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06MAT41

Fourth Semester B.E. Degree Examination, June / July 08

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, choosing at least two from each part.***

PART - A

1 a. Solve $\frac{dy}{dx} = x^2y - 1$ with y(0) = 1 using Taylor's series method and find y(0.1). Consider up to fourth degree terms. **(06 Marks)**

b. Use Runge – Kutta 4th order method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 and find y for x = 0.2 and 0.4. Take h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2,\ y(0) = 1,\ y(0.1) = 1.1169,\ y(0.2) = 1.2773,\ y(0.3) = 1.5049$, find $y(0.4)$ accurate up to 3 decimal places using Milne's predictor – corrector method. **(07 Marks)**

2 a. If f(z) = u + i υ is analytic, show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$. **(06 Marks)**

b. Show that $u = e^{2x}(x\cos 2y - y\sin 2y)$ is harmonic. Find the analytic function f(z) = u + i v. **(07 Marks)**

c. Find the bilinear transformation that maps $Z_1 = i$, $Z_2 = 1$, $Z_3 = -1$ onto the points $W_1 = 1$, $W_2 = 0$, $W_3 = \infty$ respectively. Also find the image of $|Z| < 1$ in w–plane under this transformation. **(07 Marks)**

3 a. If f(z) = u + I v is an analytic function and f′(z) is continuous at each point with in and on a closed curve C, then show that $\int_C f(z)dz = 0$. **(06 Marks)**

b. Expand $f(z) = \frac{z+1}{(z+2)(z+3)}$ in a Laurent's series valid for

i) $|z| > 3$ ii) $2 < |z| < 3$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$. **(07 Marks)**

4 a. Obtain a series solution for the differential equation : $y'' - xy' + y = 0$ **(06 Marks)**

b. Prove the Rodrigue's formula $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. **(07 Marks)**

c. Express $x^3 + 2x^2 - x = 3$ in terms of Legendre polynomials. **(07 Marks)**

PART - B

5 a. Fit a 2nd degree polynomial of the form $y = a + bx + cx^2$ for the data : **(06 Marks)**

x	0	1	2	3	4	5
y	1	3	7	13	21	31

b. Find the coefficient of correlation, line of regression of x on y and line of regression of y on x ; given **(07 Marks)**

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

c. Three machines a, b, c produce respectively 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. An item selected at random is found to be defective. Find the probability that it is from machine A. **(07 Marks)**

6 a. The probability that an individual suffers a bad reaction from an injection is 0.001. Find the probability that out of 2000 individuals, more than 2 will get a bad reaction. **(06 Marks)**

b. The sale per day in a shop is exponentially distributed with average sale amounting to Rs. 100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on a day. **(07 Marks)**

c. The life of an electric bulb is normally distributed with average life of 2000 hours and standard deviation of 60 hours. Out of 2500 such bulbs, find the number of bulbs that are likely to last between 1900 and 2100 hours. Given $P[0 \le z \le 1.67] = 0.4525$. **(07 Marks)**

7 a. The weights of 1500 ball bearings are normally distributed with a mean of 635 gms and standard deviation of 1.36 gms. If 300 samples of size 36 each are drawn from this population, find the expected mean and standard deviation of the sampling distribution of means, if sampling is done

i) With replacement ii) What out replacement. When sampling is done with replacement, how many samples will have their mean, greater than 635.5 gms. Given $P[0 \le z \le 2.203] = 0.4861$. **(06 Marks)**

b. Eleven students were given a test in statistics. They were provided additional coaching and then a second test of equal difficulty was held at the end of coaching. Marks scored by them in the two tests are given below :

Test – 1	23	20	19	21	18	20	18	17	23	16	19
Test – 2	24	19	22	18	20	22	20	20	23	20	17

Do the marks give evidence that the students have benefited by extra coaching? Given $t_{0.05}^{(\gamma=10)}$ is 2.228. Test the hypothesis at 5% level of significance. **(07 Marks)**

c. Fit a Poisson distribution to the following data and test for its goodness of fit at 5% level of significance.

x	0	1	2	3	4
f	419	352	154	56	19

Given $\chi^2_{0.05}$ for $\gamma = 3$ is 7.82. **(07 Marks)**

8 a. The joint probability distribution for two random variables X and Y is as given below.

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find the marginal distributions of X, Y. Also find the covariance of X and Y. **(06 Marks)**

b. Find the fixed probability vector of the regular stochastic matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$ **(07 Marks)**

c. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that

i) A has the ball ii) B has the ball iii) C has the ball, for the fourth throw. **(07 Marks)**

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