Mathematics

(Common for B.Sc. / BA)

Paper 1: Differential Equations, Abstract Algebra and Real Analysis

Time: 3 hours Max Marks: 80

SECTION - A

Answer all the FOUR questions. Each question carries 15 marks. 4X15=60

1. a) i) Solve $x \cos x \dfrac{dy}{dx} + \left(x \sin x + \cos x\right) y = 1$ (8marks)

   ii) Solve $p^2 + 2 p y \cot x = y^2$ (7marks)

   (or)

   b) i) Solve $\dfrac{dy}{dx}(x^2y^3 + xy) = 1$ (8marks)

   ii) Solve $y + px = p^2x^4$ (7marks)

2. a) i) Solve $(D^2 - 3D + 2) y = \cos(e^{-x})$ by the method of variation of parameters. (8marks)

   ii) Solve $(D^2 + 9)y = \cos^3 x$ (7marks)

   (or)

   b) i) Solve $(D^2 + 4) y = x \sin x$ (8marks)

   ii) Solve $3x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = x$ (7marks)

3. a) i) State and prove Lagrange's theorem on Groups (8marks)

   ii) Show that a finite semi group satisfying cancellation laws is a group. (7marks)

   (or)

   b) i) State and prove Cayley's theorem on Permutation Groups (8marks)

   ii) Show that G = $\left\{x = 2^a 3^b / a, b \in Z\right\}$ is a group under multiplication. (7marks)

4. a) i) Prove that the sequence $\{S_n\}$ defined by

   $S_n = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \cdots\cdots\cdots\cdots + \dfrac{1}{n!}$ is convergent. (8 marks)

   ii) If $\{S_n\}$ is a Cauchy sequence then show that $\{S_n\}$ is convergent. (7marks)

   (or)

b) i) State and prove Cauchy's n^{th} root test.
(8marks)

ii) Test for convergence $\sum \frac{x^n}{x^n + a^n}$ (x > 0, a >0) (7marks)

SECTION – B

Answer any FOUR Questions 4x5=20

5. Solve y (xy + 2 $x^2 y^2$) dx + x (xy - $x^2 y^2$) dy = 0.
6. Find the orthogonal trajectories of the family of rectangular hyperbolas xy = a^2 where a is the parameter.
7. Solve (D^2 -3D+2) y = cosh x.
8. Solve (D^2 -2D +1) y = $x^2 e^{3x}$
9. State and prove fundamental theorem on groups.
10. If f is a homomorphism of a group G in to a group G' then prove that kernel of f is a normal sub group of G.
11. If $\{a_n\}$ is a sequence defined by a_1 = 1, $a_{n+1} = \frac{2a_n + 3}{4}$ for n$\geq$1. Show that $\{a_n\}$ is increasing sequence and find its limit.
12. Test for convergence $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$