BTS – III – 11.14 – 0942 Reg. No. [| | | | | | | |]

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B.Tech. Degree III Semester Examination November 2014

IT/CS/CE/SE/ME/EE/EC/EB/EI/FT 1301 ENGINEERING MATHEMATICS II

(2012 Scheme)

Time: 3 Hours Maximum Marks: 100

PART A

(Answer *ALL* questions)

(8 × 5 = 40)

I. (a) Reduce the following matrix to row echelon form and find its rank.

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b) Let F and G be subspaces of a vector space V. $F+G=\{f+g : f \in F, g \in G\}$. Show that $F+G$ is a subspace of V assuming usual definition of vector addition and scalar multiplication.

(c) Express $f(x)=\begin{cases} -x & for\ -\pi < x < 0 \\ x & for\ 0 < x < \pi \end{cases}$ as a Fourier series.

(d) Using the Fourier integral representation show that $\int_0^{\infty} \frac{\omega \sin x\omega}{1+\omega^2} dw = \frac{\pi}{2} e^{-x} (x > 0)$.

(e) Find the Laplace transform of $t \sin 3t \cos 2t$.

(f) Apply convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$

(g) Show that the function defined by

$F = (Z \cos x + \sin y) i + (x \cos y + \sin z) j + (y \cos z + \sin x) k$ is irrotational and find its scalar potential.

(h) If $F = (5xy - 6x^2) i + (2y - 4x) j$ evaluate $\int_c F \bullet dR$ along the curve C in the xy plane, $y = x^3$ from the point $(1,1)$ to $(2,8)$.

PART B

(4 × 15 = 60)

II. (a) Test for consistency and solve the system of equations (7)

$x+2y+z=3$

$2x+3y+2z=5$

$3x-5y+5z=2$

$3x+9y-z=4$

(b) Let T be a linear transformation R^3 to R^2 where (8)

$Tx = Ax, A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. $X = (x\ y\ z)^T$. Find Ker(*T*), *Ran(T)* and their dimensions.

OR

III. (a) Show that the transformation (7)

$y_1 = 2x_1 + x_2 + x_3,\ y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation.

(b) State Cayley Hamilton theorem and by using it find the inverse of the matrix. (8)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix}$$

(P.T.O.)

IV. (a) $f(x) = \pi x, \quad 0 \le x \le 1$ (8)

$\pi(2-x), \ 1 \le x \le 2$

Hence show that in the interval $(0,2)$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + ...\right]$$

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

(b) Find the Fourier transform of (7)

$f(x) = 1 \ \ for \ |x| < 1$

$0 \ \ for \ |x > 1|$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

OR

V. (a) Find the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 \le x \le 1$. Hence show that $\pi^2 = 8\left[\frac{1}{12} + \frac{1}{3^2} + \frac{1}{5^2} ....\right]$ (8)

(b) Solve the integral equation $\int_0^\infty f(\theta) \cos x\theta d\theta = 1 - \alpha \ \ for \ \ 0 \le \alpha \le 1$ (7)

$= 0, \qquad for \ \ \alpha > 1$.

VI. (a) Find the inverse Laplace transform of (7)

(i) $\frac{s+8}{s^2+4s+5}$

(ii) $\tan^{-1}(2/S)$

(b) Use Laplace transform to solve $y'' - 3y' + 2y = 4t + e^{3t}$ when $y(0) = 1, y'(0) = -1$. (8)

OR

VII. (a) Find the Laplace transform of (7)

(i) $U(t-a)$, unit slip function

(ii) Using it find the Laplace transform of $f(t) = t-1, \ 1 < t < 2$

$3-t, \ 2 < t < 3$.

(b) If $L^{-1}\left[\bar{f}(s)\right] = f(t)$, show that $L^{-1}\left[\frac{1}{s}\bar{f}(s)\right] = \int_0^t f(u) du$ (8)

Use this result to obtain

$L^{-1}\left[\frac{1}{s(s+a)}\right]$ and $L^{-1}\left[\frac{1}{s^2(s+a)}\right]$.

VIII. (a) Verify stoke's theorem for $F = (x^2+y^2)i - 2xyj$ taken round the rectangle bounded by the series $x = \pm a, \ y = 0, \ y = b$. (8)

(b) Prove that if ϕ is a scalar function, f is a vector function then curl $(\phi f) = (grad \phi) \times f + \phi(curl f)$ (7)

OR

IX. (a) Evaluate $\iint \vec{a}.\vec{n} ds$ where $\vec{a} = (x+y^2)i - 2xi + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. (8)

(b) If $\vec{r}$ be the position vector of a variable point (x,y,z) and $|\vec{r}| = r$ then show that (7)

$\nabla \cdot (f(r)\vec{r}) = rf'(r) + 3f(r)$ Also if $\nabla \cdot (f(r)\vec{r}) = 0$ show that $f(r) = \frac{c}{r^3}$, c being a constant.

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