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Inviailator's Signature :	

CS/B.Sc. (H), (BT)/SEM-2/BMT-204/2011 2011 BIOMATHEMATICS – II

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

 $1. \quad \hbox{Choose the correct alternatives for any $\it ten$ of the following:}$

 $10 \times 1 = 10$

- i) The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is
 - a) divergent
 - b) convergent
 - c) absolutely convergent
 - d) none of these.
- ii) The sequence $\{x_n\}$, where $x_n = \frac{2n-1}{n+1}$, $n \in \mathbb{N}$ is bounded by
 - a) 2

b) 3

c) $\frac{1}{2}$

d) none of these.

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- The eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are iii)
 - 6, 1 a)

b) 3, 2

6, 3 c)

- none of these. d)
- A square matrix A is called orthogonal if iv)
 - $A = A^2$ a)
- b) $A^T = A^{-1}$
- $AA^{-1} = I$ c)
- d) none of these.
- If two eigenvalues of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the

third eigenvalue is

0 a)

b) 1

c) 4

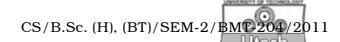
- d) none of these.
- Three lines are co-planer if vi)
 - they are concurrent a)
 - a line is perpendicular to each of them b)
 - they are concurrent and a line in perpendicular to c) each of them
 - d) none of these.
- vii) The equation of a straight line parallel to the X-axis is given by

a)
$$\frac{x-a}{1} = \frac{y-b}{1} = \frac{z-c}{1}$$
 b) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{1}$

b)
$$\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{1}$$

c)
$$\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$$
 d) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$.

d)
$$\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$$



viii) If two non-zero vectors \overrightarrow{A} and \overrightarrow{B} are parallel then

a)
$$\vec{A} \times \vec{B} = \vec{0}$$

b)
$$|\vec{A} \times \vec{B}| = 1$$

c)
$$\overrightarrow{A} \cdot \overrightarrow{B} = 0$$

d)
$$\left| \overrightarrow{A} \right| = \left| \overrightarrow{B} \right|$$
.

The differential equation satisfying the relation ix) $x = A \cos(mt - \alpha)$ is

a)
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 1 - x^2$$

a)
$$\frac{dx}{dt} = 1 - x^2$$
 b) $\frac{d^2x}{dt^2} = -\alpha^2x$

c)
$$\frac{d^2x}{dt^2} = -m^2x$$

d)
$$\frac{\mathrm{d}x}{\mathrm{d}t} = -m^2 x$$
.

The order the differential equation $\left\{1 + \frac{d^2y}{dx^2}\right\}^{\frac{1}{2}} = x^2$ is x)

- none of these. d)
- The value of Γ (3.5) is xi)

a)
$$\frac{15\sqrt{\pi}}{8}$$

b)
$$15\sqrt{\pi}$$

c)
$$\frac{3\sqrt{\pi}}{4}$$

d) none of these.

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- The complementary function of the differential equation $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 0 \text{ is}$

 - a) $(c_1 + c_2 t)e^{-3t}$ b) $(c_1 e^{-3t} + c_2 e^{-2t})$
 - c) $c_1e^{-2t} + c_2e^t$ d) none of these.
- xiii) The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{10^{n}}{\ln x^{n}}$ is
 - a) 10

b) $\frac{1}{10}$

c) 5 d) $\frac{1}{5}$.

GROUP - B (Short Answer Type Questions)

Answer any three of the following.

 $3 \times 5 = 15$

- 2. a) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} 2y = 0$
 - Find the particular integral of the differential equation $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 2x + 4.$ 3 + 2
- Find the eigenvalues and eigenvectors 3. the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.
 - Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. 2 + 3



- 4. a) Show that any square matrix A and its transpose A^T have same eigenvalues.
 - b) Find the product of the eigenvalues of the matrix $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. 3+2
- 5. a) If $\vec{a} = 3\vec{i} \vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} \vec{k}$ and $\vec{c} = \vec{i} 2\vec{j} + 2\vec{k}$ then verify $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.
 - b) If $\vec{\alpha} = 3$ $\vec{i} \vec{j} + 2\vec{k}$, $\vec{\beta} = 2$ $\vec{i} + \vec{j} \vec{k}$ and $\vec{\gamma} = \vec{i} 2$ $\vec{j} + 2\vec{k}$ then show that $(\vec{\alpha} \times \vec{\beta}) \times \vec{\gamma} \neq \vec{\alpha} \times (\vec{\beta} \times \vec{\gamma})$.

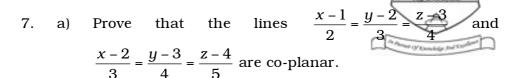
GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

- 6. a) If two mappings $f: R \to R$ and $g: R \to R$ be defined as follows: $f(x) = x^2$, g(x) = x 2, then show that $f \circ g \neq g \circ f$.
 - b) Show that the mapping $f:R\to R$ defined by $f(x)=7x+3, x\in R$ is bijective.
 - c) The binary operation * is defined on the set of integers Z as a*b=a+b-2, for all $a,b\in Z$. Show that (Z, *) is a group.
 - d) In a group G, if every element is its own inverse, then show that the group G is commutative. 2 + 3 + 5 + 5

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- b) Prove that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{1+z}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect and find the co-ordinates of their point of intersection.
- c) Find the equation of the line through (1, 2, -1) perpendicular to each of the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.
- 8. a) Examine the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \infty$
 - b) Discuss the convergence of the following series :

i)
$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots \infty$$

ii)
$$1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \dots \infty$$

c) Discuss the conditional convergence of

$$\frac{1}{2^3} - \frac{1}{3^3} (1+2) + \frac{1}{4^3} (1+2+3) - \frac{1}{5^3} (1+2+3+4) + \dots \infty$$

3 + 4 + 4 + 4

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- a) Let f(x) = |x| in [-1, 1]. Is Rolle's theorem applicable 9. to f(x) in [-1, 1]? Justify your answer
 - b) Lagrange's mean value f(b) - f(a) = (b - a) f'(c) where a < c < b, find *c* if $f(x) = \sqrt{x}$, a = 4, b = 9.
 - Express the following integrals in terms of gamma c) function:
 - i) $\int_{0}^{\infty} e^{-x^2} dx$
 - ii) $\int_{0}^{\infty} \sqrt{x} e^{-x^3} dx.$

- 3 + 4 + 4 + 4
- 10. a) Solve any three of the following:
 - $(D^3 6D^2 + 11D 6)y = e^{-2x} + e^{-3x}$
 - ii) $\left(D^4 2D^2 + 1\right)y = x^2 \cos x$
 - iii) $(D^2 + a^2)y = \sec ax$
 - iv) $(D-2)^2 = 8(e^{2x} + \sin 2x + x^2).$
 - Show that the sequence $\left\{\frac{3n-1}{2n+1}\right\}$, $n \in \mathbb{N}$ is convergent. b)

 $3 \times 4 + 3$

[Turn over