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BT-2/JX **8251**

Mathematics—II (2006 to onward)

Paper : MATH-102 (E)

Time : Three Hours] [Maximum Marks : 100

Note :— Attempt **FIVE** questions, selecting at least **ONE** question from each unit.

UNIT—I

1. (a) Using Gauss-Jordan method, find the inverse of the matrix A

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

(b) Determine the value of λ for which the following set of equations may possess non-trivial solution

$$3x + y - \lambda z = 0,$$
$$4x - 2y - 3z = 0,$$
$$2\lambda x + 4\lambda + \lambda z = 0$$

for each permissible value of λ, determine the general solution.

2. (a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}.$$

(b) Prove that every Hermitian matrix can be written as $A \in iB$, where A is real and symmetric and B is real and skew-symmetric.

UNIT—II

3. (a) Solve :

$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$$ 6

(b) Find the orthogonal trajectories of a system of parabola $y^2 = 4a(x + a)$. 6

(c) Find the complete solution of

$$(D^4 + 4D - 3)y = X\, e^{3x}.$$ 8

4. (a) Using the method of variation of parameter, solve :

$$\frac{d^2y}{dx^2} + a^2y = \sec ax.$$

(b) An e.m.f. E sin pt is applied at t = 0 to a circuit consisting a capacitance C and inductance L. The current i satisfies the equation $L\frac{di}{dt} + \frac{1}{C}\int i\, dt = E \sin pt$, if $p^2 = \frac{1}{LC}$ and initially the current i and charge q are zero. Show that the current at time t is $\left(\frac{Et}{2L}\right)\sin pt$, where $i = \frac{dq}{dt}$.

UNIT—III

5. (a) Evaluate :

$$L\left\{\int_0^t \frac{\sin t}{t}\, dt\right.$$ 6

(b) Evaluate :

$$L^{-1}\left\{\frac{s^2+s}{(s^2+1)(s^2+2s+2)}\right\}.$$ 6

(c) Use Convolution theorem to evaluate :

$$L^{-1}\frac{1}{s^3(s^2+1)}.$$ 8

6. (a) Using Laplace transform method, solve

$$ty'' + 2y' + ty = \sin t, \text{ when } y(0) = 1.$$

(b) In an electrical circuit with e.m.f. E(t) resistance R and inductance L, the current i build up at the rate given by $L\frac{di}{dt}+Ri=E(t)$. If the switch is connected at t = 0 and disconnected at t = a, find the current i at any instant.

UNIT—IV

7. (a) Solve the partial differential equation

$$(z^2 - 2yz - y^2)\,p + (xy + zx)q = xy - zx.$$

(b) Solve :

$$2z + p^2 + qy + 2y^2 = 0.$$

8. (a) Solve :

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x.$$

(b) Solve the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, subject to $u(0, t) = u(l, t) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi x}{l}$.