2020-21 Onwards (MR-20)	MALLA REDDY ENGINEERING COLLEGE (Autonomous)	B.Tech. I Semester		
Code: A0B01	Linear Algebra and Numerical Methods I B. Tech I Sem (Common For CSE & IT)	L	T	P
Credits: 4		3	1	-

Prerequisites: Matrices, Differentiation and Integration.

Course Objectives:

1. To learn types of matrices, Concept of rank of a matrix and applying the concept of rank to know the consistency of linear equations and to find all possible solutions, if exist.
2. To learn concept of Eigen values and Eigen vectors of a matrix, diagonalization of a matrix, Cayley Hamilton theorem and reduce a quadratic form into a canonical form through a linear transformation.
3. To learn various methods to find roots of an equation.
4. To learn Concept of finite differences and to estimate the value for the given data using interpolation.
5. To learn Solving ordinary differential equations and evaluation of integrals using numerical techniques.

MODULE I: Matrix algebra **[12 Periods]**

Vector Space, basis, linear dependence and independence (Only Definitions)

Matrices: Types of Matrices, Symmetric; Hermitian; Skew-symmetric; Skew- Hermitian; orthogonal matrices; Unitary Matrices; Rank of a matrix by Echelon form and Normal form, Inverse of Non-singular matrices by Gauss-Jordan method; solving system of Homogeneous and Non-Homogeneous linear equations, LU – Decomposition Method.

MODULE II: Eigen Values and Eigen Vectors **[12 Periods]**

Eigen values , Eigen vectors and their properties; Diagonalization of a matrix; Cayley-Hamilton Theorem (without proof); Finding inverse and power of a matrix by Cayley-Hamilton Theorem; Singular Value Decomposition.

Quadratic forms: Nature, rank, index and signature of the Quadratic Form, Linear Transformation and Orthogonal Transformation, Reduction of Quadratic form to canonical forms by Orthogonal Transformation Method.

MODULE III: : Algebraic &Transcendental equations **[12 Periods]**

(A) Solution of Algebraic and Transcendental Equations: Introduction-Errors, types of errors. Bisection Method, Method of False Position, Newton-Raphson Method.

(B) The Iteration Method ,Ramanujan's method to find smallest root of Equation. Jacobi's Iteration method. Gauss seidel Iteration method.

MODULE IV: Interpolation **[12 Periods]**

Introduction- Errors in Polynomial Interpolation – Finite differences- Forward Differences-Backward differences-Central differences - Symbolic relations and separation of symbols. Differences of a polynomial-Newton's formulae for interpolation; Central difference interpolation Formulae – Gauss Central Difference Formulae ; Interpolation with unevenly spaced points-Lagrange's Interpolation formula.

MODULE V: Numerical solution of Ordinary Differential Equations and Numerical Integration **[12 Periods]**

Numerical solution of Ordinary Differential Equations : Introduction-Solution of Ordinary Differential Equation by Taylor's series method - Picard's Method of successive Approximations - Euler's Method-Modified Euler's Method – Runge-Kutta Methods.

Numerical Integration: Trapezoidal Rule, Simpson's 1/3rd Rule, Simpson's 3/8 Rule.

TEXT BOOKS

1. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010.
2. Erwin kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
3. D. Poole, Linear Algebra: A Modern Introduction, 2nd Edition, Brooks/Cole, 2005.
4. M . K Jain, S R K Iyengar, R.K Jain, Numerical Methods for Scientific and Engineering Computation, New age International publishers.
5. S.S.Sastry, Introductory Methods of Numerical Analysis,5th Edition,PHI Learning Private Limited

REFERENCES

1. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9th Edition, Pearson, Reprint,2002.
2. N.P. Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications,
   Reprint, 2008.
3. V. Krishnamurthy, V.P. Mainra and J.L. Arora, An introduction to Linear Algebra, AffiliatedEast–West press, Reprint 2005.
4. Ramana B.V., Higher Engineering Mathematics, Tata McGraw Hill New Delhi, 11th Reprint,2010.

Matrix Algebra

Matrix: A matrix is a rectangular array of numbers (may be real or complex). its order is the number of rows and columns that define the array.

ex: $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ 2x2 matrix

$\begin{bmatrix} 1 & 3 & 4 \\ 5 & 0 & 2 \end{bmatrix}$ 2x3 matrix

$\begin{bmatrix} 1 & 3 & -2 \\ 3 & 2 & 5 \\ 7 & -1 & 0 \end{bmatrix}$ 3x3 matrix, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ 3x1 matrix

$[1, 1-i, 0]$ → 1x3 matrix. $[0]$ 1x1 matrix.

Types of matrices:

(i) Square matrix: A matrix A in which the number of rows is equal to the number of columns is called a square matrix. thus for the elements a_{ij} of a square matrix $A = [a_{ij}]_{n\times n}$ for which $i=j$. The elements $a_{11}, a_{22} \ldots a_{nn}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of the matrix

ex: $A = \begin{bmatrix} 0 & 5 & 4 \\ 3 & -1 & 7 \\ 4 & 3 & 1 \end{bmatrix}_{3\times 3}$ is a square matrix of order 3. The elements $0, -1, 1$ constitute the principal diagonal of The matrix A.

(ii) Row matrix – column matrix

Any $1 \times n$ matrix which has only one row and n column is called a row matrix or a row vector. Similarly any $m \times 1$ matrix which has m rows and only one column is called a column matrix or a column vector.

Ex: $A = [3, 4, 5, 6]_{1\times 4}$ is a row matrix of type 1×4.

$$B = \begin{bmatrix} 7 \\ 8 \\ 9 \\ 5 \end{bmatrix}_{4\times 1}$$ is a column matrix of type 4×1

(iii) Unit matrix or Identity matrix:

A square matrix in which each diagonal element is one and each non-diagonal element is equal to zero is called a unit matrix or an identity matrix and is denoted by I. It will denote a unit matrix of order n. – I_n.

A square matrix $A = [a_{ij}]$ is a unit matrix if $a_{ij} = 1$ if $i = j$ and $a_{ij} = 0$ if $i \neq j$.

ex. $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

(iv) Null matrix or Zero matrix

The $m \times n$ matrix whose elements are all zero is called the null matrix or zero matrix of type $m \times n$. It is denoted by $O_{m \times n}$.

ex: $O_{3\times4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times4}$ and $O_{3\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

are matrices of type 3×4, and 3×3.

(v) Diagonal matrix:

A square matrix $A = [a_{ij}]_{n\times n}$ whose elements above and below the principal diagonal are all zero is called a diagonal matrix ie. $a_{ij} = 0 \ \forall \ i \neq j$. ie

Thus a diagonal matrix has both upper and lower triangular matrices.

ex: $A_{3\times3} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $B_{3\times3} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

are diagonal matrices.

(vi) Scalar matrix:

A diagonal matrix whose diagonal elements are all equal to a scalar is called a Scalar matrix

ex: $A_{4\times4} = \begin{bmatrix} k & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & k \end{bmatrix}$ is a Scalar matrix each of whose diagonal elements is equal to k.

(VII) Upper and lower Triangular Matrices

A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ whenever $i > j$, Thus, in an upper triangular matrix, all the ~~mat~~ elements below the principal diagonal are zero.

IIly a square matrix $A = [a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ whenever $i < j$, Thus, in a lower triangular matrix all the elements above the principal diagonal are zero.

ex: $A = \begin{bmatrix} 3 & 5 & 7 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3}$ and $B = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 7 \end{bmatrix}_{4\times4}$

are upper triangular matrices.

$P = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 5 & 7 \end{bmatrix}_{3\times3}$ and $Q = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 4 & 3 & 6 & 0 \\ 6 & -1 & 0 & 8 \end{bmatrix}_{4\times4}$

are lower triangular matrices.

(VIII) Orthogonal matrix

A Square matrix A is said to be orthogonal if $A^T A = I$

ex: $A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$

IX Idempotent matrix :

A matrix A is said to be idempotent if $A^2 = A$

ex: $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

(X) Involutary matrix :

A matrix A is said to be involutary if $A^2 = I$, where I is the identity matrix

ex: $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

(XI) Nilpotent matrix :

A matrix A is said to be nilpotent if $A^k = 0$ (ncell matrix) where k is a positive integer. However if k is a least positive integer for which $A^k = 0$. Then k is called the Index of the nilpotent matrix.

ex $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

(XII) Trace of a matrix

Let A be a square matrix of order n. The sum of the elements of A lying along the principal diagonal is called the trace of the matrix A. Trace of the matrix A is denoted as trA

Thus if $A = [a_{ij}]_{n \times n}$ then

$$tr\ A = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + a_{33} + \cdots$$

Note : let A and B be two square matrices of order n and λ be a scalar then

i) $tr(\lambda A) = \lambda\ tr A$

ii) $tr(A+B) = tr A + tr B$

iii) $tr(AB) = tr(BA)$.