Code No. : **6002**

FACULTY OF ENGINEERING AND INFORMATICS

B.E. I Year (Suppl.) (Common to All Branches) Examination, Dec. 2009/Jan. 2010

MATHEMATICS – I

Time: 3 Hours] [Max. Marks: 75

*Note : 1) Answer **all** questions of Part **A**.*
*2) Answer **five** questions from Part **B**.*

PART – A **(25 Marks)**

1. Find all values of a such that the rank of the matrix $\begin{bmatrix} a & -1 & 0 & 0 \\ 0 & a & -1 & 0 \\ 0 & 0 & a & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is 4. **3**

2. Show that the vectors (0, 1, –1), (–1, 0, –1), (3, 1, 3) are linearly independent. **2**

3. Define absolute convergent series and conditionally convergent series. **2**

4. Test the series $\sum\left(1+\frac{1}{n}\right)^n$ for convergence. **3**

5. Expand $f(x) = \log_e x$ in powers of $(x - 1)$. **3**

6. Find the envelope of the family of straight-lines $y = ax + a^2$, a is a parameter. **2**

7. Find the oblique asymptote of $x^3 + y^3 - 3xy = 0$. **3**

8. Determine $\lim\limits_{(x,y)\to(1,1)} \dfrac{x(y-1)}{y(x-1)}$, if it exists. **2**

9. Evaluate $\int_1^2 \int_1^3 xy^2 dxdy$. **3**

10. Find the value of a so that the vector $(x + 3y)\, i + (y - 2z)\, j + (x + az)\, k$ is solenoidal. **2**

PART – B (50 Marks)

11. a) If $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$, find the matrix $A^5-4A^4-7A^3+11A^2-A-10\,I$ using Caley - Hamilton theorem. 5

b) Reduce the quadratic form $Q = 2xy + 2yz + 2zx$ to canonical form. 5

12. a) Discuss the convergence of the series $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+..........$ 5

b) Test the series $\sum\left(\frac{n}{n+1}\right)^n x^n, x>0$ for convergence. 5

13. a) State and prove Lagrange's mean value theorem. 5

b) Find the evolute of $x^2 = 4ay$. 5

14. a) If $f(x,y)=\begin{cases}\frac{xy^3}{x+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$, compute $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ at (0, 0). 5

b) If $u = f(y-z, z-x, x-y)$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. 5

15. a) State and prove Gauss's divergence theorem. 5

b) Find the directional derivative of the function $f(x, y, z) = xyz$ at (–1, 1, 3) in the direction of $i - 2j + 2k$. 5

16. a) Find the rank of the matrix $\begin{bmatrix}1 & 2 & -3\\ 2 & 1 & -1\\ 1 & -1 & 2\\ 5 & 4 & -5\end{bmatrix}$. 5

b) Discuss the convergence of the series $\sum\frac{\cos n\pi}{n^2+1}$. 5

17. a) Discuss the maxima and minima of $f(x, y) = 2x^2 - 2y^2 - x^4 + y^4$. 5

b) If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$, find grad $(\bar{a}\cdot\bar{r})$. 5