Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Sc. (H), (BT)/SEM-2/BMT-204/2011

2011

BIOMATHEMATICS – II

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is

a) divergent

b) convergent

c) absolutely convergent

d) none of these.

ii) The sequence $\{x_n\}$, where $x_n = \frac{2n-1}{n+1}$, $n \in N$ is bounded by

a) 2

b) 3

c) $\frac{1}{2}$

d) none of these.

iii) The eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

a) 6, 1 b) 3, 2

c) 6, 3 d) none of these.

iv) A square matrix A is called orthogonal if

a) $A = A^2$ b) $A^T = A^{-1}$

c) $AA^{-1} = I$ d) none of these.

v) If two eigenvalues of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the third eigenvalue is

a) 0 b) 1

c) 4 d) none of these.

vi) Three lines are co-planer if

a) they are concurrent

b) a line is perpendicular to each of them

c) they are concurrent and a line in perpendicular to each of them

d) none of these.

vii) The equation of a straight line parallel to the X-axis is given by

a) $\frac{x-a}{1} = \frac{y-b}{1} = \frac{z-c}{1}$ b) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{1}$

c) $\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$ d) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$.

viii) If two non-zero vectors $\vec{A}$ and $\vec{B}$ are parallel then

a) $\vec{A} \times \vec{B} = \vec{0}$
b) $\left| \vec{A} \times \vec{B} \right| = 1$
c) $\vec{A} \cdot \vec{B} = 0$
d) $\left| \vec{A} \right| = \left| \vec{B} \right|$.

ix) The differential equation satisfying the relation $x = A \cos \left(mt - \alpha \right)$ is

a) $\frac{dx}{dt} = 1 - x^2$
b) $\frac{d^2 x}{dt^2} = -\alpha^2 x$
c) $\frac{d^2 x}{dt^2} = -m^2 x$
d) $\frac{dx}{dt} = -m^2 x$.

x) The order the differential equation $\left\{ 1 + \frac{d^2 y}{dx^2} \right\}^{\frac{1}{2}} = x^2$ is

a) 1
b) 2
c) 3
d) none of these.

xi) The value of Γ (3·5) is

a) $\frac{15\sqrt{\pi}}{8}$
b) $15\sqrt{\pi}$
c) $\frac{3\sqrt{\pi}}{4}$
d) none of these.

xii) The complementary function of the differential equation $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$ is

a) $(c_1 + c_2 t)e^{-3t}$ b) $\left(c_1 e^{-3t} + c_2 e^{-2t}\right)$

c) $c_1 e^{-2t} + c_2 e^{t}$ d) none of these.

xiii) The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{10^n}{\lfloor n} x^n$ is

a) 10 b) $\frac{1}{10}$

c) 5 d) $\frac{1}{5}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. a) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

b) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$. 3 + 2

3. a) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. 2 + 3

4. a) Show that any square matrix A and its transpose A^T have same eigenvalues.

b) Find the product of the eigenvalues of the matrix $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. 3 + 2

5. a) If $\vec{a} = 3\vec{i} - \vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} - 2\vec{j} + 2\vec{k}$ then verify $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

b) If $\vec{\alpha} = 3\vec{i} - \vec{j} + 2\vec{k}$, $\vec{\beta} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{\gamma} = \vec{i} - 2\vec{j} + 2\vec{k}$ then show that $(\vec{\alpha} \times \vec{\beta}) \times \vec{\gamma} \neq \vec{\alpha} \times (\vec{\beta} \times \vec{\gamma})$. 2 + 3

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

6. a) If two mappings $f : R \to R$ and $g : R \to R$ be defined as follows : $f(x) = x^2$, $g(x) = x - 2$, then show that $f \circ g \neq g \circ f$.

b) Show that the mapping $f : R \to R$ defined by $f(x) = 7x + 3$, $x \in R$ is bijective.

c) The binary operation * is defined on the set of integers Z as $a * b = a + b - 2$, for all $a, b \in Z$. Show that $(Z, *)$ is a group.

d) In a group G, if every element is its own inverse, then show that the group G is commutative. 2 + 3 + 5 + 5

7. a) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are co-planar.

b) Prove that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{1+z}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the co-ordinates of their point of intersection.

c) Find the equation of the line through (1, 2, − 1) perpendicular to each of the lines $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$.

5 + 5 + 5

8. a) Examine the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+........\infty$

b) Discuss the convergence of the following series :

i) $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+.......\infty$

ii) $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+........\infty$

c) Discuss the conditional convergence of

$$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4)+........\infty$$

3 + 4 + 4 + 4

9. a) Let $f(x) = |x|$ in $[-1, 1]$. Is Rolle's theorem applicable to $f(x)$ in $[-1, 1]$? Justify your answer.

b) In the Lagrange's mean value theorem $f(b) - f(a) = (b-a)\, f'(c)$ where $a < c < b$, find c if $f(x) = \sqrt{x}$, $a = 4$, $b = 9$.

c) Express the following integrals in terms of gamma function :

i) $\int_0^\infty e^{-x^2} dx$

ii) $\int_0^\infty \sqrt{x}\, e^{-x^3} dx$. 3 + 4 + 4 + 4

10. a) Solve any *three* of the following :

i) $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$

ii) $(D^4 - 2D^2 + 1)y = x^2 \cos x$

iii) $(D^2 + a^2)y = \sec ax$

iv) $(D-2)^2 = 8(e^{2x} + \sin 2x + x^2)$.

b) Show that the sequence $\left\{\frac{3n-1}{2n+1}\right\}$, $n \in N$ is convergent.

3 × 4 + 3