c) Solve the differential equation $x \frac{dy}{dx} - 2y = xy^4$.

3 + 5 + 7

9. a) Solve $y = px + \frac{a}{p}$ and also obtain the singular solution.

b) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$.

c) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.

5 + 6 + 4

10. a) State D' Alembert's ratio test. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$.

b) Prove that the series

$$\left(1+\frac{1}{2}\right)-\left(1+\frac{1}{4}\right)+\left(1+\frac{1}{8}\right)-\left(1+\frac{1}{16}\right)+\ldots$$ is an oscillating series.

c) Verify for the following example that the converse of the statement "If $\sum a_n$ be a convergent series then $\lim_{n\to\infty} a_n = 0$" is not true, where $\sum a_n = \sum_{n=1}^{\infty}\frac{1}{n}$.

5 + 6 + 4

11. a) Define basis of a vector space.

b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis for R^3.

c) Determine the value of k so that the set $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$ is linearly dependent in R^3. 3 + 6 + 6

12. a) Let $T : R^3 \to R^2$ be a linear transformation defined by $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$ for $(x, y, z) \in R^3$. Obtain a matrix representation for the linear transformation T.

b) Let V = set of all 2 × 2 matrices and $T : V \to V$ be defined by $T(X) = AX - XA$ where $X \in V$ and $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

Find the basis of ker (T) and the nullity.

c) Let $T : R^3 \to R^2$ be a linear mapping defined by $T(x, y, z) = (x + 2y, z)$. Verify that

dimension (kernel (T)) + dimension (Image (T))

= dim (R^3). 5 + 6 + 4

13. a) Define a subspace of a vector space.

b) State a necessary and sufficient condition for $W \subseteq V$ to be a subspace of $V(F)$.

c) Test whether the series

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \ldots (0 < x < 1)$$

is convergent or not. 3 + 3 + 9

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