ME8492 - KINEMATICS OF MACHINERY

L T P C
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Objectives:

- To understand the basic components and layout of linkages in the assembly of a system / machine.
- To understand the principles in analysing the assembly with respect to the displacement, velocity, and acceleration at any point in a link of a mechanism.
- To understand the motion resulting from a specified set of linkages, design few linkage mechanisms and cam mechanisms for specified output motions.
- To understand the basic concepts of toothed gearing and kinematics of gear trains and the effects of friction in motion transmission and in machine components.

UNIT I BASICS OF MECHANISMS 9

Classification of mechanisms – Basic kinematic concepts and definitions – Degree of freedom, Mobility – Kutzbach criterion, Gruebler's criterion – Grashof's Law – Kinematic inversions of four-bar chain and slider crank chains – Limit positions – Mechanical advantage – Transmission Angle – Description of some common mechanisms – Quick return mechanisms, Straight line generators, Universal Joint – rocker mechanisms.

UNIT II KINEMATICS OF LINKAGE MECHANISMS 9

Displacement, velocity and acceleration analysis of simple mechanisms – Graphical method– Velocity and acceleration polygons – Velocity analysis using instantaneous centres – kinematic analysis of simple mechanisms – Coincident points – Coriolis component of Acceleration – Introduction to linkage synthesis problem.

UNIT III KINEMATICS OF CAM MECHANISMS

Classification of cams and followers – Terminology and definitions – Displacement diagrams – Uniform velocity, parabolic, simple harmonic and cycloidal motions – Derivatives of follower motions – Layout of plate cam profiles – Specified contour cams – Circular arc and tangent cams – Pressure angle and undercutting – sizing of cams.

UNIT IV GEARS AND GEAR TRAINS

Law of toothed gearing – Involutes and cycloidal tooth profiles –Spur Gear terminology and definitions –Gear tooth action – contact ratio – Interference and undercutting. Helical, Bevel, Worm, Rack and Pinion gears [Basics only]. Gear trains – Speed ratio, train value – Parallel axis gear trains – Epicyclic Gear Trains.

UNIT V FRICTION IN MACHINE ELEMENTS 9

Surface contacts – Sliding and Rolling friction – Friction drives – Friction in screw threads – Bearings and lubrication – Friction clutches – Belt and rope drives – Friction in brakes- Band and Block brakes.

TOTAL: 45 PERIODS

OUTCOMES:

Upon the completion of this course the students will be able to

CO1 Discuss the basics of mechanism
CO2 Calculate velocity and acceleration in simple mechanisms
CO3 Develop CAM profiles
CO4 Solve problems on gears and gear trains
CO5 Examine friction in machine elements

TEXT BOOKS:

1. F.B. Sayyad, " ", MacMillan Publishers Pvt Ltd., Tech-max Educational resources, 2011.
2. Rattan, S.S, "Theory of Machines", 4th Edition, Tata McGraw-Hill, 2014.
3. Uicker, J.J., Pennock G.R and Shigley, J.E., "Theory of Machines and Mechanisms", 4th Edition, Oxford University Press, 2014.

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1. Thomas Bevan, "Theory of Machines", 3rd Edition, CBS Publishers and Distributors, 2005.
2. Cleghorn. W. L, "Mechanisms of Machines", Oxford University Press, 2005
3. Robert L. Norton, "Kinematics and Dynamics of Machinery", Tata McGraw-Hill, 2009.
4. Allen S. Hall Jr., "Kinematics and Linkage Design", Prentice Hall, 1961
5. Ghosh. A and Mallick, A.K., "Theory of Mechanisms and Machines", Affiliated East-West Pvt Ltd., New Delhi, 1988.
6. Rao.J.S. and Dukkipati.R.V. "Mechanisms and Machine Theory", Wiley-Eastern Ltd., New Delhi, 1992.
7. John Hannah and Stephens R.C., "Mechanics of Machines", Viva Low-Prices Student Edition, 1999.
8. Ramamurthi. V, "Mechanics of Machines", Narosa Publishing House, 2002.
9. Khurmi, R.S., "Theory of Machines",14th Edition, S Chand Publications, 2005
10. Sadhu Sigh : Theory of Machines, "Kinematics of Machine", Third Edition, Pearson Education, 2012

UNIT I BASICS OF MECHANISMS

1.1 Introduction:

Link or Element, Pairing of Elements with degrees of freedom, Grubler's criterion (without derivation), Kinematic chain, Mechanism, Mobility of Mechanism, Inversions, Machine.

1.1.1 Kinematic Chains and Inversions:

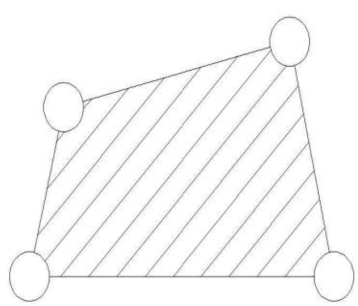
Kinematic chain with three lower pairs, Four bar chain, Single slider crank chain and Double slider crank chain and their inversions.

1.1.2Terminology and Definitions-Degree of Freedom, Mobility

- **Kinematics:** The study of motion (position, velocity, acceleration). A major goal of understanding kinematics is to develop the ability to design a system that will satisfy specified motion requirements. This will be the emphasis of this class.
- **Kinetics:** The effect of forces on moving bodies. Good kinematic design should produce good kinetics.
- **Mechanism:** A system design to transmit motion. (low forces)
- **Machine:** A system designed to transmit motion and energy. (forces involved)
- **Basic Mechanisms:** Includes geared systems, cam-follower systems and linkages (rigid links connected by sliding or rotating joints). A mechanism has multiple moving parts (for example, a simple hinged door does not qualify as a mechanism).

1.1.3 Key concepts:

- **Degrees of freedom:** The number of inputs required to completely control a system.
  Examples: A simple rotating link. A two-link system. A four-bar linkage. A five-bar linkage.
- **Types of motion:** Mechanisms may produce motions that are pure rotation, pure translation, or a combination of the two. We reduce the degrees of freedom of a mechanism by restraining the ability of the mechanism to move in translation (x-y directions for a 2D mechanism) or in rotation (about the z-axis for a 2-D mechanism).
- **Link:** A rigid body with two or more nodes (joints) that are used to connect to other rigid bodies. (WM examples: binary link, ternary link (3 joints), quaternary link (4 joints)
- **Joint:** A connection between two links that allows motion between the links. The motion allowed may be rotational (revolute joint), translational (sliding or prismatic joint), or a combination of the two (roll-slide joint).
- **Kinematic chain:** An assembly of links and joints used to coordinate an output motion with an input motion.
- **Link or element:**



A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a link.

A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them, thus a link may consist of one or more resistant bodies. A link is also known as Kinematic link or an element.

Links can be classified into 1) Binary, 2) Ternary, 3) Quaternary, etc.

- **Kinematic Pair:**

A Kinematic Pair or simply a pair is a joint of two links having relative motion between them.

1.1.4 Types of Kinematic Pairs:

Kinematic pairs can be classified according to
i) Nature of contact.
ii) Nature of mechanical constraint.
iii) Nature of relative motion.

i) Kinematic pairs according to nature of contact:

a) Lower Pair: A pair of links having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Examples: Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint.

b) Higher Pair: When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples: Wheel rolling on a surface cam and follower pair, tooth gears, ball and roller bearings, etc.

ii) Kinematic pairs according to nature of mechanical constraint.

a) Closed pair: When the elements of a pair are held together mechanically, it is known as a closed pair. The contact between the two can only be broken only by the destruction of at least one of the members. All the lower pairs and some of the higher pairs are closed pairs.

b) Unclosed pair: When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this the links are not held together mechanically. Ex.: Cam and follower pair.

iii) Kinematic pairs according to nature of relative motion.

a) Sliding pair: If two links have a sliding motion relative to each other, they form a sliding pair. A rectangular rod in a rectangular hole in a prism is an example of a sliding pair.

b) Turning Pair: When on link has a turning or revolving motion relative to the other, they constitute a turning pair or revolving pair.

c) Rolling pair: When the links of a pair have a rolling motion relative to each other, they form a rolling pair. A rolling wheel on a flat surface, ball ad roller bearings, etc are some of the examples for a Rolling pair.

d) Screw pair (Helical Pair): if two mating links have a turning as well as sliding motion between them, they form a screw pair. This is achieved by cutting matching threads on the two links.

The lead screw and the nut of a lathe is a screw Pair

e) Spherical pair: When one link in the form of a sphere turns inside a fixed link, it is a spherical pair. The ball and socket joint is a spherical pair.

1.2 Degrees of Freedom:

An unconstrained rigid body moving in space can describe the following independent motions.

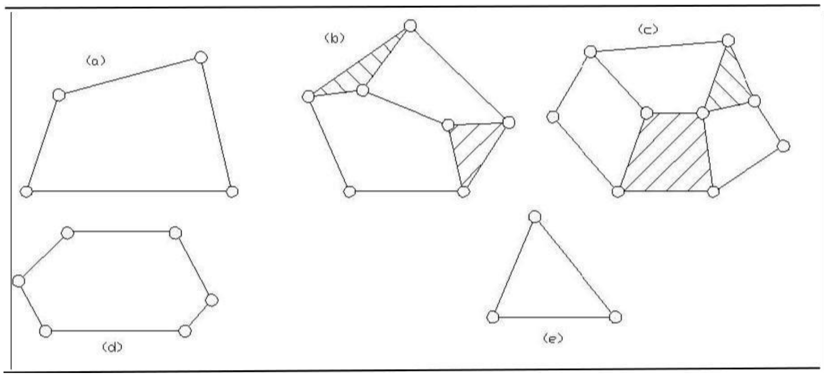
1. Translational Motions along any three mutually perpendicular axes x, y and z,
2. Rotational motions along these axes.

To find the number of degrees of freedom for a plane mechanism we have an equation known as
Grubler's equation and is given by $F = 3(n - 1) - 2j1 - j2$

F = Mobility or number of degrees of freedom, n = Number of links including frame.
j1 = Joints with single (one) degree of freedom, J2 = Joints with two degrees of freedom.
If F > 0, results in a mechanism with 'F' degrees of freedom.
F = 0, results in a statically determinate structure.
F < 0, results in a statically indeterminate structure.

1.2.1Kinematic Chain:

A Kinematic chain is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the others is definite (fig. a, b, and c.)



In case, the motion of a link results in indefinite motions of other links, it is a non-kinematic chain. However, some authors prefer to call all chains having relative motions of the links as kinematic chains.

1.2.2Linkage, Mechanism and structure:

- A linkage is obtained if one of the links of kinematic chain is fixed to the ground. If motion of each link results in definite motion of the others, the linkage is known as mechanism.
- If one of the links of a redundant chain is fixed, it is known as a structure.

The three main types of constrained motion in kinematic pair are,

1.**Completely constrained motion**: If the motion between a pair of links is limited to a definite direction, then it is completely constrained motion.

2. Incompletely Constrained motion : If the motion between a pair of links is not confined to a definite direction, then it is incompletely constrained motion

3. **Successfully constrained motion or Partially constrained motion:**If the motion in a definite direction is not brought about by itself but by some other means, then it is known as successfully constrained motion.

Machine:

It is a combination of resistant bodies with successfully constrained motion which is used to transmit or t

1.3.Kutzbach criterion, Grashoff's law

Kutzbach criterion:

- **Fundamental Equation for 2-D Mechanisms:** M = 3(L – 1) – 2J1 – J2
  Can we intuitively derive Kutzbach's modification of Grubler's equation? Consider a rigid link constrained to move in a plane.

1.3.1 Grashoff's law:

- **Grashoff 4-bar linkage:** A linkage that contains one or more links capable of undergoing a full rotation. A linkage is Grashoff if: S + L < P + Q (where: S = shortest link length, L = longest, P, Q = intermediate length links).
- **Non Grashoff 4 bar:** No link can rotate 360 if: S + L > P + Q

Inversions:

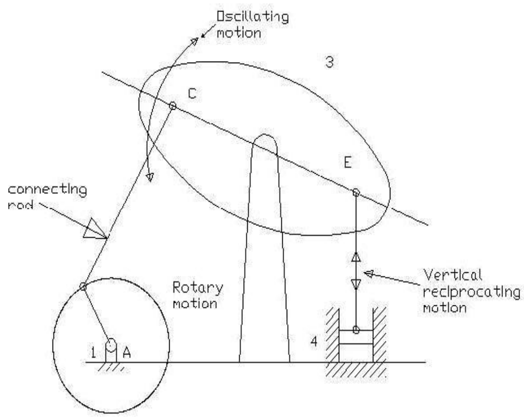
By fixing each link at a time we get as many mechanisms as the number of links, then each mechanism is called 'Inversion' of the original Kinematic Chain.

Inversions of four bar chain mechanism:

There are three inversions: 1) Beam Engine or Crank and lever mechanism. 2) Coupling rod of locomotive or double crank mechanism. 3) Watt's straight-line mechanism or double lever mechanism.

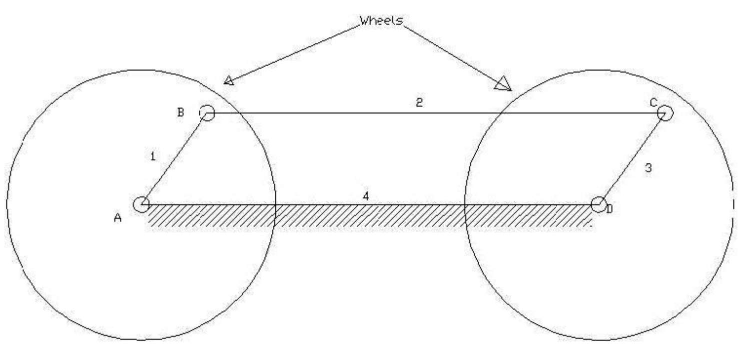
1. Beam Engine: - 1st Inversion or 3rd Inversion

When the crank AB rotates about A, the link CE pivoted at D makes vertical reciprocating motion at end E. This is used to convert rotary motion to reciprocating motion and vice versa. It is also known as Crank and lever mechanism.



2. Coupling rod of locomotive:

In this mechanism the length of link AD = length of link C. Also, length of link AB = length of link CD. When AB rotates about A, the crank DC rotates about D. this mechanism is used for coupling locomotive wheels. Since links AB and CD work as cranks, this mechanism is also known as double crank mechanism. This is shown in the figure below.



3. **Watt's straight-line mechanism or Double lever mechanism:** In this mechanism, the links AB & DE act as levers at the ends A & E of these levers are fixed. The AB & DE are parallel in the mean position of the mechanism and coupling rod BD is perpendicular to the levers AB & DE. On any small displacement of the mechanism the tracing point 'C' traces the shape of number '8', a

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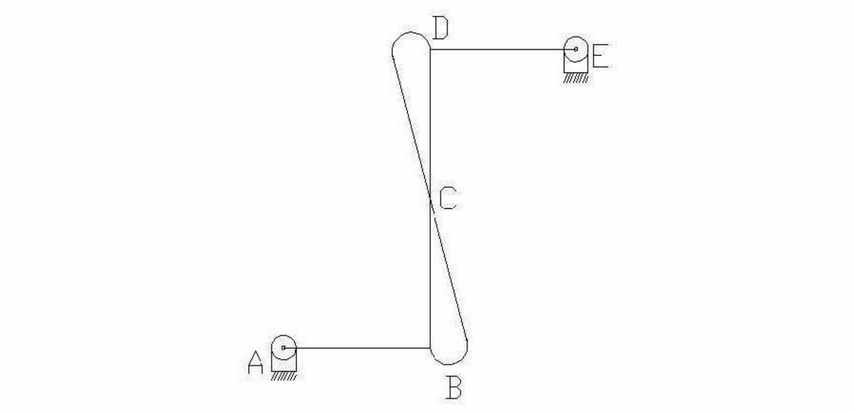
portion of which will be approximately straight. Hence this is also an example for the approximate straight-line mechanism. This mechanism is shown below.



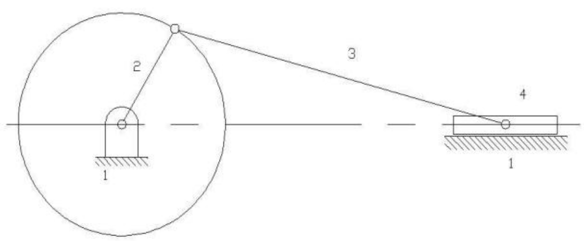
2. Slider crank Chain:

It is a four-bar chain having one sliding pair and three turning pairs. It is shown in the figure below the purpose of this mechanism is to convert rotary motion to reciprocating motion and vice versa. Inversions of a Slider crank chain:

There are four inversions in a single slider chain mechanism. They are:

1. Reciprocating engine mechanism:

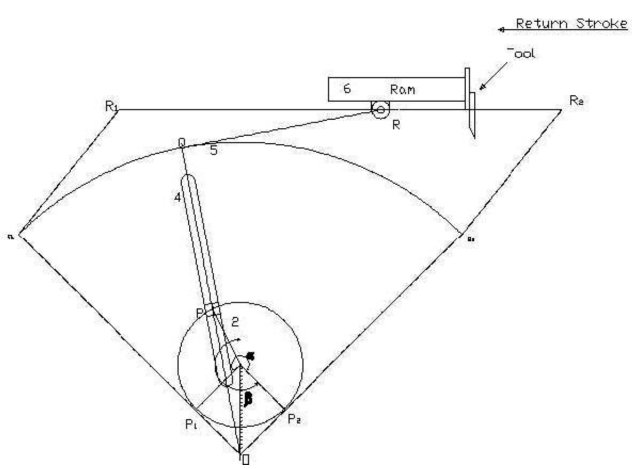
In the first inversion, the link 1 i.e., the cylinder and the frame are kept fixed. The fig below shows a reciprocating engine.



A slotted link 1 is fixed. When the crank 2 rotates about O, the sliding piston 4 reciprocates in the slotted link 1. This mechanism is used in steam engine, pumps, compressors, I.C. engines, etc.

2. Crank and slotted lever mechanism:

It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.

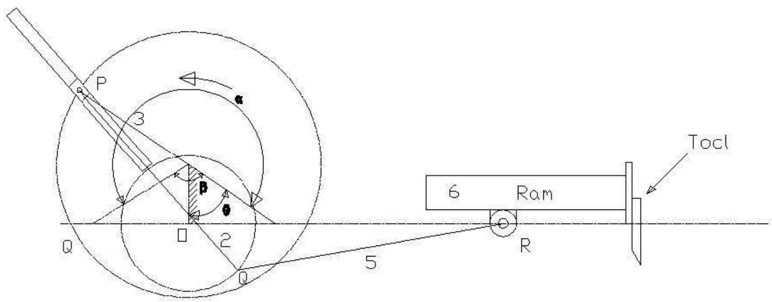


- In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6).
- The ram with the cutting tool reciprocates perpendicular to the fixed link 3.
- The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4.
- Thus, the cutting stroke is executed during the rotation of the crank through angle α and the return stroke is executed when the crank rotates through angle β or 360 – α. Therefore, when the crank rotates uniformly, we get

$$\frac{\text{Time to cutting}}{\text{Time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360 - \alpha}$$

- This mechanism is used in shaping machines, slotting machines and in rotary engines.

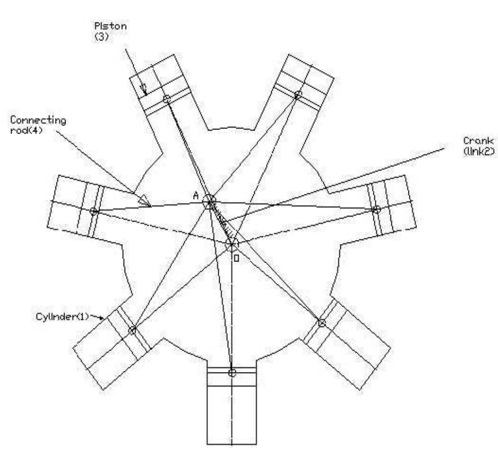
1.4 . 1 Whitworth quick return motion mechanism:



- Third inversion is obtained by fixing the crank i.e. link 2. Whitworth quick return mechanism is an application of third inversion.
- The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6).
- The rotary motion of P is taken to the ram R which reciprocates. The quick return motion mechanism is used in shapers and slotting machines.
- The angle covered during cutting stroke from P1 to P2 in counter clockwise direction is α or 360 -2θ. During the return stroke, the angle covered is 2θ or β.

1. Rotary engine mechanism or Gnome Engine:

- Rotary engine mechanism or gnome engine is another application of third inversion. It is a rotary cylinder V – type internal combustion engine used as an aero – engine.
- The Gnome engine has generally seven cylinders in one plane. The crank OA is fixed and all the connecting rods from the pistons are connected to A.
- In this mechanism when the pistons reciprocate in the cylinders, the whole assembly of cylinders, pistons and connecting rods rotate about the axis O, where the entire mechanical power developed, is obtained in the form of rotation of the crank shaft. This mechanism is shown in the figure below.



Double Slider Crank Chain:

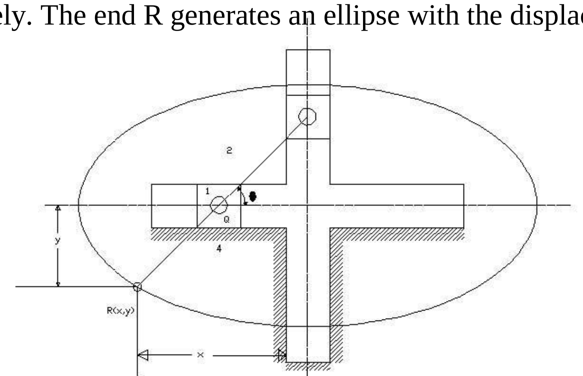
A four bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as double slider crank chain.

3 Inversions of Double slider Crank chain:

It consists of two sliding pairs and two turning pairs. They are three important inversions of double slider crank chain. 1) Elliptical trammel. 2) Scotch yoke mechanism. 3) Oldham's Coupling.

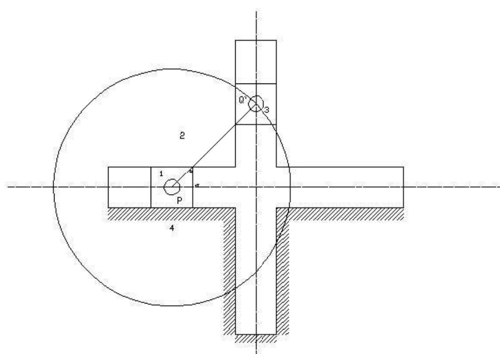
4. Elliptical Trammel:

This is an instrument for drawing ellipses. Here the slotted link is fixed. The sliding block P and Q in vertical and horizontal slots respectively. The end R generates an ellipse with the displacement of sliders P and Q.

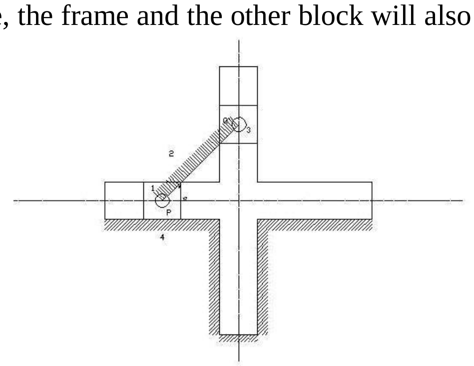


The co-ordinates of the point R are x and y. From the fig. cos θ = x.PR
and Sin θ = y. QR
Squaring and adding (i) and (ii) we get $x\ y^2 = \cos^2\theta + \sin^2\theta$
The equation is that of an ellipse, Hence the instrument traces an ellipse. Path traced by mid-point of
It is an equation of circle with PR = QR = radius of a circle.

5. Scotch yoke mechanism: This mechanism, the slider P is fixed. When PQ rotates above P, the slider Q reciprocates in the vertical slot. The mechanism is used to convert rotary to reciprocating mechanism.



5. Oldham's coupling: The third inversion of obtained by fixing the link connecting the 2 blocks P & Q. If one block is turning through an angle, the frame and the other block will also turn through the same angle. It is shown in the figure below.



- An application of the third inversion of the double slider crank mechanism is Oldham's coupling shown in the figure. This coupling is used for connecting two parallel shafts when the distance between the shafts is small.

- The two shafts to be connected have flanges at their ends, secured by forging. Slots are cut in the flanges. These flanges form 1 and 3.
- An intermediate disc having tongues at right angles and opposite sides is fitted in between the flanges. The intermediate piece forms the link 4 which slides or reciprocates in flanges 1 & 3.
- The link two is fixed as shown. When flange 1 turns, the intermediate disc 4 must turn through the same angle and whatever angle 4 turns, the flange 3 must turn through the same angle.
- Hence 1, 4 & 3 must have the same angular velocity at every instant. If the distance between the axis of the shaft is x, it will be the diameter if the circle traced by the centre of the intermediate piece. The maximum sliding speed of each tongue along its slot is given by
    - $v=x\omega$ where, ω = angular velocity of each shaft in rad/sec v = linear velocity in m/sec

1.6 Mechanical Advantage

The mechanical advantage (MA) is defined as the ratio of output torque to the input torque. (or) ratio of load to output.

Transmission angle.

The extreme values of the transmission angle occur when the crank lies along the line of frame.

1.7 Description of common mechanisms-Single, Double and offset slider mechanisms - Quick return mechanisms:

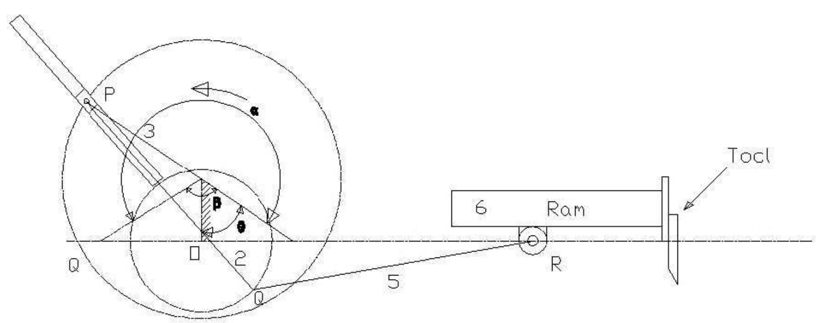
1.Quick Return Motion Mechanisms:

Many times mechanisms are designed to perform repetitive operations. During these operations for a certain period the mechanisms will be under load known as working stroke and the remaining period is known as the return stroke, the mechanism returns to repeat the operation without load. The ratio of time of working stroke to that of the return stroke is known a time ratio.

Quick return mechanisms are used in machine tools to give a slow cutting stroke and a quick return stroke. The various quick return mechanisms commonly used are i) Whitworth ii) Drag link. iii) Crank and slotted lever mechanism

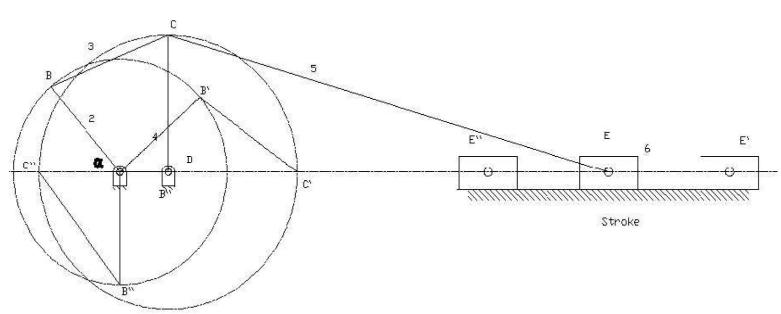
2. Whitworth quick return mechanism:

- Whitworth quick return mechanism is an application of third inversion of the single slider crank chain. This mechanism is shown in the figure below.
- The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6). The rotary motion of P is taken to the ram R which reciprocates.
- The quick return motion mechanism is used in shapers and slotting machines.



- The angle covered during cutting stroke from P1 to P2 in counter clockwise direction is α or 360 -2θ.
- During the return stroke, the angle covered is 2θ or β.

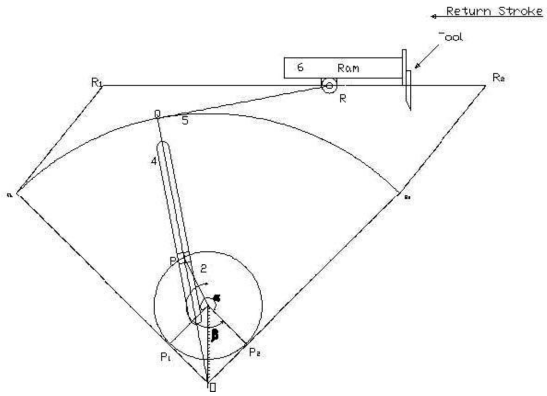
3. Drag link mechanism:



- This is four bar mechanism with double crank in which the shortest link is fixed. If the crank AB rotates at a uniform speed, the crank CD rotate at a non -uniform speed.
- This rotation of link CD is transformed to quick return reciprocatory motion of the ram E by the link CE as shown in figure.
- When the crank AB rotates through an angle α in Counter clockwise direction during working stroke, the link CD rotates through 180. We can observe that ∠ α >∠ β.
- Hence time of working stroke is α /β times more or the return stroke is α /β times quicker. Shortest link is always stationary link.
- Sum of the shortest and the longest links of the four links 1, 2, 3 and 4 are less than the sum of the other two. It is the necessary condition for the drag link quick return mechanism.

4.Crank and slotted lever mechanism:

It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.



- In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6).
- The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4.
- Thus, the cutting stroke is executed during the rotation of the crank through angle α and the return stroke is executed when the crank rotates through angle β or 360 – α. Therefore, when the crank rotates uniformly, we get,

5. Ratchets and escapements - Indexing Mechanisms - Rocking Mechanisms:

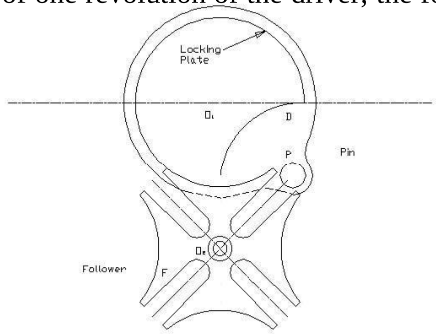
Intermittent motion mechanism:

Ratchet and Pawl mechanism:

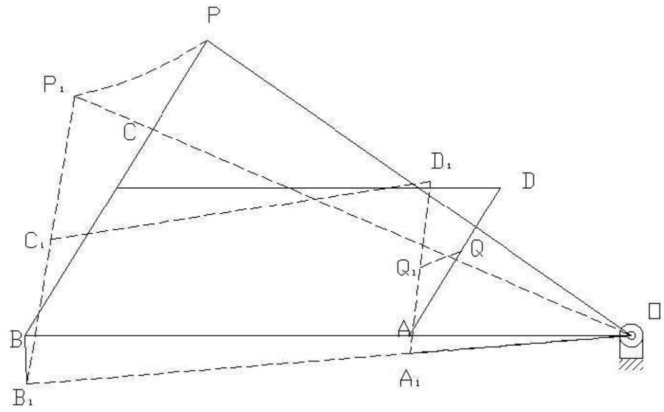
- This mechanism is used in producing intermittent rotary motion member. A ratchet and Pawl mechanism consist of a ratchet wheel 2 and a pawl 3 as shown in the figure.
- When the lever 4 carrying pawl is raised, the ratchet wheel rotates in the counter clock wise direction (driven by pawl). As the pawl lever is lowered the pawl slides over the ratchet teeth. One more pawl 5 is used to prevent the ratchet from reversing.
- Ratchets are used in feed mechanisms, lifting jacks, clocks, watches and counting devices.

6. Geneva mechanism: Geneva mechanism is an intermittent motion mechanism. It consists of a driving wheel D carrying a pin P which engages in a slot of follower F as shown in figure.

- During one quarter revolution of the driving plate, the Pin and follower remain in contact and hence the follower is turned by one quarter of a turn.
- During the remaining time of one revolution of the driver, the follower remains in rest locked in position by the circular arc.

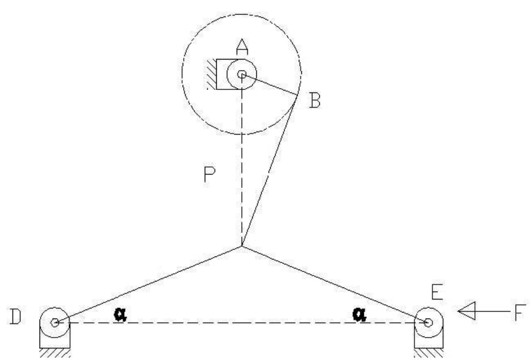


7. Pantograph: Pantograph is used to copy the curves in reduced or enlarged scales. Hence this mechanism finds its use in copying devices such as engraving or profiling machines.



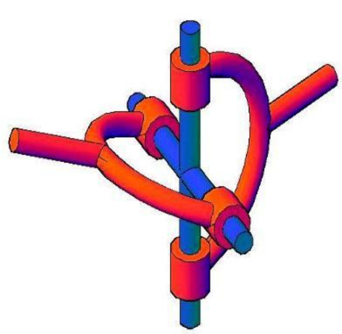
- This is a simple figure of a Pantograph. The links are pin jointed at A, B, C and D. AB is parallel to DC and AD is parallel to BC. Link BA is extended to fixed pin O. Q is a point on the link AD.
- If the motion of Q is to be enlarged then the link BC is extended to P such that O, Q and P are in a straight line. Then it can be shown that the points P and Q always move parallel and similar to each other over any path straight or curved.
- Their motions will be proportional to their distance from the fixed point. Let ABCD be the initial position. Suppose if point Q moves to Q1, then all the links and the joints will move to the new positions (such as A moves to A1, B moves to Q1, C moves to Q1, D moves to D1 and P to P1) and the new configuration of the mechanism is shown by dotted lines. The movement of Q (Q Q1) will be enlarged to PP1 in a definite ratio.

8. Toggle Mechanism:



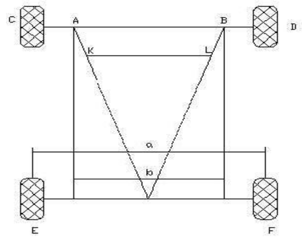
- In slider crank mechanism as the crank approaches one of its dead centre position, the slider approaches zero. The ratio of the crank movement to the slider movement approaching infinity is proportional to the mechanical advantage. This is the principle used in toggle mechanism.
- A toggle mechanism is used when large forces act through a short distance is required. The figure below shows a toggle mechanism. Links CD and CE are of same length. Resolving the forces at C vertically **F Sin α =P Cos α 2**
- Therefore, F = P . (because Sin α/Cos α = Tan α) 2 tan α Thus for the given value of P, as the links CD and CE approaches collinear position (αO), the force F rises rapidly.

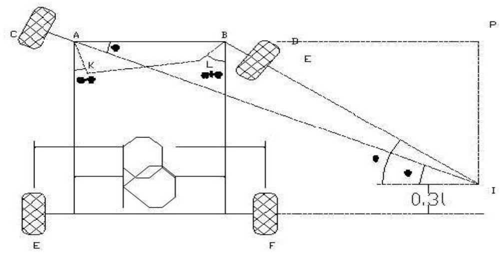
9. Hooke's joint:



- Hooke's joint used to connect two parallel intersecting shafts as shown in figure. This can also be used for shaft with angular misalignment where flexible coupling does not serve the purpose.
- Hence Hooke's joint is a means of connecting two rotating shafts whose axes lie in the same plane and their directions making a small angle with each other.
- It is commonly known as Universal joint. In Europe it is called as Cardan joint.

10. Ackermann steering gear mechanism:





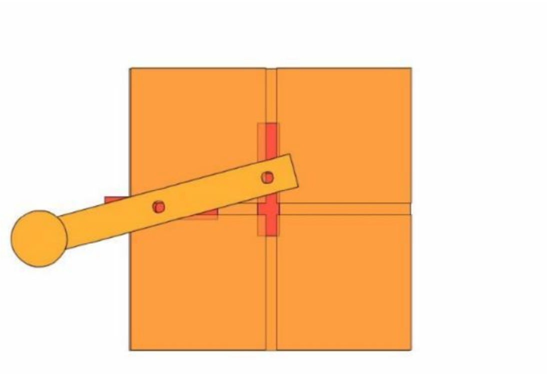
- This mechanism is made of only turning pairs and is made of only turning pairs wear and tear of the parts is less and cheaper in manufacturing.
- The cross-link KL connects two short axles AC and BD of the front wheels through the short links AK and BL which forms bell crank levers CAK and DBL respectively as shown in fig, the longer links AB and KL are parallel and the shorter links AK and BL are inclined at an angle α.
- When the vehicles steer to the right as shown in the figure, the short link BL is turned so as to increase α, whereas the link LK causes the other short link AK to turn so as to reduce α. The fundamental equation for correct steering is,

$$\mathbf{Cot\Phi – Cos\theta} = \boldsymbol{b / l}$$

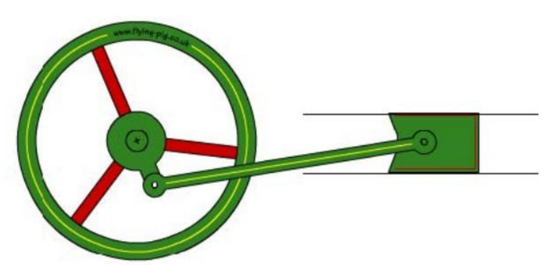
- In the above arrangement it is clear that the angle Φ through which AK turns is less than the angle θ through which the BL turns and therefore the left front axle turns through a smaller angle than the right front axle. For different angle of turn θ, the corresponding value of Φ and (Cot Φ – Cos θ) are noted.
- This is done by actually drawing the mechanism to a scale or by calculations. Therefore, for different value of the corresponding value of and are tabulated.

Three correct steering positions will be:
1) When moving straight.
2) When moving one correct angle to the right corresponding to the link ratio AK/AB and angle α.
3) Similar position when moving to the left. **In all other positions pure rolling is not obtainable.**

ELLIPTICAL TRAMMEL



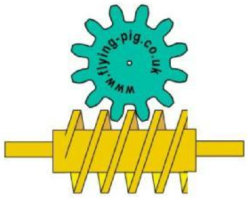
PISTON ARRANGEMENT



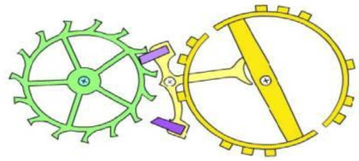
ELLIPTICAL TRAMMEL: This fascinating mechanism converts rotary motion to reciprocating motion in two axis. Notice that the handle traces out an ellipse rather than a circle. A similar mechanism is used in ellipse drawing tools.

PISTON ARRANGEMENT: This mechanism is used to convert between rotary motion and reciprocating motion, it works either way. Notice how the speed of the piston changes. The piston starts from one end, and increases its speed. It reaches maximum speed in the middle of its travel then gradually slows down until it reaches the end of its travel.

RACK AND PINION



RATCHET



RACK AND PINION: The rack and pinion is used to convert between rotary and linear motion. The rack is the flat, toothed part, the pinion is the gear.

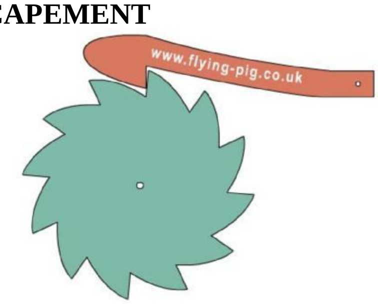
- Rack and pinion can convert from rotary to linear of from linear to rotary. The diameter of the gear determines the speed that the rack moves as the pinion turns.
- Rack and pinions are commonly used in the steering system of cars to convert the rotary motion of the steering wheel to the side to side motion in the wheels. Rack and pinion gears give a positive motion especially compared to the friction drive of a wheel in tarmac.
- In the rack and pinion railway a central rack between the two rails engages with a pinion on the engine allowing the train to be pulled up very steep slopes.

RATCHET: The ratchet can be used to move a toothed wheel one tooth at a time. The part used to move the ratchet is known as the pawl.

- The ratchet can be used as a way of gearing down motion. By its nature motion created by a ratchet is intermittent. By using two pawls simultaneously this intermittent effect can be almost, but not quite, removed.
- Ratchets are also used to ensure that motion only occurs in only one direction, useful for winding gear which must not be allowed to drop. Ratchets are also used in the freewheel mechanism of a bicycle.

WORM GEAR

WATCH ESCAPEMENT



1.8 Straight line generators, Design of Crank-rocker Mechanisms:

- **Straight Line Motion Mechanisms:**

The easiest way to generate a straight-line motion is by using a sliding pair but in precision machines sliding pairs are not preferred because of wear and tear. Hence in such cases different methods are used to generate straight line motion mechanisms:

1. Exact straight-line motion mechanism.

a. Peaucellier mechanism, b. Hart mechanism, c. Scott Russell mechanism

2. Approximate straight-line motion mechanisms

a. Watt mechanism, b. Grasshopper's mechanism, c. Robert's mechanism, d. Tchebicheff's mechanism

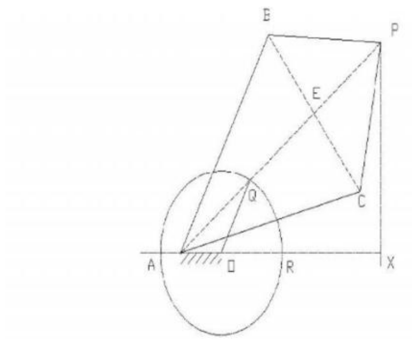
a. Peaucillier mechanism:

The pin Q is constrained to move long the circumference of a circle by means of the link OQ. The link OQ and the fixed link are equal in length. The pins P and Q are on opposite corners of a four-bar chain which has all four links QC, CP, PB and BQ of equal length to the fixed pin A. i.e., link AB = link AC. The product AQ x AP

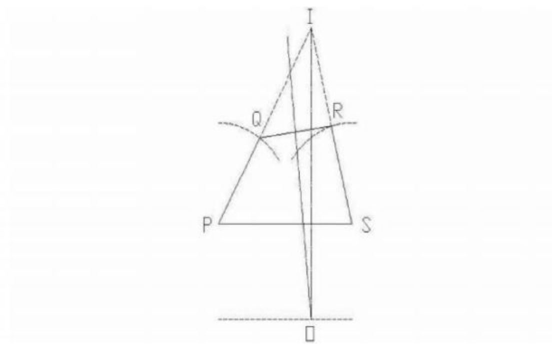
remain constant as the link OQ rotates may be proved as follows: Join BC to bisect PQ at F; then, from the right angled triangles AFB, BFP, we have AB=AF+FB and BP=BF+FP. Subtracting,
AB-BP= AF-FP=(AF–FP) (AF+FP) = AQ x AP .
Since AB and BP are links of a constant length, the product AQ x AP is constant. Therefore, the point P traces out a straight path normal to AR.

b. Robert's mechanism:

This is also a four bar chain. The link PQ and RS are of equal length and the tracing pint 'O' is rigidly attached to the link QR on a line which bisects QR at right angles. The best position for O may be found by making use of the instantaneous centre of QR. The path of O is clearly approximately horizontal in the Robert's mechanism.



a. Peaucillier mechanism



b. Hart mechanism

UNIT II - KINEMATICS OF LINKAGE MECHANISMS

2.1 Displacement, velocity and acceleration analysis in simple mechanisms:

Important Concepts in Velocity Analysis

1. The absolute velocity of any point on a mechanism is the velocity of that point with reference to ground.
2. Relative velocity describes how one point on a mechanism moves relative to another point on the mechanism.
3. The velocity of a point on a moving link relative to the pivot of the link is given by the equation: $V = \omega r$, where = angular velocity of the link and r = distance from pivot.

Acceleration Components

- ***Normal Acceleration:*** $\mathbf{A}^n$ = Points toward the centre of rotation
- ***Tangential Acceleration:*** $\mathbf{A}^t$ = In a direction perpendicular to the link
- ***Coriolis Acceleration:*** $\mathbf{A}^c$ = In a direction perpendicular to the link
- ***Sliding Acceleration:*** $\mathbf{A}^s$ = In the direction of sliding.

A rotating link will produce normal and tangential acceleration components at any point a distance, r, from the rotational pivot of the link. The total acceleration of that point is the vector sum of the components. A slider attached to ground experiences only sliding acceleration.

The total acceleration of a point is the vector sum of all applicable acceleration components:

$$\mathbf{A} = \mathbf{A}^n + \mathbf{A}^t + \mathbf{A}^c + \mathbf{A}^s$$

These vectors and the above equation can be broken into x and y components by applying sines and cosines to the vector diagrams to determine the x and y components of each vector. In this way, the x and y components of the total acceleration can be found.

2.2 Graphical Method, Velocity and Acceleration polygons:

*** Graphical velocity analysis:**

It is a very short step (using basic trigonometry with sines and cosines) to convert the graphical results into numerical results. The basic steps are these:

1. Set up a velocity reference plane with a point of zero velocity designated.
2. Use the equation, $V = \omega r$, to calculate any known linkage velocities.
3. Plot your known linkage velocities on the velocity plot. A linkage that is rotating about ground gives an absolute velocity. This is a vector that originates at the zero-velocity point and runs perpendicular to the link to show the direction of motion. The vector, **V**A, gives the velocity of point A.
4. Plot all other velocity vector directions. A point on a grounded link (such as point B) will produce an absolute velocity vector passing through the zero-velocity point and perpendicular to the link. A point on a floating link (such as B relative to point A) will produce a relative velocity vector. This vector will be perpendicular to the link AB and pass through the reference point (A) on the velocity diagram.
5. One should be able to form a closed triangle (for a 4-bar) that shows the vector equation: **V**B = **V**A + **V**B/A where **V**B = absolute velocity of point B, **V**A = absolute velocity of point A, and **V**B/A is the velocity of point B relative to point A.

2.3 Velocity and Acceleration analysis of mechanisms (Graphical Methods):

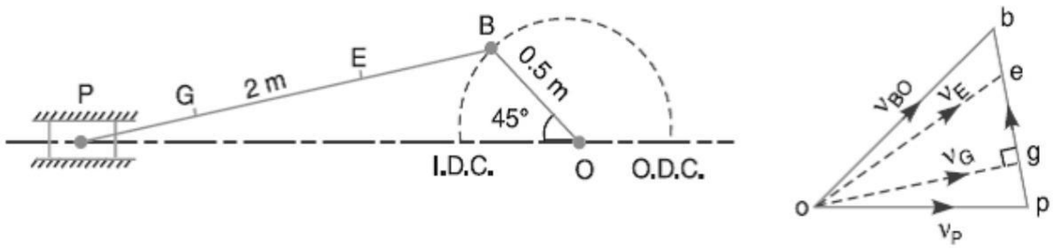
Velocity and acceleration analysis by vector polygons: Relative velocity and accelerations of particles in a common link, relative velocity and accelerations of coincident particles on separate link, Coriolis component of acceleration.

Velocity and acceleration analysis by complex numbers: Analysis of single slider crank mechanism and four bar mechanism by loop closure equations and complex numbers.

2.4 Coincident points, Coriolis Acceleration:

- ***Coriolis Acceleration:*** $\mathbf{A}^c = 2\,(dr/dt)$. In a direction perpendicular to the link. A slider attached to ground experiences only sliding acceleration.

Example: 1 The crank and connecting rod of a steam engine are 0.5m and 2 m long. The crank makes 180 rpm in CW direction. When it turned 45º from IDC determine 1. Velocity of piston. 2.Angular velocity of connecting rod, 3. Velocity of point E on connecting rod, 1.5 m from gudgeon pin 4.Velocities of rubbing at the pins of crank shaft, crank and crosshead when the diameters of the pins are 50mm, 60mm, 30mm respectively



(*a*) Space diagram. (*b*) Velocity diagram.

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$

$$v_D = \text{vector } od = 1.6 \text{ m/s}$$

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

28

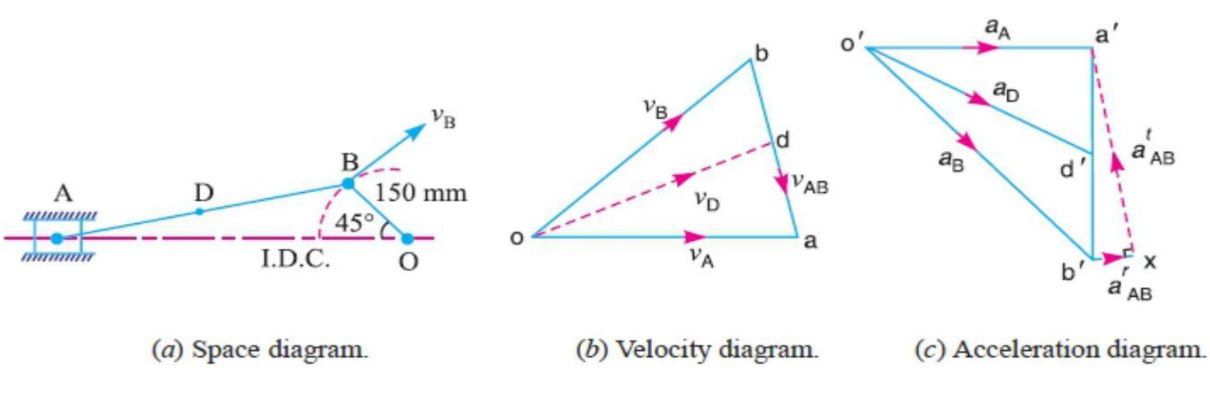
$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B)$$

Example *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :* **1.** *linear velocity and acceleration of the midpoint of the connecting rod, and* **2.** *angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

Solution. Given : $N_{BO} = 300$ r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; $OB = 150$ mm 0.15 m ; $BA = 600$ mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

vector $ob = v_{BO} = v_B = 4.713$ m/s

v_{AB} = vector $ba = 3.4$ m / s

Velocity of A, v_A = vector $oa = 4$ m / s

v_D = vector $od = 4.1$ m/s

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

vector $o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$

a_D = vector $o'd' = 117 \text{ m/s}^2$

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B)$$

$a^t_{AB} = 103 \text{ m/s}^2$

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B)$$

An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s². Find: ***1.*** *velocity of G and angular velocity of AB, and* ***2.*** *acceleration of G and angular acceleration of AB.*

Solution. Given : $\omega_{BC} = 75$ rad/s ; $\alpha_{BC} = 1200$ rad/s², $CB = 100$ mm $= 0.1$ m; $BA = 300$ mm $= 0.3$ m

We know that velocity of B with respect to C or velocity of B,

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \text{ m/s} \quad \text{...(Perpendicular to } BC)$$

Since the angular acceleration of the crankshaft, $\alpha_{BC} = 1200$ rad/s², therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \text{ m/s}^2$$

Note: When the angular acceleration is not given, then there will be no tangential component of the acceleration.

1. *Velocity of G and angular velocity of AB*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.6 (*a*). Now the velocity diagram, as shown in Fig. 8.6 (*b*), is drawn as discussed below:

1. Draw vector cb perpendicular to CB, to some suitable scale, to represent the velocity of B with respect to C or velocity of B (*i.e.* v_{BC} *or* v_B), such that

vector $cb = v_{BC} = v_B = 7.5$ m/s

2. From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B *i.e.* v_{AB}, and from point c, draw vector ca parallel to the path of motion of A (which is along AC) to represent the velocity of A *i.e.* v_A. The vectors ba and ca intersect at a.

3. Since the point G lies on AB, therefore divide vector ab at g in the same ratio as G divides AB in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector cg represents the velocity of G.

By measurement, we find that velocity of G,

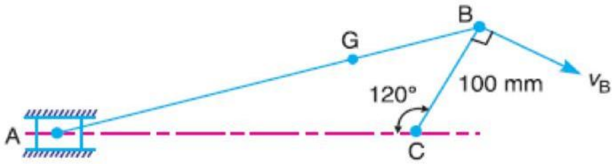
v_G = vector $cg = 6.8$ m/s **Ans.**

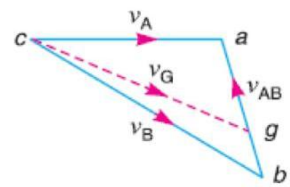
From velocity diagram, we find that velocity of A with respect to B,

$$v_{AB} = \text{vector } ba\ 324 \text{ m/s}$$

We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3 \text{ rad/s (Clockwise)} \textbf{ Ans.}$$





2. *Acceleration of G and angular acceleration of AB*

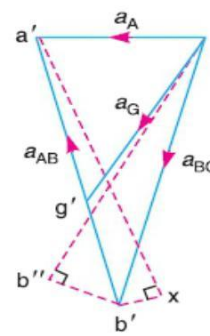
We know that radial component of the acceleration of B with respect to C,

$$^* a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \text{ m/s}^2$$

and radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.6 (c), is drawn as discussed below:



(c) Acceleration diagram.

Fig. 8.6

1. Draw vector $c'b''$ parallel to CB, to some suitable scale, to represent the radial component of the acceleration of B with respect to C, *i.e.* a^r_{BC}, such that

$$\text{vector } c'b'' = a^r_{BC} = 562.5 \text{ m/s}^2$$

2. From point b'', draw vector $b''b'$ perpendicular to vector $c'b''$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}, such that

$$\text{vector } b''b' = a^t_{BC} = 120 \text{ m/s}^2 \qquad \text{... (Given)}$$

3. Join $c'b'$. The vector $c'b'$ represents the total acceleration of B with respect to C *i.e.* a_{BC}.

4. From point b', draw vector $b'x$ parallel to BA to represent radial component of the acceleration of A with respect to B *i.e.* a^r_{AB} such that

$$\text{vector } b'x = a^r_{AB} = 53.3 \text{ m/s}^2$$

5. From point x, draw vector xa' perpendicular to vector $b'x$ or BA to represent tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}, whose magnitude is not yet known.

6. Now draw vector $c'a'$ parallel to the path of motion of A (which is along AC) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $c'a'$ intersect at a'. Join $b'a'$. The vector $b'a'$ represents the acceleration of A with respect to B *i.e.* a_{AB}.

7. In order to find the acceleratio of G, divide vector $a'b'$ in g' in the same ratio as G divides BA in Fig. 8.6 (a). Join $c'g'$. The vector $c'g'$ represents the acceleration of G.

By measurement, we find that acceleration of G,

$$a_G = \text{vector } c'g' = 414 \text{ m/s}^2 \textbf{ Ans.}$$

From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = \text{vector } xa' = 546 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

∴ Angular acceleration of AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820 \text{ rad/s}^2 \text{ (Clockwise)} \textbf{ Ans.}$$

Example 8.3. *In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s².*

The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: **1.** *the magnitude of vertical and horizontal component of the acceleration of the point B, and* **2.** *the angular acceleration of the links AB and BC.*

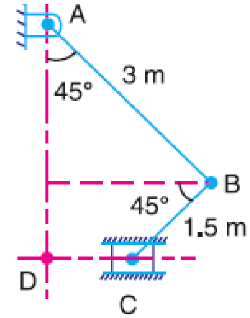


Fig. 8.7

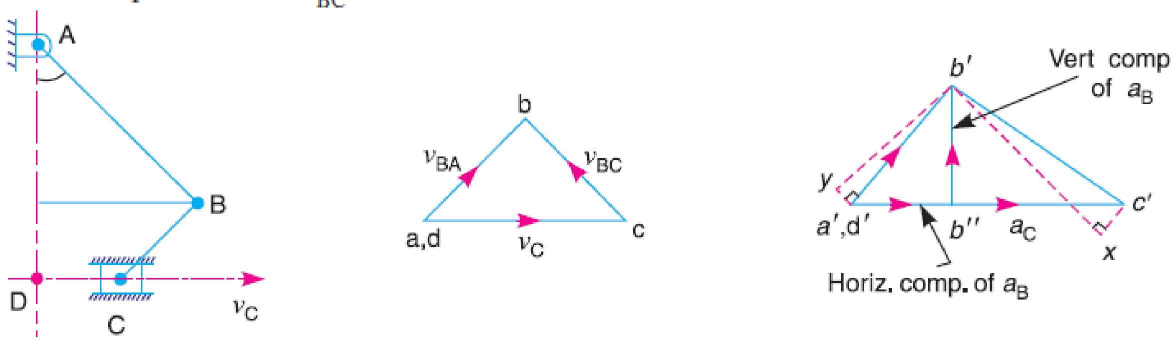
Solution. Given : $v_C = 1$ m/s ; $a_C = 2.5$ m/s²; $AB = 3$ m ; $BC = 1.5$ m

First of all, draw the space diagram, as shown in Fig. 8.8 (*a*), to some suitable scale. Now the velocity diagram, as shown in Fig. 8.8 (*b*), is drawn as discussed below:

1. Since the points *A* and *D* are fixed points, therefore they lie at one place in the velocity diagram. Draw vector *dc* parallel to *DC*, to some suitable scale, which represents the velocity of slider *C* with respect to *D* or simply velocity of *C*, such that

$$\text{vector } dc = v_{CD} = v_C = 1 \text{ m/s}$$

2. Since point *B* has two motions, one with respect to *A* and the other with respect to *C*, therefore from point *a*, draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A*, *i.e.* v_{BA} and from point *c* draw vector *cb* perpendicular to *CB* to represent the velocity of *B* with respect to *C* *i.e.* v_{BC}. The vectors *ab* and *cb* intersect at *b*.



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.8

By measurement, we find that velocity of *B* with respect to *A*,

By measurement, we find that velocity of *B* with respect to *A*,

$$v_{BA} = \text{vector } ab = 0.72 \text{ m/s}$$

and velocity of *B* with respect to *C*,

$$v_{BC} = \text{vector } cb = 0.72 \text{ m/s}$$

Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

Solution. Given : $\omega_{QP} = 10$ rad/s; $PQ = 62.5$ mm $= 0.0625$ m ; $QR = 175$ mm $= 0.175$ m ; $RS = 112.5$ mm $= 0.1125$ m ; $PS = 200$ mm $= 0.2$ m

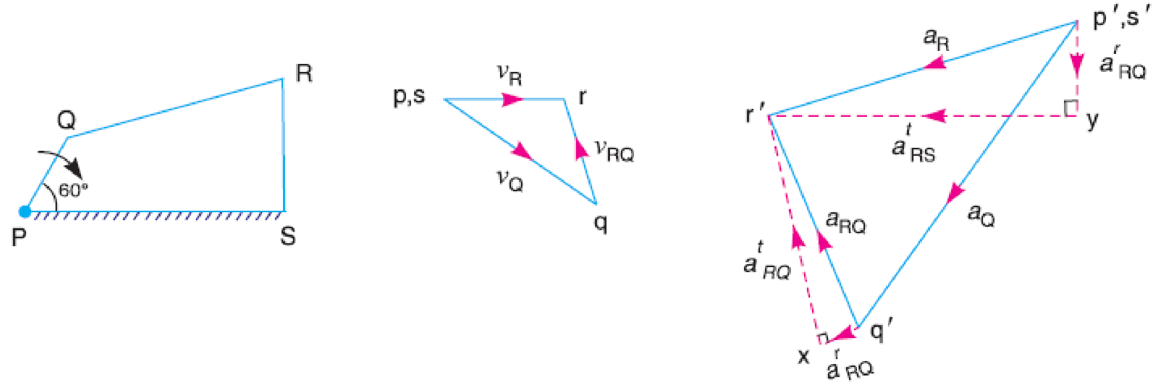
We know that velocity of *Q* with respect to *P* or velocity of *Q*,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

...(Perpendicular to *PQ*)

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale, as shown in Fig. 8.9 (*a*). Now the velocity diagram as shown in Fig. 8.9 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.9

1. Since *P* and *S* are fixed points, therefore these points lie at one place in velocity diagram. Draw vector *pq* perpendicular to *PQ*, to some suitable scale, to represent the velocity of *Q* with respect to *P* or velocity of *Q i.e.* v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point *q*, draw vector *qr* perpendicular to *QR* to represent the velocity of *R* with respect to *Q* (*i.e.* v_{RQ}) and from point *s*, draw vector *sr* perpendicular to *SR* to represent the velocity of *R* with respect to *S* or velocity of *R* (*i.e.* v_{RS} or v_R). The vectors *qr* and *sr* intersect at *r*. By measurement, we find that

$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

We know that angular velocity of link *QR*,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)} \textbf{ Ans.}$$

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. 8.9 (*c*) is drawn as follows :

1. Since P and S are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $p'q'$ parallel to PQ, to some suitable scale, to represent the radial component of acceleration of Q with respect to P or acceleration of Q *i.e* a^r_{QP} or a_Q such that

$$\text{vector } p'q' = a^r_{QP} = a_Q = 6.25 \text{ m/s}^2$$

2. From point q', draw vector $q'x$ parallel to QR to represent the radial component of acceleration of R with respect to Q *i.e.* a^r_{RQ} such that

$$\text{vector } q'x = a^r_{RQ} = 0.634 \text{ m/s}^2$$

3. From point x, draw vector xr' perpendicular to QR to represent the tangential component of acceleration of R with respect to Q *i.e* a^t_{RQ} whose magnitude is not yet known.

4. Now from point s', draw vector $s'y$ parallel to SR to represent the radial component of the acceleration of R with respect to S *i.e.* a^r_{RS} such that

$$\text{vector } s'y = a^r_{RS} = 1.613 \text{ m/s}^2$$

5. From point y, draw vector yr' perpendicular to SR to represent the tangential component of acceleration of R with respect to S *i.e.* a^t_{RS}.

6. The vectors xr' and yr' intersect at r'. Join $p'r$ and $q'r'$. By measurement, we find that

$$a^t_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a^t_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link QR,

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

and angular acceleration of link RS,

$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Example 8.6. *In the mechanism, as shown in Fig. 8.12, the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm.*

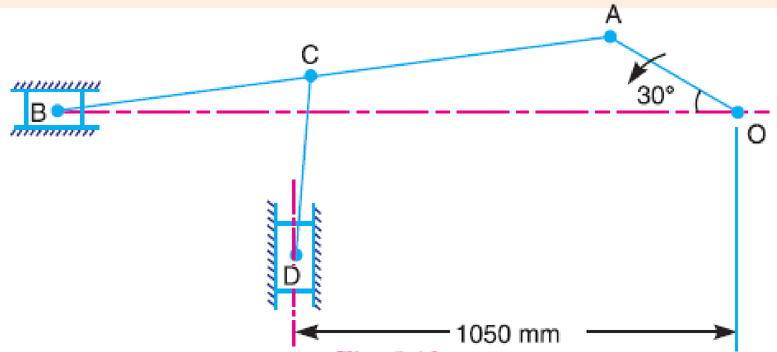


Fig. 8.12

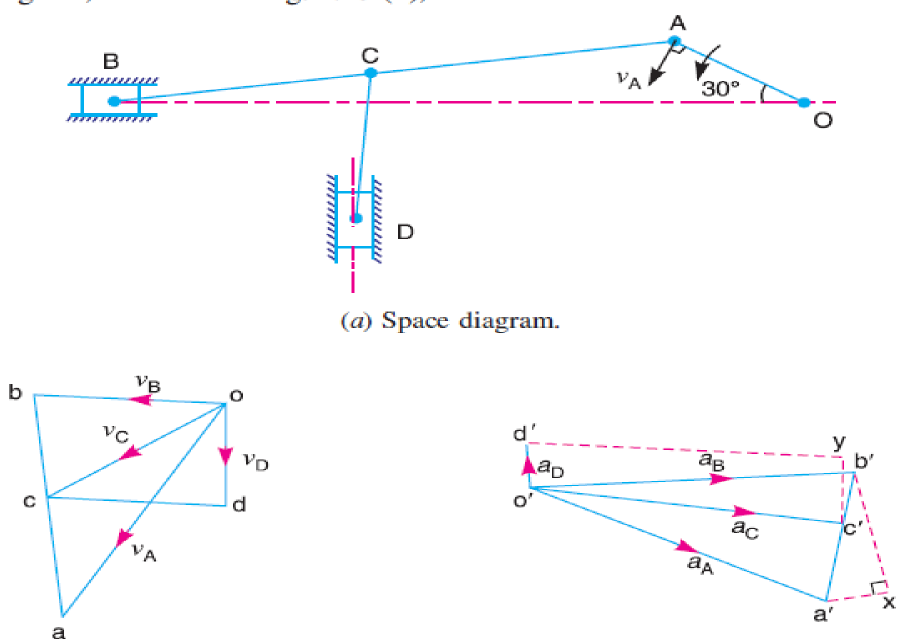
For the given configuration, determine : 1. velocities of sliding at B and D, 2. angular velocity of CD, 3. linear acceleration of D, and 4. angular acceleration of CD.

Solution. Given : N_{AO} = 20 r.p.m. or $\omega_{AO} = 2\pi \times 20/60 = 2.1$ rad/s ; OA = 300 mm = 0.3 m ; AB = 1200 mm = 1.2 m ; $BC = CD$ = 450 mm = 0.45 m

U

1. ***Velocities of sliding at B and D***

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.13 (*a*). Now the velocity diagram, as shown in Fig. 8.13 (*b*), is drawn as discussed below:



(*a*) Space diagram.

(*b*) Velocity diagram.

(*c*) Acceleration diagram.

Fig. 8.13

1. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* (or simply velocity of *A*), such that

$$\text{vector } oa = v_{AO} = v_A = 0.63 \text{ m/s}$$

2. From point *a*, draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* (*i.e.* v_{BA}) and from point *o* draw vector *ob* parallel to path of motion *B* (which is along *BO*) to represent the velocity of *B* with respect to *O* (or simply velocity of *B*). The vectors *ab* and *ob* intersect at *b*.

3. Divide vector *ab* at *c* in the same ratio as *C* divides *AB* in the space diagram. In other

4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C (*i.e.* v_{DC}) and from point o draw vector od parallel to the path of motion of D (which along the vertical direction) to represent the velocity of D.

By measurement, we find that velocity of sliding at B,

$$v_B = \text{vector } ob = 0.4 \text{ m/s} \quad \textbf{Ans.}$$

and velocity of sliding at D,

$$v_D = \text{vector } od = 0.24 \text{ m/s} \quad \textbf{Ans.}$$

2. Angular velocity of CD

By measurement from velocity diagram, we find that velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 0.37 \text{ m/s}$$

∴ Angular velocity of CD,

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise). } \textbf{Ans.}$$

3. Linear acceleration of D

We know that the radial component of the acceleration of A with respect to O or acceleration of A,

$$a^r_{AO} = a_A = \frac{v^2_{AO}}{OA} = \omega^2_{AO} \times OA = (2.1)^2 \times 0.3 = 1.323 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.54)^2}{1.2} = 0.243 \text{ m/s}^2$$

...(By measurement, $v_{BA} = 0.54$ m/s)

Radial component of the acceleration of D with respect to C,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(0.37)^2}{0.45} = 0.304 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.13 (*c*), is drawn as discussed below:

1. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 1.323 \text{ m/s}^2$$

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'x = a^r_{BA} = 0.243 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (*i.e.* a^t_{BA}) whose magnitude is not yet known.

4. From point o', draw vector $o'b'$ parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' *and* $o'b'$ intersect at b'. Join $a'b'$. The vector $a'b'$ represents the acceleration of B with respect to A.

5. Divide vector $a'b'$ at c' in the same ratio as C divides AB in the space diagram. In other words,

$$BC / BA = b'c'/b'a'$$

6. From point c', draw vector $c'y$ parallel to CD to represent the radial component of the acceleration of D with respect to C, such that

4. Angular acceleration of CD

From the acceleration diagram, we find that the tangential component of the acceleration of D with respect to C,

$$a^t_{DC} = \text{vector } yd' = 1.28 \text{ m/s}^2$$

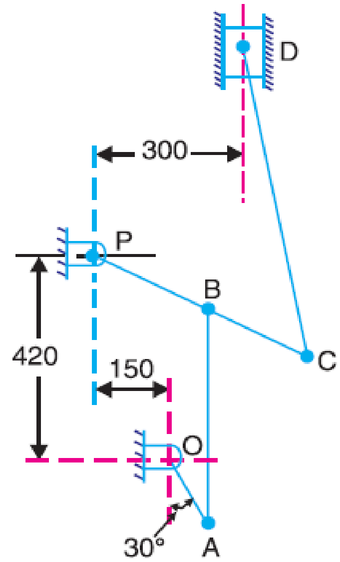
...(By measurement)

∴ Angular acceleration of *CD*,

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2 \text{ (Clockwise) } \textbf{Ans.}$$

Example 8.7. *Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown in Fig. 8.14.*

The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150 mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.



All dimensions in mm.

Fig. 8.14

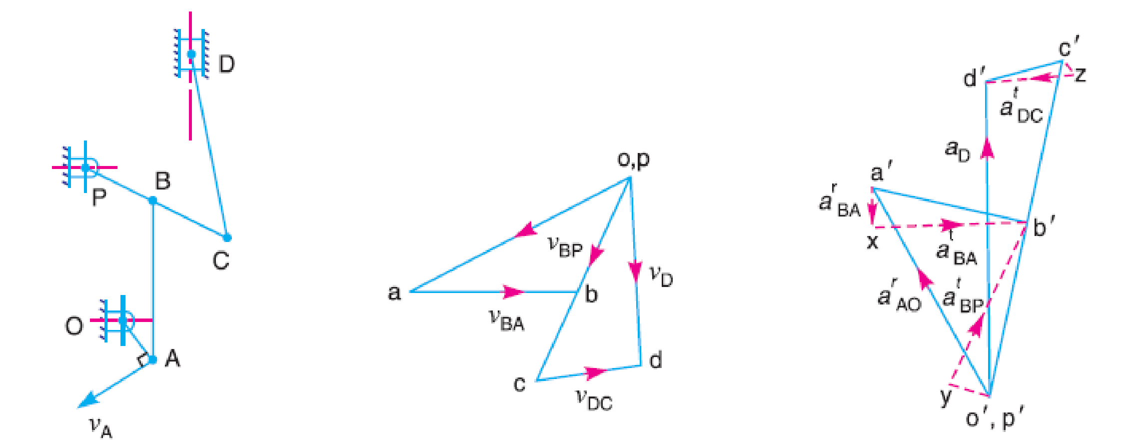
Solution. Given: $N_{AO} = 180$ r.p.m., or $\omega_{AO} = 2\pi \times 180/60 = 18.85$ rad/s ; $OA = 150$ mm $= 0.15$ m ; $AB = 450$ mm $= 0.45$ m ; $PB = 240$ mm $= 0.24$ m ; $CD = 660$ mm $= 0.66$ m

We know that velocity of *A* with respect to *O* or velocity of *A*,

$$v_{AO} = v_A = \omega_{AO} \times OA$$
$$= 18.85 \times 0.15 = 2.83 \text{ m/s}$$

...(Perpendicular to *OA*)

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.15 (*a*). Now the velocity diagram, as shown in Fig. 8.15 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.15

1. Since *O* and *P* are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or velocity of *A* (*i.e.* v_{AO} or v_A), such that

$$\text{vector } oa = v_{AO} = v_A = 2.83 \text{ m/s}$$

2. Since the point *B* moves with respect to *A* and also with respect to *P*, therefore draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A i.e.* v_{BA}, and from point *p* draw vector *pb* perpendicular to *PB* to represent the velocity of *B* with respect to *P* or velocity of *B* (*i.e.* v_{BP} or v_B). The vectors *ab* and *pb* intersect at *b*.

3. Since the point *C* lies on *PB* produced, therefore divide vector *pb* at *c* in the same ratio as *C* divides *PB* in the space diagram. In other words, *pb/pc* = *PB/PC*.

4. From point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C and from point o draw vector od parallel to the path of motion of the slider D (which is vertical), to represent the velocity of D, *i.e.* v_D.

By measurement, we find that velocity of the slider D,

$$v_D = \text{vector } od = 2.36 \text{ m/s}$$

Velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 1.2 \text{ m/s}$$

Velocity of B with respect to A,

$$v_{BA} = \text{vector } ab = 1.8 \text{ m/s}$$

and velocity of B with respect to P, v_{BP} = vector pb = 1.5 m/s

Acceleration of the slider D

We know that radial component of the acceleration of A with respect to O or acceleration of A,

$$a^r_{AO} = a_A = \omega^2_{AO} \times AO = (18.85)^2 \times 0.15 = 53.3 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(1.8)^2}{0.45} = 7.2 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to P,

$$a^r_{BP} = \frac{v^2_{BP}}{PB} = \frac{(1.5)^2}{0.24} = 9.4 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to C,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(1.2)^2}{0.66} = 2.2 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.15 (c), is drawn as discussed below:

1. Since O and P are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or the acceleration of A (*i.e.* a^r_{AO} or a_A), such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 53.3 \text{ m/s}^2$$

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A (*i.e.* a^r_{BA}), such that

$$\text{vector } a'x = a^r_{BA} = 7.2 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to the vector $a'x$ or AB to represent the tangential component of the acceleration of B with respect to A *i.e.* a^t_{BA} whose magnitude is yet unknown.

4. Now from point p', draw vector $p'y$ parallel to PB to represent the radial component of the acceleration of B with respect to P (*i.e.* a^r_{BP}), such that

$$\text{vector } p'y = a^r_{BP} = 9.4 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to vector $b'y$ or PB to represent the tangential component of the acceleration of B, *i.e.* a^t_{BP}. The vectors xb' and yb' intersect at b'. Join $p'b'$. The vector $p'b'$ represents the acceleration of B, *i.e.* a_B.

6. Since the point C lies on PB produced, therefore divide vector $p'b'$ at c' in the same ratio as C divides PB in the space diagram. In other words, $p'b'/p'c' = PB/PC$

7. From point c', draw vector $c'z$ parallel to CD to represent the radial component of the acceleration of D with respect to C *i.e.* a^r_{DC}, such that

$$\text{vector } c'z = a^r_{DC} = 2.2 \text{ m/s}^2$$

8. From point z, draw vector zd' perpendicular to vector $c'z$ or CD to represent the tangential component of the acceleration of D with respect to C *i.e.* a^t_{DC}, whose magnitude is yet unknown.

9. From point o', draw vector $o'd'$ parallel to the path of motion of D (which is vertical) to represent the acceleration of D, *i.e.* a_D. The vectors zd' and $o'd'$ intersect at d'. Join $c'd'$.

By measurement, we find that acceleration of D,

$$a_D = \text{vector } o'd' = 69.6 \text{ m/s}^2 \quad \textbf{Ans.}$$

Angular acceleration of CD

From acceleration diagram, we find that tangential component of the acceleration of D with respect to C,

$$a^t_{DC} = \text{vector } zd' = 17.4 \text{ m/s}^2 \quad \text{...(By measurement)}$$

We know that angular acceleration of CD,

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2 \text{ (Anticlockwise)} \quad \textbf{Ans.}$$

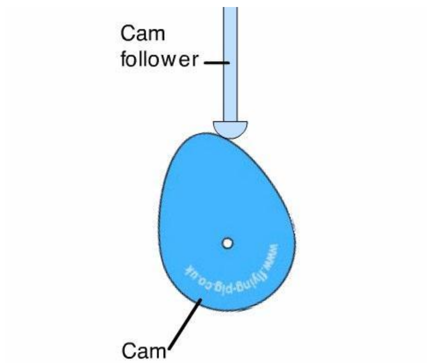
UNIT III KINEMATICS OF CAM MECHANISMS

3.1 INTRODUCTION

A cam is a mechanical device used to transmit motion to a follower by direct contact. The driver is called the cam and the driven member is called the follower. In a cam follower pair, the cam normally rotates while the follower may translate or oscillate.

Cams:

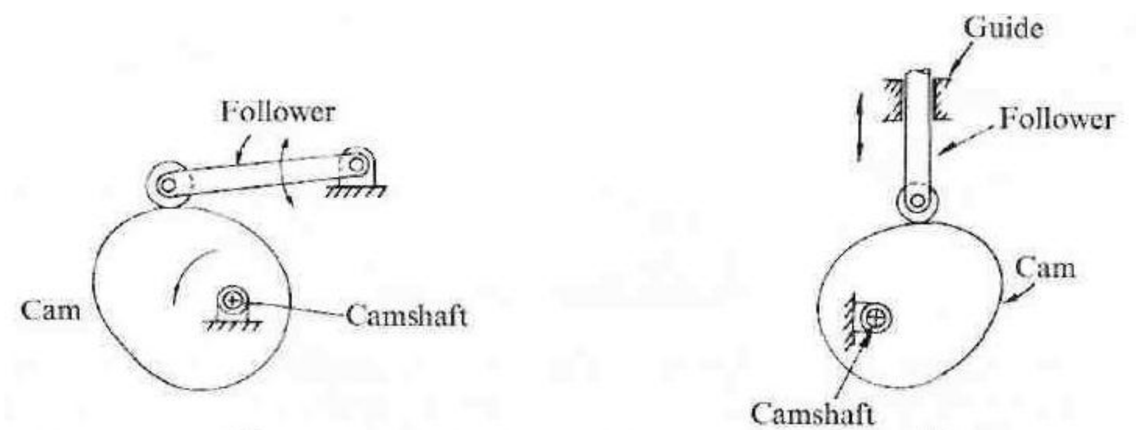
Type of cams, Type of followers, Displacement, Velocity and acceleration time curves for cam profiles, Disc cam with reciprocating follower having knife edge, roller follower, Follower motions including SHM, Uniform velocity, Uniform acceleration and retardation and Cycloidal motion.



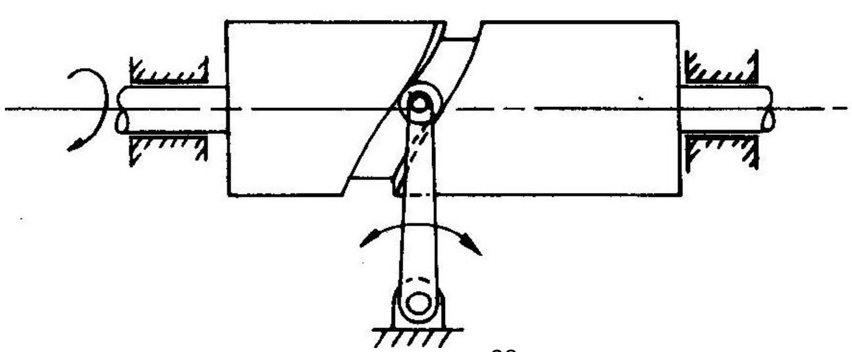
3.1.1 Types of cams

Cams can be classified based on their physical shape.

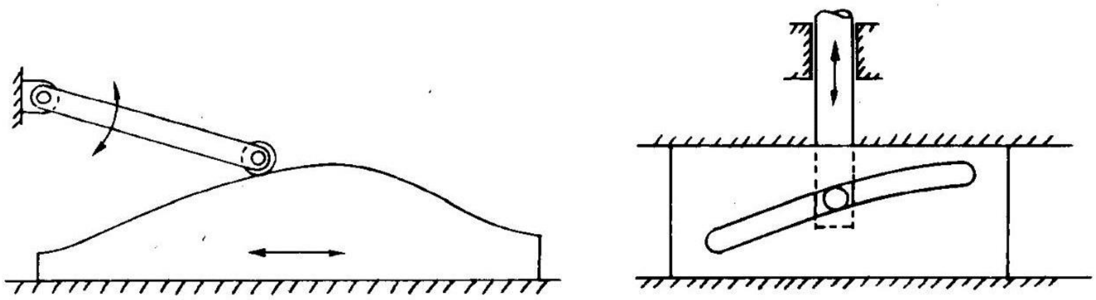
a) Disk or plate cam The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis of rotation of the cam shaft and is held in contact with the cam by springs or gravity.



b) Cylindrical cam: The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder.



c) Translating cam. The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate (Fig.3.3a) or reciprocate (Fig.3.3b). The contour or the shape of the groove is determined by the specified motion of the follower.

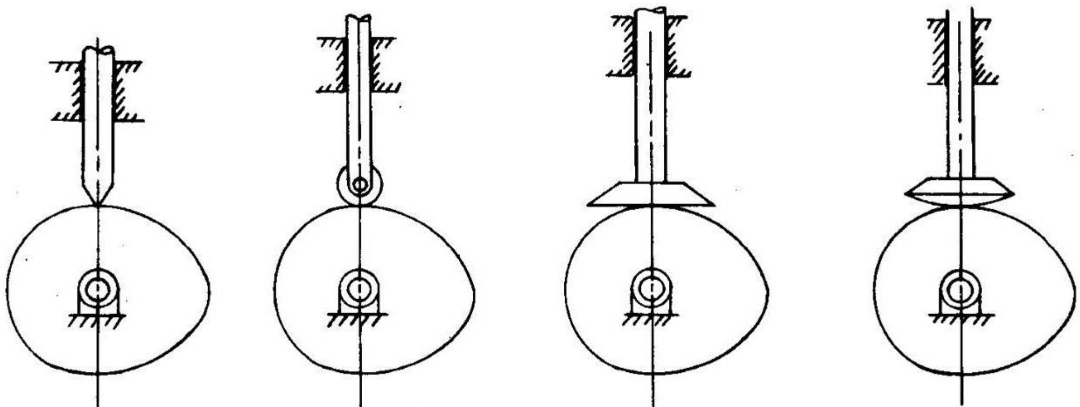


(a) (b)

Types of followers (Fig3.4):

(i)Based on surface in contact.

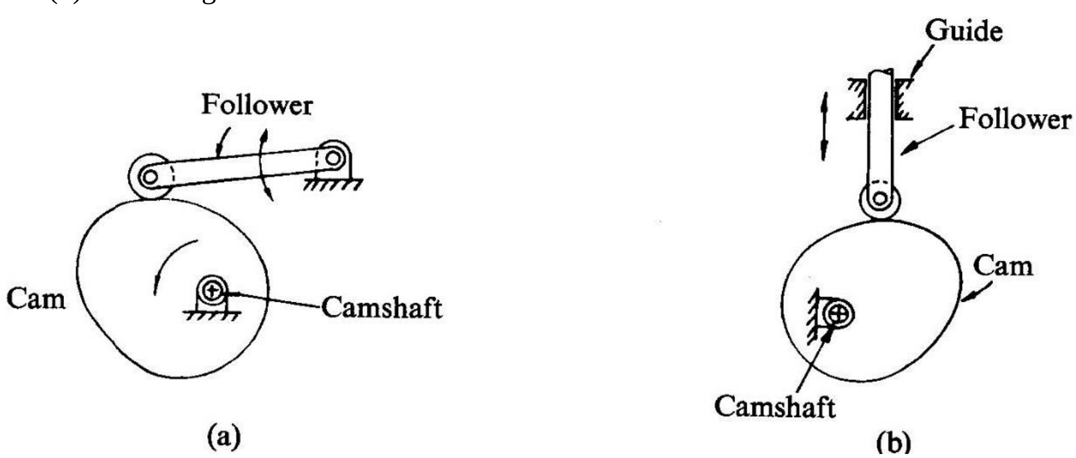
(a)Knife edge follower
(b)Roller follower
(c)Flat faced follower
(d)Spherical follower



(a) (b) (c) (d)

(ii)Based on type of motion

(a)Oscillating follower
(b)Translating follower



(a) (b)

(a)Radial follower: The lines of movement of in-line cam followers pass through the centre of the camshafts

(b) Off-set follower: For this type the lines of movement are offset from the centres of the camshafts

Cam Nomenclature (Fig.3.7):

Cam Profile The contour of the working surface of the cam.

Tracer Point The point at the knife edge of a follower, or the centre of a roller, or the centre of a spherical face.

Pitch Curve The path of the tracer point.

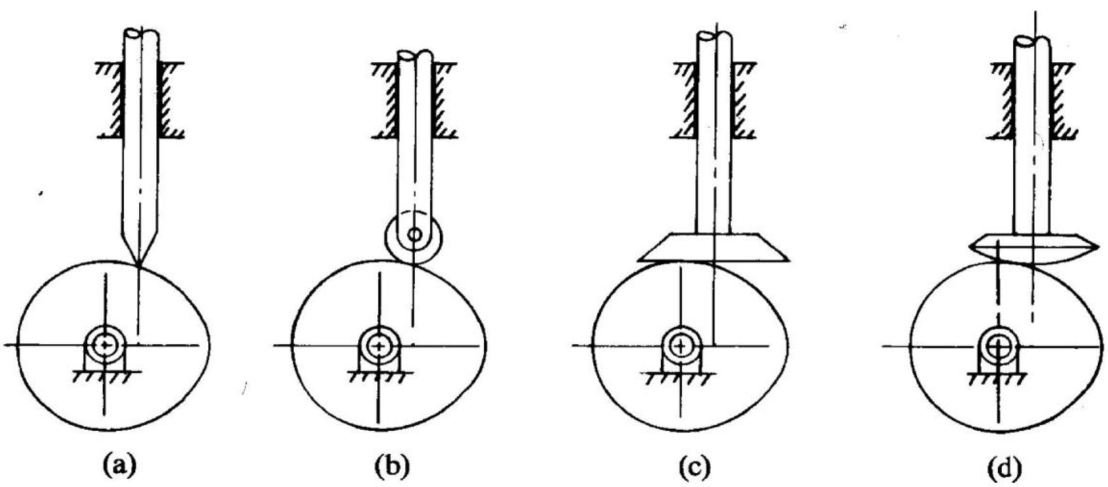
Base Circle The smallest circle drawn, tangential to the cam profile, with its centre on the axis of the camshaft. The size of the base circle determines the size of the cam.

Prime Circle The smallest circle drawn, tangential to the pitch curve, with its centre on the axis of the cam shaft.

Pressure Angle The angle between the normal to the pitch curve and the direction of motion of the follower at the point of contact

3.2 Types of follower motion:

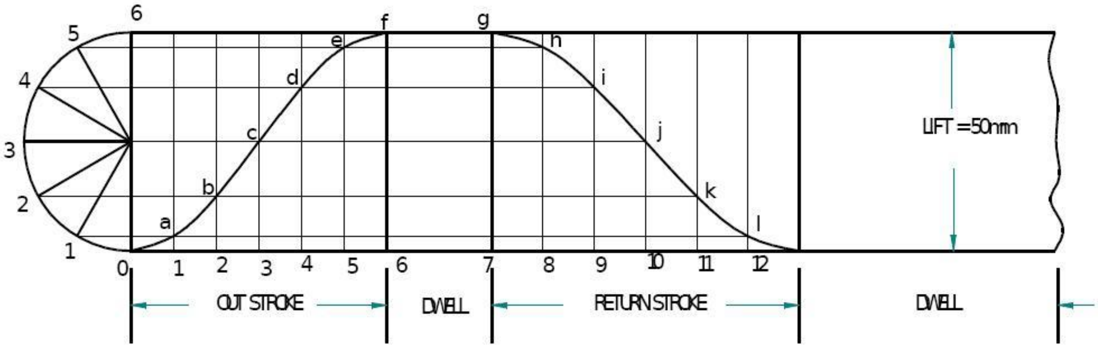
(a)Uniform velocity
(b)Modified uniform velocity
(c)Uniform acceleration and deceleration
(d)Simple harmonic motion
(e)Cycloidal motion



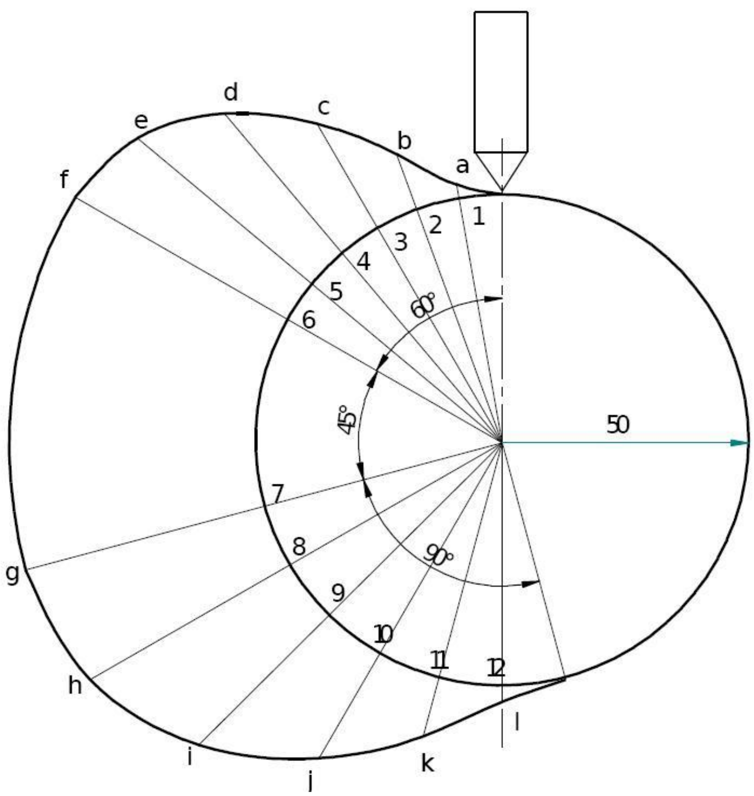
1. ***Draw the cam profile for following conditions:***

Follower type=Knife edged in-line; lift=50mm; base circle radius =50mm; outstroke with SHM,for60^0 Cam rotation; dwellfor45^{0}camrotation; return stroke with SHM, for90^{o}cam rotation; dwell for the remaining period. Draw the cam profile for the same operating condition so f with the follower offset by 10mm to the left of cam centre.

Displacement diagram:

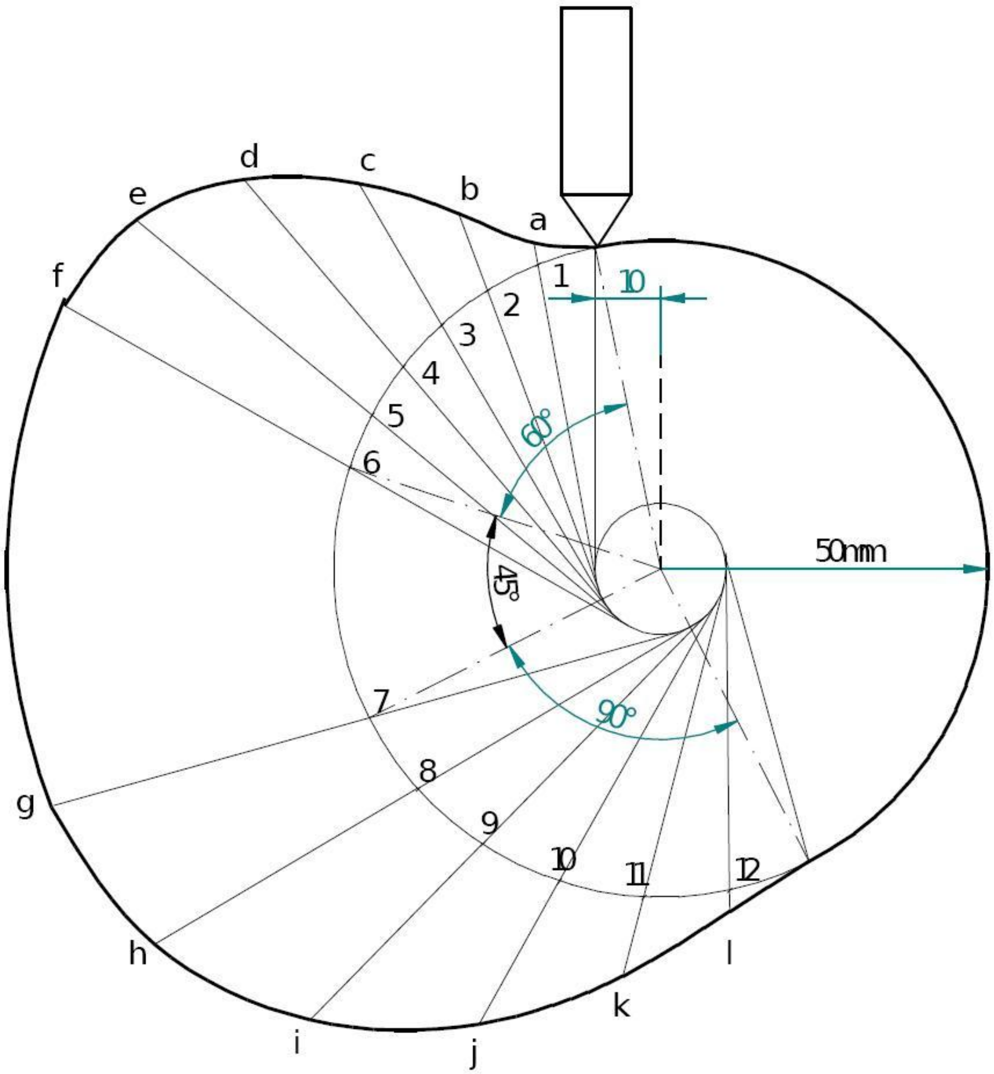


Cam profile:



46

Cam profile with 10 mm offset:

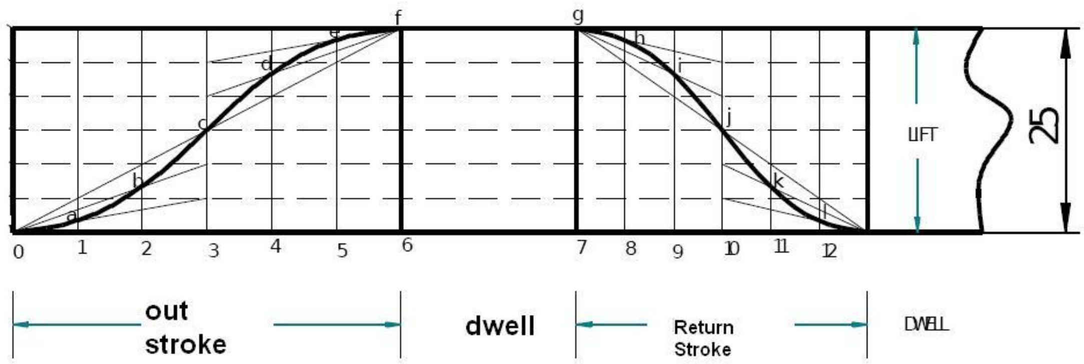


***2.* Draw the cam profile for following conditions:**

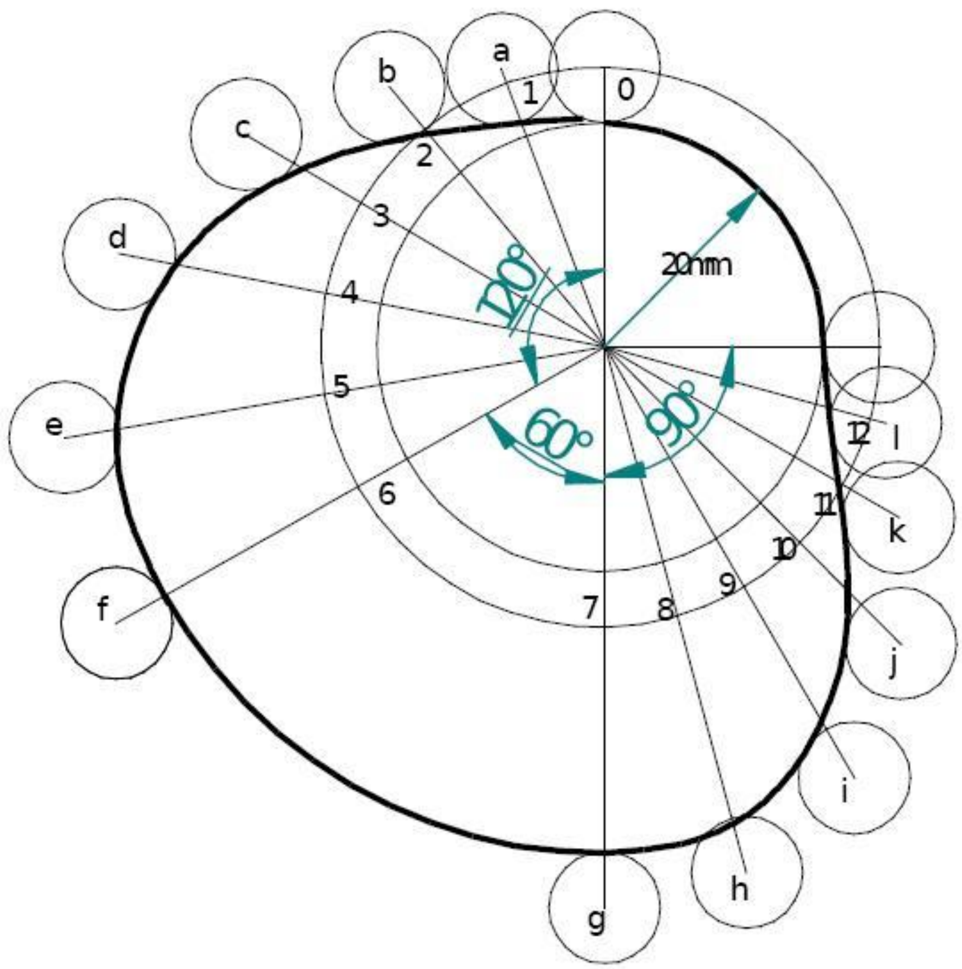
Follower type=roller follower, in-line; lift=25mm; base circle radius=20mm; roller radius= 5mm; out stroke with Uniform acceleration and retardation, for 120^0 cam rotation; dwell for 60^0 cam rotation; return stroke with Uniform acceleration and retardation, for90^0 cam rotation; dwell for the remaining period (4) Draw the cam profile for conditions same with follower off set to right of cam centre by 5mm and cam rotating counter clockwise.

52

Displacement Diagram:

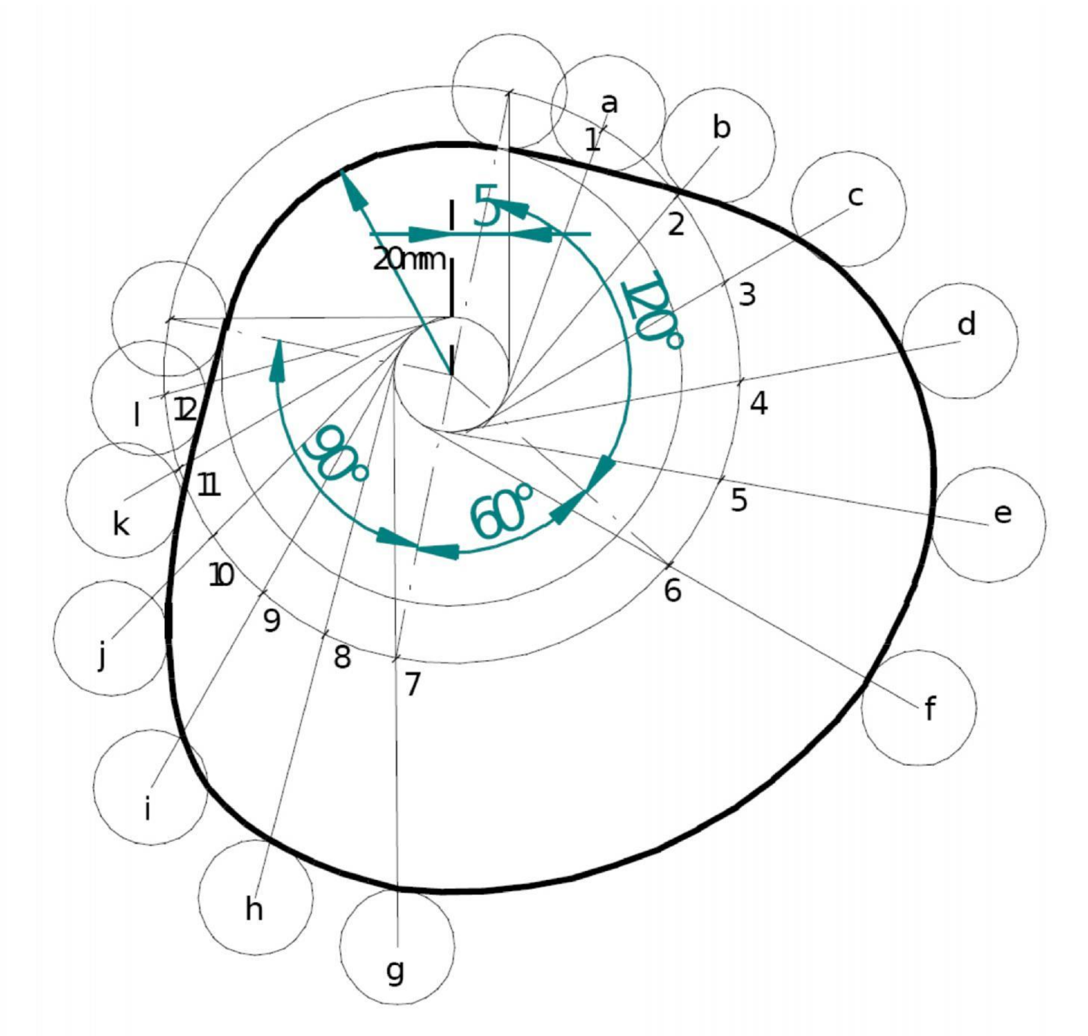


Cam profile;



52

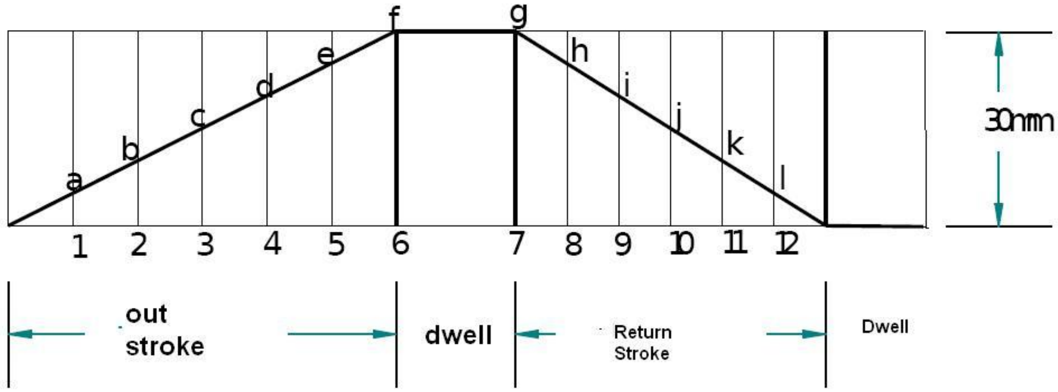
Cam profile with 5 mm offset



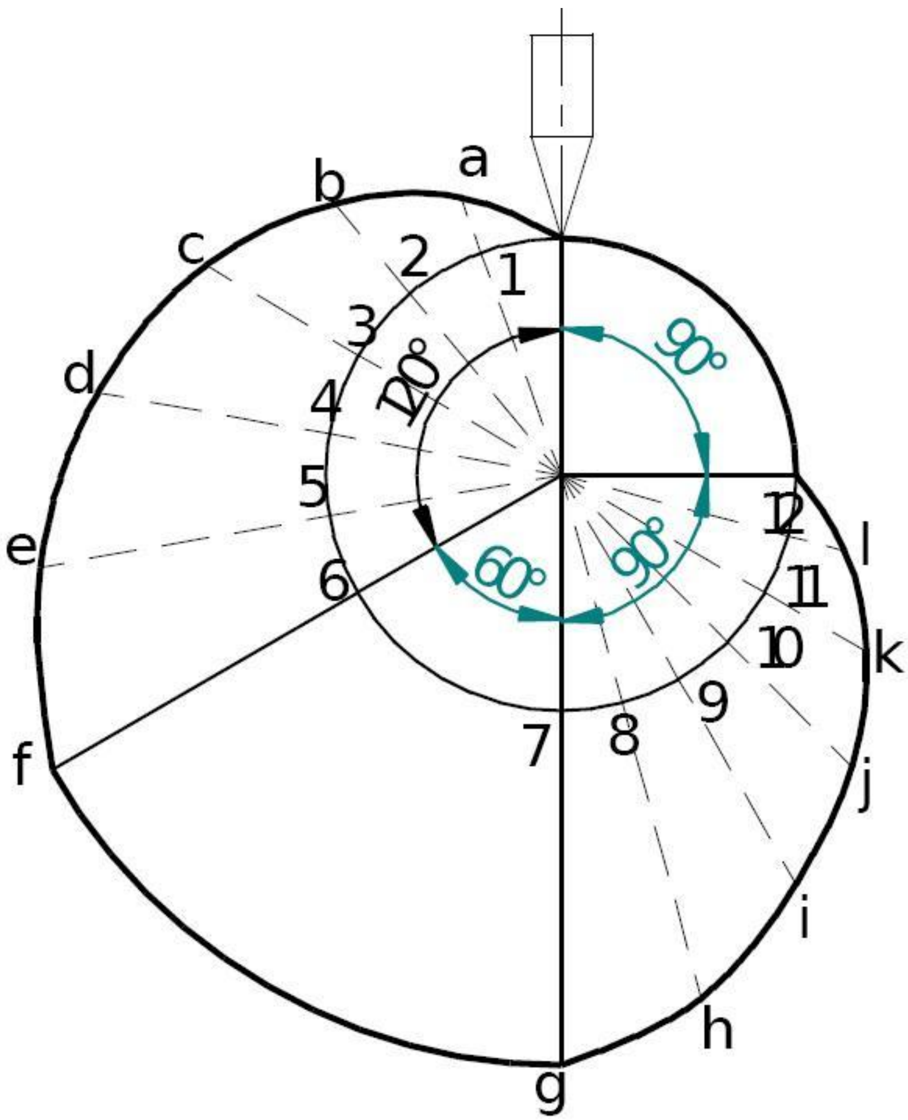
(3) Draw the cam profile for following conditions:

Follower type=knife edge d follower, in line; lift=30mm; base circle radius =20mm; outstroke with uniform velocity in120^{0}of cam rotation; dwell for 60^0; return stroke with uniform velocity, during90^{0}ofcam rotation; dwell for the remaining period.

Displacement Diagram



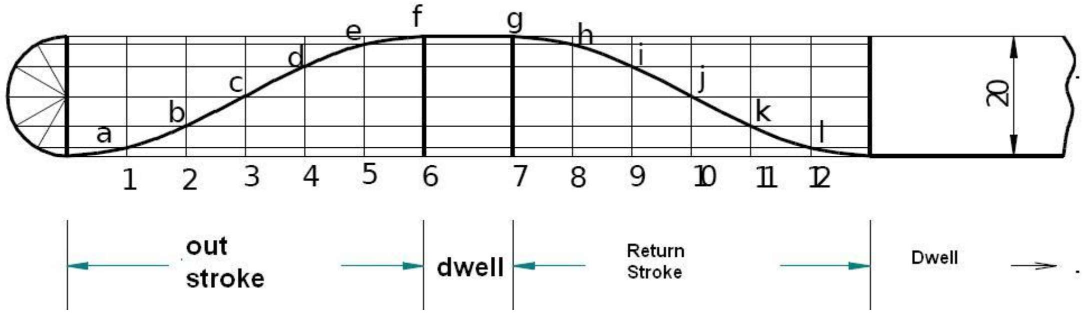
Cam profile



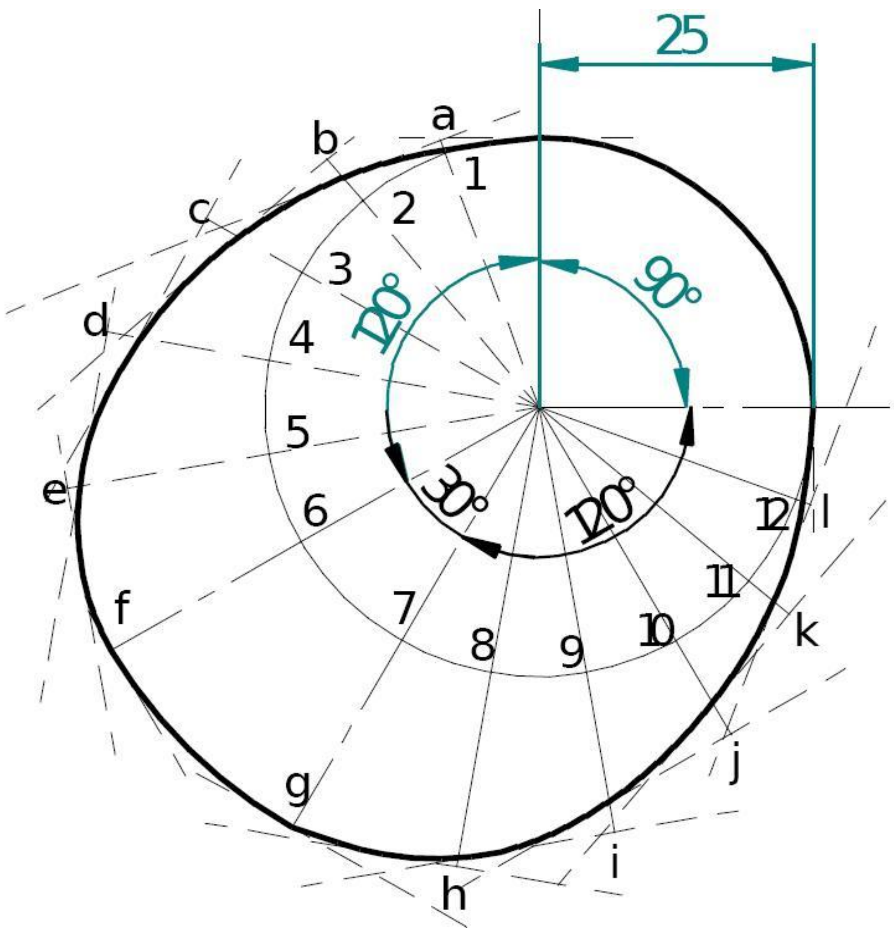
52

(4) Draw the cam profile for following conditions: Follower type = flat faced follower, inline; follower rises by 20mm with SHM in 120^0 of cam rotation, dwells for 30^0 of cam rotation; returns with SHM in 120^0 of cam rotation and dwells during the remaining period. Base circle radius =25mm.

Displacement Diagram:



Cam profile



52

3.5 Layout of plate cam profiles:

- Drawing the displacement diagrams for the different kinds of the motions and the plate cam profiles for these different motions and different followers.
- SHM, Uniform velocity, Uniform acceleration and retardation and Cycloidal motions
- Knife-edge, Roller, Flat-faced and Mushroom followers.

3.6Derivatives of Follower motion:

- Velocity and acceleration of the followers for various types of motions.
- Calculation of Velocity and acceleration of the followers for various types of motions.

3.7Circular arc and Tangent cam s:

- Circular arc
- Tangent cam

Standard cam motion:

- Simple Harmonic Motion
- Uniform velocity motion
- Uniform acceleration and retardation motion
- Cycloidal motion

3.8 Pressure angle and undercutting:

- Pressure angle
- Undercutting

. A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :

1. To raise the valve through 50 mm during 120° rotation of the cam ;

2. To keep the valve fully raised through next 30°;

3. To lower the valve during next 60°; and

4. To keep the valve closed during rest of the revolution i.e. 150° ;

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.

Draw the profile of the cam when (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and (b) the line of the stroke is offset 15 mm from the axis of the cam shaft.

The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m.

Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

Solution. Given : $S = 50$ mm $= 0.05$ m ; $\theta_O = 120° = 2\pi/3$ rad $= 2.1$ rad ; $\theta_R = 60° = \pi/3$ rad $= 1.047$ rad ; $N = 100$ r.p.m.

(a) *Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft*

The profile of the cam, as shown in Fig. 20.17, is drawn as discussed in the following steps :

1. Draw a base circle with centre O and radius equal to the minimum radius of the cam (*i.e.* 25 mm).
2. Draw a prime circle with centre O and radius,

   $$OA = \text{Min. radius of cam} + \frac{1}{2}\text{ Dia. of roller} = 25 + \frac{1}{2} \times 20 = 35 \text{ mm}$$
3. Draw angle $AOS = 120°$ to represent raising or out stroke of the valve, angle $SOT = 30°$ to represent dwell and angle $TOP = 60°$ to represent lowering or return stroke of the valve.
4. Divide the angular displacements of the cam during raising and lowering of the valve (*i.e.* angle AOS and TOP) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3, etc. with the centre O and produce the lines beyond prime circle as shown in Fig. 20.17.
6. Set off $1B$, $2C$, $3D$ etc. equal to the displacements from displacement diagram.
7. Join the points A, B, C ... N, P, A. The curve drawn through these points is known as *pitch curve.*

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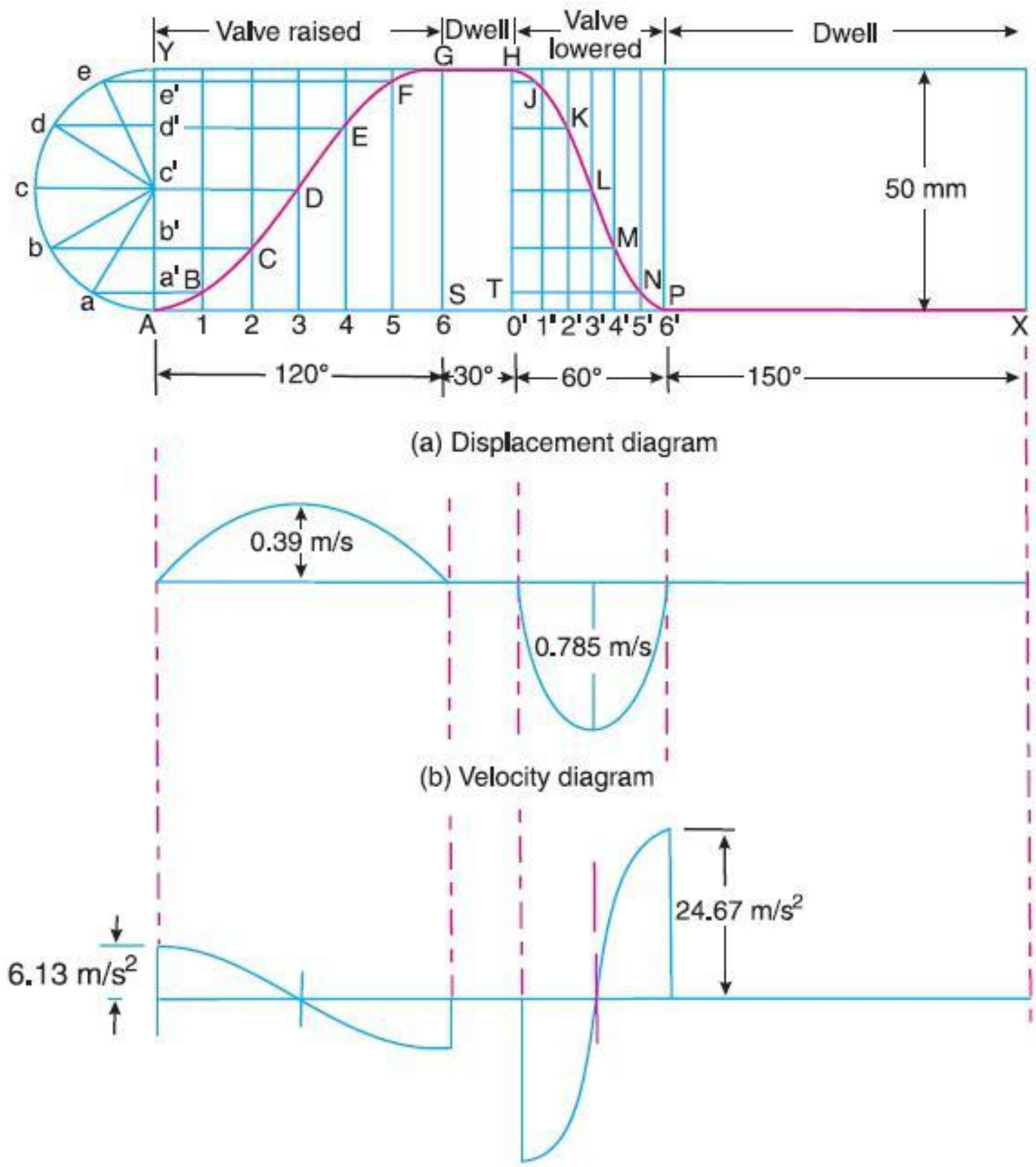
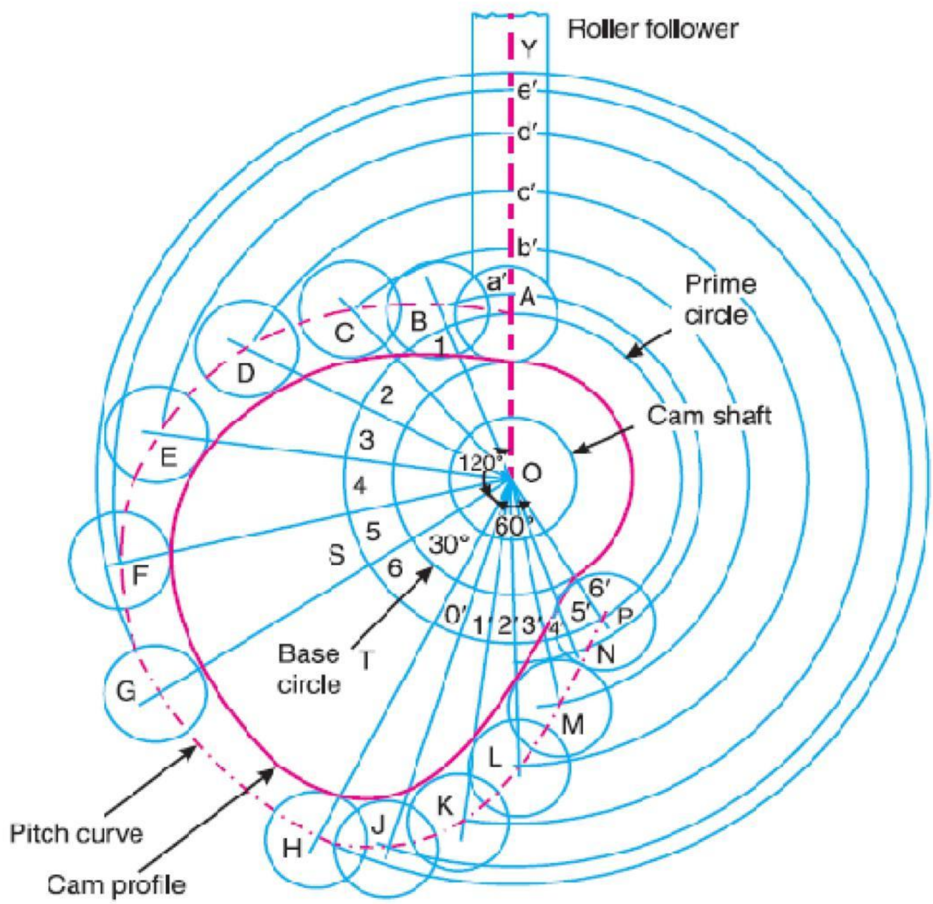
7. Join the points A, B, C ... N, P, A. The curve drawn through these points is known as *pitch curve.*



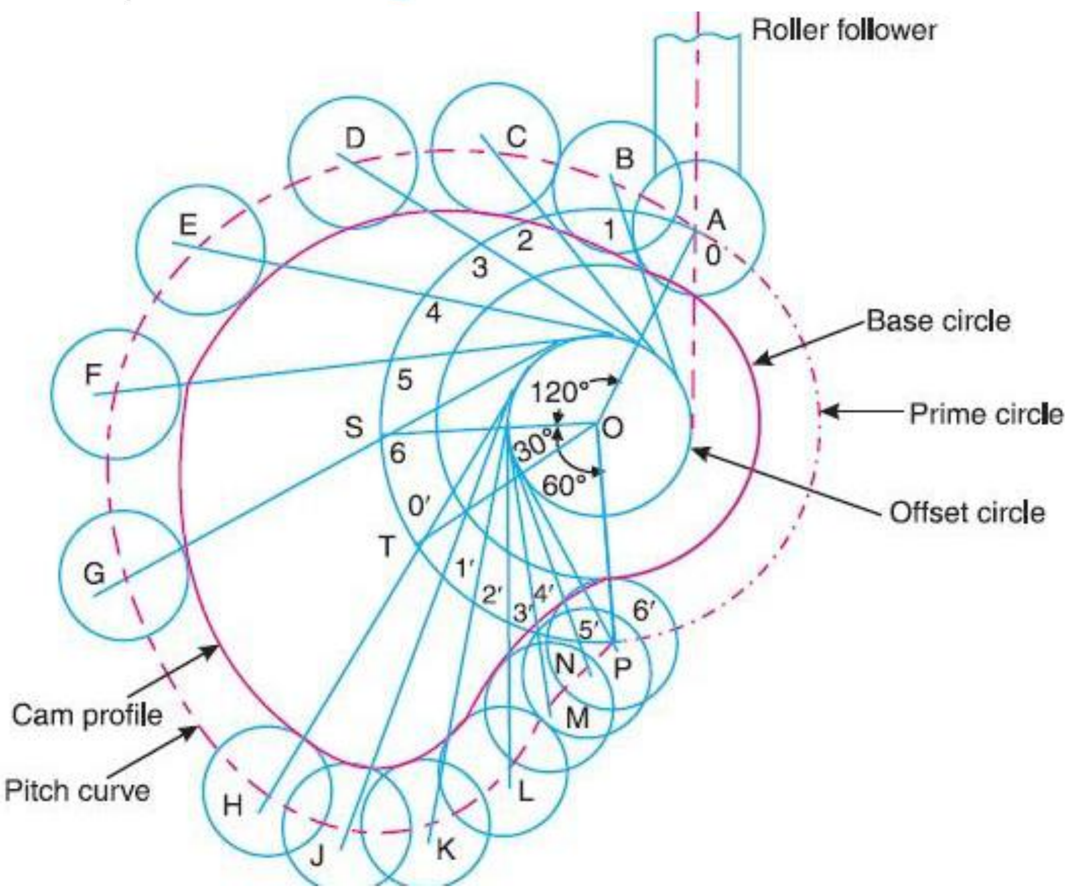
(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram



Roller follower
Y
e′
d′
c′
b′
a′
A
B
C
D
E
F
G
H
J
K
L
M
N
P
1
2
3
4
5
6
0′
1′
2′
3′
4′
5′
6′
S
T
O
120°
60°
30°
Prime circle
Cam shaft
Base circle
Pitch curve
Cam profile



Roller follower
A
B
C
D
E
F
G
H
J
K
L
M
N
P
0
1
2
3
4
5
6
0′
1′
2′
3′
4′
5′
6′
S
T
O
120°
30°
60°
Base circle
Prime circle
Offset circle
Cam profile
Pitch curve

(b) Profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft

The profile of the cam when the line of stroke is offset from the axis of the cam shaft, as shown in Fig. 20.18, may be drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius equal to 25 mm.
2. Draw a prime circle with centre *O* and radius *OA* = 35 mm.
3. Draw an off-set circle with centre *O* and radius equal to 15 mm.
4. Join *OA*. From *OA* draw the angular displacements of cam *i.e.* draw angle *AOS* = 120°, angle *SOT* = 30° and angle *TOP* = 60°.
5. Divide the angular displacements of the cam during raising and lowering of the valve into the same number of equal even parts (*i.e.* six parts) as in displacement diagram.
6. From points 1, 2, 3 .... etc. and $0', 1', 3'$, ...etc. on the prime circle, draw tangents to the offset circle.
7. Set off 1*B*, 2*C*, 3*D*... etc. equal to displacements as measured from displacement diagram.
8. By joining the points *A*, *B*, *C* ... *M*, *N*, *P*, with a smooth curve, we get a ***pitch curve.***
9. Now *A*, *B*, *C*...etc. as centre, draw circles with radius equal to the radius of roller.
10. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.18. This is the required profile of the cam.

Maximum acceleration of the valve rod

We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

We also know that maximum velocity of the valve rod to raise valve,

$$v_O = \frac{\pi \omega S}{2\theta_O} = \frac{\pi \times 10.47 \times 0.05}{2 \times 2.1} = 0.39 \text{ m/s}$$

and maximum velocity of the valve rod to lower the valve,

$$v_R = \frac{\pi \omega S}{2\theta_R} = \frac{\pi \times 10.47 \times 0.05}{2 \times 1.047} = 0.785 \text{ m/s}$$

The velocity diagram for one complete revolution of the cam is shown in Fig. 20.16 (***b***).

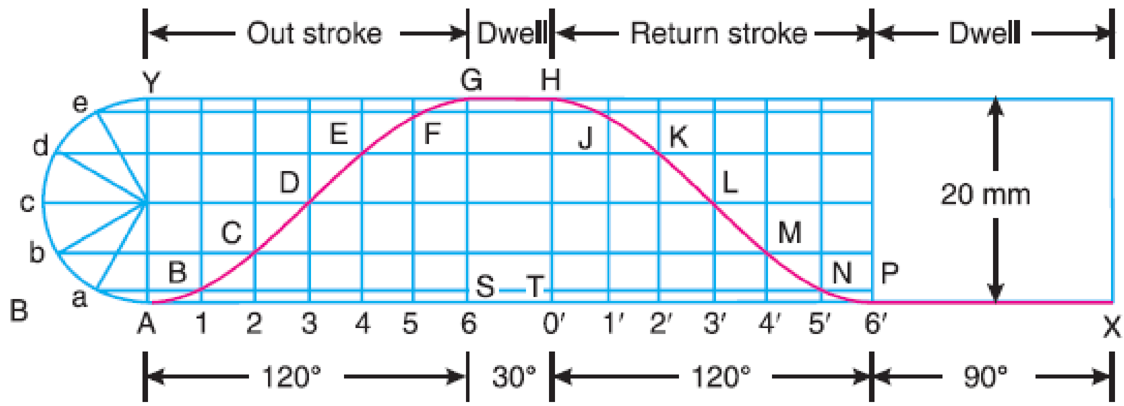
We know that the maximum acceleration of the valve rod to raise the valve,

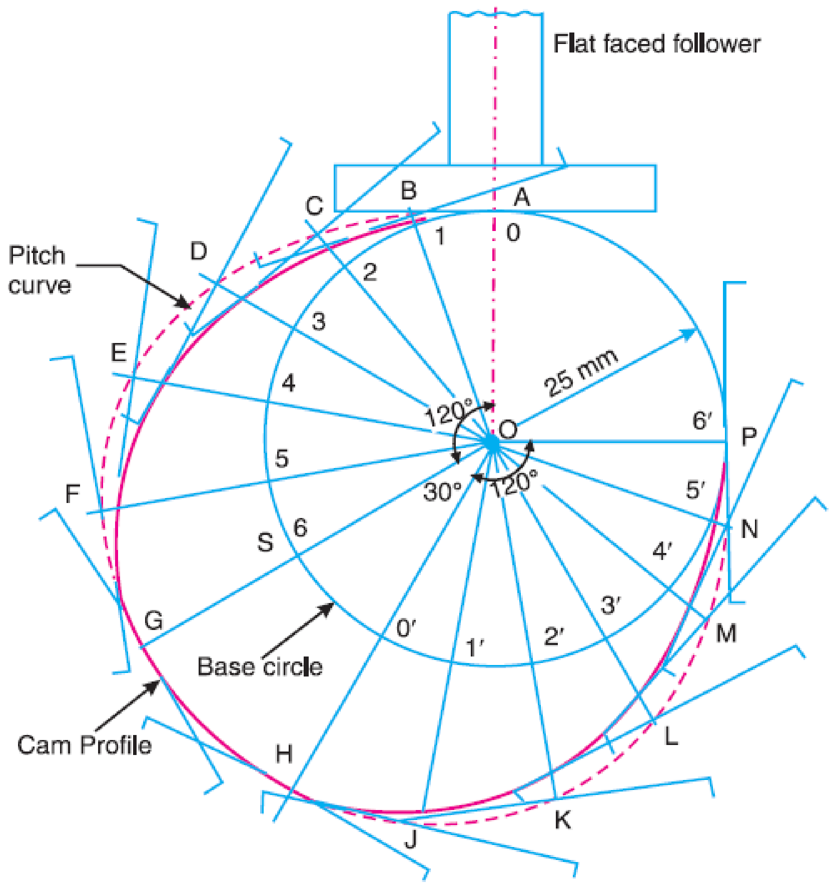
$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_0)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(2.1)^2} = 6.13 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the valve rod to lower the valve,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(1.047)^2} = 24.67 \text{ m/s}^2 \textbf{ Ans.}$$

Example: A cam drives a flat reciprocating follower in the following manner: During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam.





4.1 Introduction

A **gear** is a rotating machine part having cut *teeth*, or *cogs*, which *mesh* with another toothed part in order to transmit torque.

The gears in a transmission are analogous to the wheels in a pulley. An advantage of gears is that the teeth of a gear prevent slipping.

When two gears of unequal number of teeth are combined a mechanical advantage is produced, with both the rotational speeds and the torques of the two gears differing in a simple relationship.

In transmissions which offer multiple gear ratios, such as bicycles and cars, the term **gear**, as in *first gear*, refers to a gear ratio rather than an actual physical gear.

4.1.1 Fundamental Law of Gear-Tooth

Pitch point divides the line between the line of centres and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

Formation of teeth:

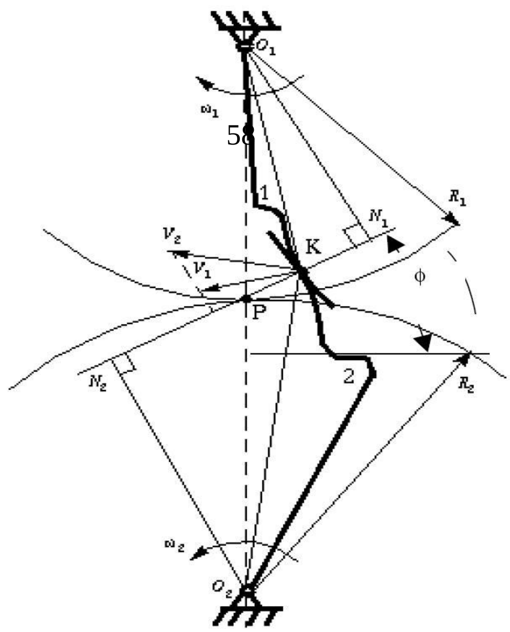
Involute teeth

Cycloidal teeth

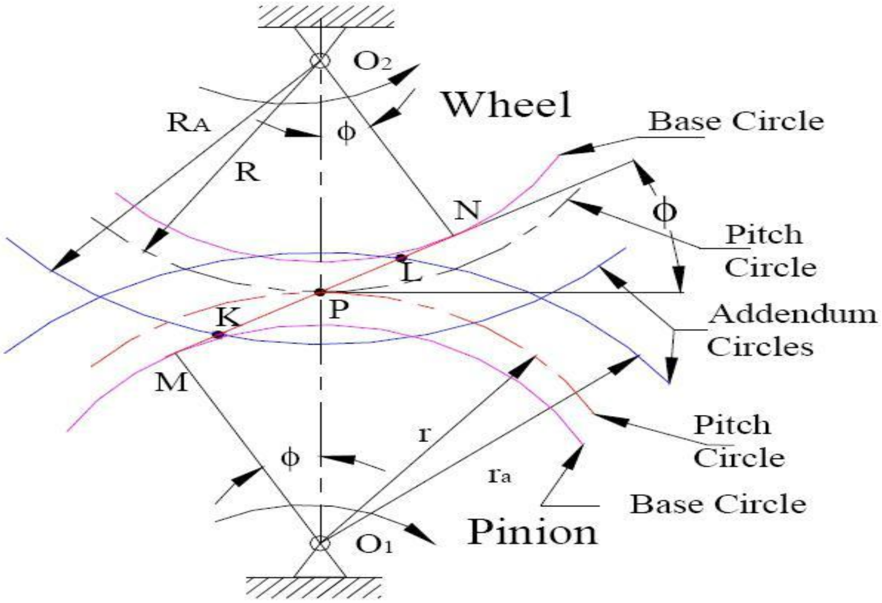
Involute curve:

The curve most commonly use d for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line Rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is un wrapped from the circle. The circle from which the involute is derived is called the **base circle**

.



Path of contact:



- Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begin sat *K* (on the near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).
- *MN* is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.
- The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion.
- Thus, the length of part of contact is *KL* which is the sum of the parts of path of Contacts *KP* and *PL*. Contact length *KP* is called as **path of approach** and contact length *PL* is called as **path of recess**.

Path of approach: *KP*

$$\begin{aligned} KP &= KN - PN \\ &= \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \end{aligned}$$

Path of recess: *PL*

$$\begin{aligned} PL &= ML - MP \\ &= \sqrt{(r_a)^2 - r^2\cos^2\phi} - r\sin\phi \end{aligned}$$

Length of path of contact :

]

$$\begin{aligned} KL &= KP + PL \\ &= \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R+r)\sin\phi \end{aligned}$$

Arc of contact: Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Figure, the arc of contact is *EPF* or *GPH*.

The arc *GP* is known as *arc of approach and* the arc *PH* is called *arc of recess*. The angles subtended by the SE arcs at O1arecalled *angle of approach* and *angle of recess* respectively.

$$\text{Length of arc of approach} = \text{arc } GP = \frac{Lenght\ of\ path\ of\ approach}{\cos\phi} = \frac{KP}{\cos\phi}$$

$$\text{Length of arc of recess} = \text{arc } PH = \frac{Lenght\ of\ path\ of\ recess}{\cos\phi} = \frac{PL}{\cos\phi}$$

$$\text{Length of arc contact} = \text{arc } GPH = arc\ GP + arc\ PH$$

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi} = \frac{Length\ of\ path\ of\ contact}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

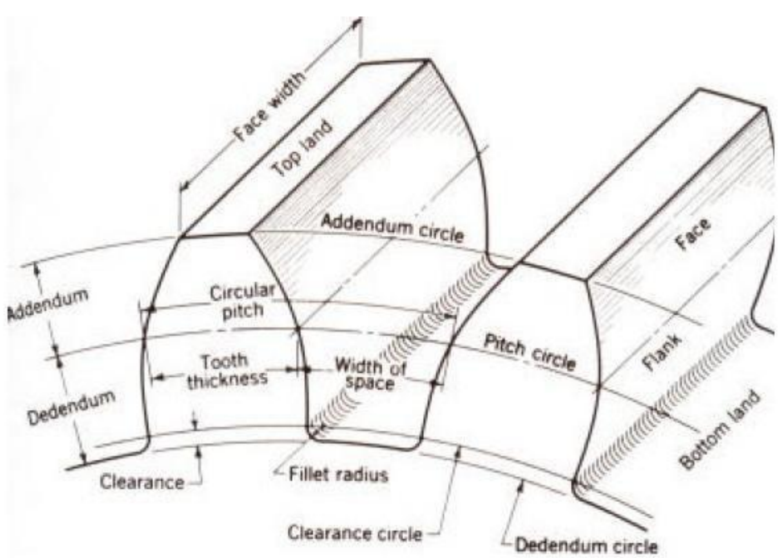
The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

$$Contat\ ratio = \frac{Length\ of\ the\ arc\ of\ contact}{P_C}$$

$P_C = Circular\ \ pitch = \pi \times m$ and m = Module.

4.2 Spur Gear Terminology

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as an actual gear



2. ***Pitch circle diameter***. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. ***Pitch point***. It is a common point of contact between two pitch circles.

4. ***Pitch surface***. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. ***Pressure angle or angle of obliquity***. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. ***Addendum***. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. ***Dedendum***. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. ***Addendum circle***. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. ***Dedendum circle***. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. ***Circular pitch***. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. ***Diametral pitch.*** It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d . Mathematically,

Diametral pitch, $$p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \ldots \left(\because\ p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. ***Module.*** It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. ***Clearance***. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. ***Total depth***. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. *Working depth*. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. *Tooth thickness*. It is the width of the tooth measured along the pitch circle.

17. *Tooth space* . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. *Backlash*. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. *Face of tooth*. It is the surface of the gear tooth above the pitch surface.

20. *Flank of tooth*. It is the surface of the gear tooth below the pitch surface.

21. *Top land*. It is the surface of the top of the tooth.

22. *Face width*. It is the width of the gear tooth measured parallel to its axis.

23. *Profile*. It is the curve formed by the face and flank of the tooth.

24. *Fillet radius*. It is the radius that connects the root circle to the profile of the tooth.

25. *Path of contact*. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. **Length of the path of contact*. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. ***Arc of contact*. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

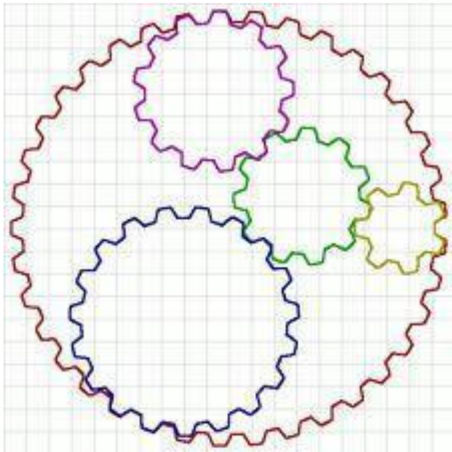
(*a*) *Arc of approach*. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) *Arc of recess*. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e.* number of pairs of teeth in contact.

Epicyclic gear trains:

- If the axis of the shafts over which the gears are mounted are moving relative to a fixed axis , the gear train is called the epicyclic gear train.
- Problems in epicyclic gear trains.



Differentials:

- Used in the rear axle of an automobile.
- To enable the rear wheels to revolve at different speeds when negotiating a curve.
- To enable the rear wheels to revolve at the same speeds when going straight.

12.20. Minimum Number of Teeth on the Pinion in Order to Avoid Interference

We have already discussed in the previous article that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points N and M (see Fig. 12.13) respectively.

Let t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2\times O_1P\times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$\ldots(\because PN = O_2P \sin\phi = R\sin\phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

∴ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_p m$ = Addendum of the pinion, where A_p is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

∴
$$A_p.m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2} \qquad \ldots(\because O_1P = r = mt/2)$$

$$= \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

or
$$A_p = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

∴
$$t = \frac{2A_p}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_p}{\sqrt{1 + G(G + 2)\sin^2\phi} - 1}$$

12.21. Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

and $A_W.m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

Using the same notations as in Art. 12.20, we have from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2 \phi - 2\, R.r \sin\phi \cos(90° + \phi)$$

$$...(\because PM = O_1P \sin\phi = r)$$

$$= R^2 + r^2 \sin^2 \phi + 2R.r \sin^2 \phi$$

$$= R^2 \left[1 + \frac{r^2 \sin^2 \phi}{R^2} + \frac{2r \sin^2 \phi}{R}\right] = R^2 \left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2 \phi\right]$$

$\therefore$ Limiting radius of wheel addendum circle,

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2 \phi} = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi}$$

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore \quad A_W m = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - \frac{m.T}{2} \qquad ...(\because O_2P = R = m.T/2)$$

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

or

$$A_W = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

$$\therefore \quad T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1}$$

Notes : 1. From the above equation, we may also obtain the minimum number of teeth on pinion.

Multiplying both sides by $\frac{t}{T}$,

$$T \times \frac{t}{T} = \frac{2A_W \times \frac{t}{T}}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1}$$

$$t = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1\right]}$$

2. If wheel and pinion have equal teeth, then $G = 1$, and

$$T = \frac{2A_W}{\sqrt{1 + 3\sin^2 \phi} - 1}$$

Example 12.11. *A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find in order that the interference is just avoided ; 1. the addenda on pinion and gear wheel ; 2. the length of path of contact ; and 3. the maximum velocity of sliding of teeth on either side of the pitch point.*

Solution. Given : $\phi = 16°$; $m = 6$ mm ; $t = 16$; $N_1 = 240$ r.p.m. or $\omega_1 = 2\pi \times 240/60 = 25.136$ rad/s ; $G = T/t = 1.75$ or $T = G.t = 1.75 \times 16 = 28$

1. Addenda on pinion and gear wheel

We know that addendum on pinion

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + \frac{28}{16}\left(\frac{28}{16} + 2\right)\sin^2 16°} - 1\right]$$

$$= 48\ (1.224 - 1) = 10.76 \text{ mm } \textbf{Ans.}$$

and addendum on wheel

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 28}{2}\left[\sqrt{1 + \frac{16}{28}\left(\frac{16}{28} + 2\right)\sin^2 16°} - 1\right]$$

$$= 84\ (1.054 - 1) = 4.56 \text{ mm } \textbf{Ans.}$$

2. Length of path of contact

We know that the pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 28/2 = 84 \text{ mm}$$

and pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 16/2 = 48 \text{ mm}$$

∴ Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76 \text{ mm}$$

and addendum circle radius of pinion,

$$r_A = r + \text{Addendum of pinion} = 48 + 4.56 = 52.56 \text{ mm}$$

We know that the length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$ ...(Refer Fig. 12.11)

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16°} - 84\sin 16°$$

$$= 49.6 - 23.15 = 26.45 \text{ mm}$$

and the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.56)^2 - (48)^2\cos^2 16°} - 48\sin 16°$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 26.45 + 11.94 = 38.39 \text{ mm } \textbf{Ans.}$$

3. Maximum velocity of sliding of teeth on either side of pitch point

Let ω_2 = Angular speed of gear wheel.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t} = 1.75$ or $\omega_2 = \frac{\omega_1}{1.75} = \frac{25.136}{1.75} = 14.28$ rad/s

∴ Maximum velocity of sliding of teeth on the left side of pitch point *i.e.* at point *K*

$$= (\omega_1 + \omega_2)\,KP = (25.136 + 14.28)\,26.45 = 1043 \text{ mm/s } \textbf{Ans.}$$

and maximum velocity of sliding of teeth on the right side of pitch point *i.e.* at point *L*

$$= (\omega_1 + \omega_2)\,PL = (25.136 + 14.28)\,11.94 = 471 \text{ mm/s } \textbf{Ans.}$$

Example 12.13. *Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form ; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 90 r.p.m. Determine : 1. The number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel, 2. The length of path and arc of contact, 3. The number of pairs of teeth in contact, and 4. The maximum velocity of sliding.*

Solution. Given : $G = T / t = 3$; $m = 6$ mm ; $A_P = A_W = 1$ module = 6 mm ; $\phi = 20°$; $N_1 = 90$ r.p.m. or $\omega_1 = 2\pi \times 90 / 60 = 9.43$ rad/s

1. Number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel

We know that number of teeth on the pinion to avoid interference,

$$t = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1} = \frac{2 \times 6}{\sqrt{1 + 3(3+2)\sin^2 20°} - 1}$$

$= 18.2$ say 19 **Ans.**

and corresponding number of teeth on the wheel,

$$T = G.t = 3 \times 19 = 57 \textbf{ Ans.}$$

2. Length of path and arc of contact

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 6 \times 19/2 = 57 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum on pinion } (A_P) = 57 + 6 = 63 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T / 2 = 6 \times 57 / 2 = 171 \text{ mm}$$

∴ Radius of addendum circle of wheel,

$$R_A = R + \text{Addendum on wheel } (A_W) = 171 + 6 = 177 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(177)^2 - (171)^2\cos^2 20°} - 171\sin 20° = 74.2 - 58.5 = 15.7 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(63)^2 - (57)^2\cos^2 20°} - 57\sin 20° = 33.17 - 19.5 = 13.67 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.7 + 13.67 = 29.37 \text{ mm} \textbf{ Ans.}$$

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{29.37}{\cos 20°} = 31.25 \text{ mm} \textbf{ Ans.}$$

3. Number of pairs of teeth in contact

We know that circular pitch,

$$p_c = \pi \times m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{p_c} = \frac{31.25}{18.852} = 1.66 \text{ say } 2 \textbf{ Ans.}$$

4. Maximum velocity of sliding

Let ω_2 = Angular speed of wheel in rad/s.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t}$ or $\omega_2 = \omega_1 \times \frac{t}{T} = 9.43 \times \frac{19}{57} = 3.14$ rad/s

∴ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\,KP \qquad \text{...}(\because KP > PL)$$

$$= (9.43 + 3.14)\,15.7 = 197.35 \text{ mm/s} \textbf{ Ans.}$$

Gear Trains

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Simple Gear Train

When there is only one gear on each shaft, as shown in Fig. 13.1, it is known as simple gear train. The gears are represented by their pitch circles.

When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 13.1 (a). Since the gear 1 drives the gear 2, therefore gear 1 is called the driver and the gear 2 is called the driven or follower.

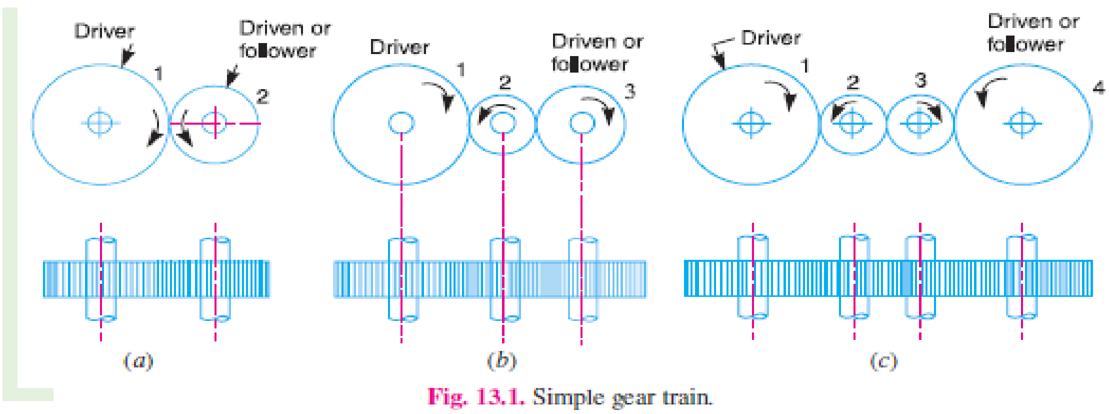


Fig. 13.1. Simple gear train.

Let N_1 = Speed of gear 1(or driver) in r.p.m.,

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as *train value* of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or 2. By providing one or more intermediate gears.

A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears are ***odd***, the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig. 13.1 (*b*).

But if the number of intermediate gears are ***even***, the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. 13.1 (*c*).

Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (*b*).

Let N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

Compound Gear Train

When there are more than one gear on a shaft, as shown in Fig. 13.2, it is called a compound train of gear.

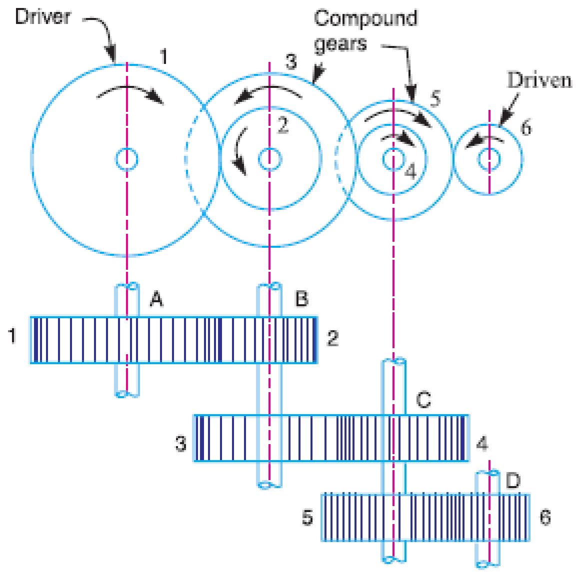


Fig. 13.2 Compound gear train.

In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

* Since gears 2 and 3 are mounted on one shaft B, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft C, therefore $N_4 = N_5$.

Speed ratio = Speed of the first driver/Speed of the last driven or follower

Train value = Speed of the last driven or follower/Speed of the first driver

= Product of number of teeth on the drivers/Product of number of teeth on the drivens

Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train.

Epicyclic Gear Train

Example 13.4. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*

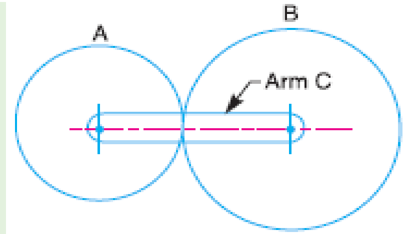


Fig. 13.7

Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

The gear train is shown in Fig. 13.7.

1. *Tabular method*

First of all prepare the table of motions as given below :

Table 13.2. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear *A* is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -150 \text{ r.p.m.}$$

$\therefore$ Speed of gear *B*, $N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270$ r.p.m.

$= 270$ r.p.m. (anticlockwise) **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear *A* makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \quad \text{or} \quad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear *B*,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$= 510$ r.p.m. (anticlockwise) **Ans.**

2. *Algebraic method*

Let N_A = Speed of gear *A*.

N_B = Speed of gear *B*, and

N_C = Speed of arm *C*.

Assuming the arm *C* to be fixed, speed of gear *A* relative to arm *C*

$$= N_A - N_C$$

and speed of gear *B* relative to arm $C = N_B - N_C$

Since the gears A and B revolve in *opposite* directions, therefore

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \quad \text{...}(i)$$

Speed of gear B when gear A is fixed

When gear A is fixed, the arm rotates at 150 r.p.m. in the anticlockwise direction, *i.e.*

$$N_A = 0, \quad \text{and} \quad N_C = +150 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{0 - 150} = -\frac{36}{45} = -0.8 \quad \text{...[From equation } (i)\text{]}$$

or $$N_B = -150 \times -0.8 + 150 = 120 + 150 = 270 \text{ r.p.m.} \quad \textbf{Ans.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m. clockwise, therefore

$$N_A = -300 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{-300 - 150} = -\frac{36}{45} = -0.8$$

or $$N_B = -450 \times -0.8 + 150 = 360 + 150 = 510 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 13.6. *An epicyclic gear consists of three gears A, B and C as shown in Fig. 13.10. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.*

Solution. Given : $T_A = 72$; $T_C = 32$; Speed of arm $EF = 18$ r.p.m.

Considering the relative motion of rotation as shown in Table 13.5.

Table 13.5. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm EF	*Gear C*	*Gear B*	*Gear A*
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$

We know that the speed of the arm is 18 r.p.m. therefore,

$$y = 18 \text{ r.p.m.}$$

and the gear A is fixed, therefore

$$y - x \times \frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$

$$\therefore \quad x = 18 \times 72 / 32 = 40.5$$

$\therefore$ Speed of gear C $= x + y = 40.5 + 18$

$= +58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

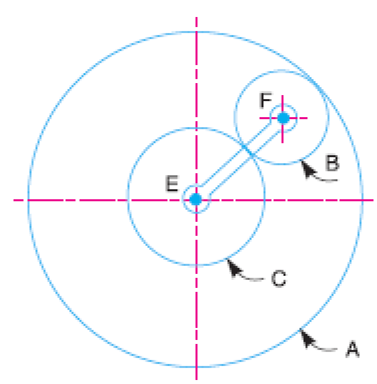


Fig. 13.10

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig. 13.10,

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \quad \text{or} \quad 2\,T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

$\therefore$ Speed of gear B $= y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -46.8$ r.p.m.

$= 46.8$ r.p.m. in the opposite direction of arm. **Ans.**

Example 13.8. *In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$.*

***1.** Sketch the arrangement ; **2.** Find the number of teeth on A and B ; **3.** If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B ; and **4.** If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise ; find the speed of wheel B.*

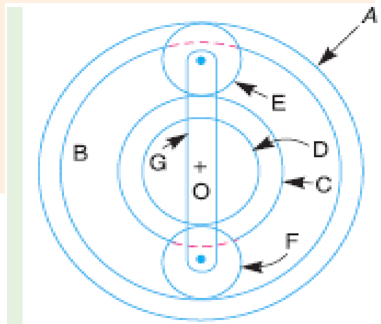


Fig. 13.12

Solution. Given : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$

1. Sketch the arrangement

The arrangement is shown in Fig. 13.12.

2. Number of teeth on wheels A and B

Let T_A = Number of teeth on wheel A, and

T_B = Number of teeth on wheel B.

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of wheels A, B, C, D, E and F respectively, then from the geometry of Fig. 13.12,

$$d_A = d_C + 2\,d_E$$

and

$$d_B = d_D + 2\,d_F$$

Since the number of teeth are proportional to their pitch circle diameters, for the same module, therefore

$$T_A = T_C + 2\,T_E = 28 + 2 \times 18 = 64 \quad \textbf{Ans.}$$

and

$$T_B = T_D + 2\,T_F = 26 + 2 \times 18 = 62 \quad \textbf{Ans.}$$

3. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed

First of all, the table of motions is drawn as given below :

Table 13.7. Table of motions.

Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $= -\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $= +\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through + x revolutions	0	$+x$	$+x\times\frac{T_A}{T_E}$	$-x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_E}$	$y-x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

Since the arm G makes 100 r.p.m. clockwise, therefore from the fourth row of the table,

$$y = -100 \quad \text{...(i)}$$

Also, the wheel A is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = 100 \quad \text{...(ii)}$$

$\therefore$ Speed of wheel $B = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} = -100 + 100 \times \frac{64}{28} \times \frac{26}{62} = -100 + 95.8$ r.p.m.

$= -4.2$ r.p.m. $= 4.2$ r.p.m. clockwise **Ans.**

4. *Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise*

Since the arm G makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \quad \text{...(iii)}$$

Also the wheel A makes 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$x + y = 10 \quad \text{or} \quad x = 10 - y = 10 + 100 = 110 \quad \text{...(iv)}$$

$\therefore$ Speed of wheel $B = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} = -100 + 110 \times \frac{64}{28} \times \frac{26}{62} = -100 + 105.4$ r.p.m.

$= +5.4$ r.p.m. $= 5.4$ r.p.m. counter clockwise **Ans.**

Example 12.11. *A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find in order that the interference is just avoided ; **1.** the addenda on pinion and gear wheel ; **2.** the length of path of contact ; and **3.** the maximum velocity of sliding of teeth on either side of the pitch point.*

Solution. Given : $\phi = 16°$; $m = 6$ mm ; $t = 16$; $N_1 = 240$ r.p.m. or $\omega_1 = 2\pi \times 240/60 = 25.136$ rad/s ; $G = T/t = 1.75$ or $T = G.t = 1.75 \times 16 = 28$

1. *Addenda on pinion and gear wheel*

We know that addendum on pinion

$$= \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + \frac{28}{16}\left(\frac{28}{16} + 2\right)\sin^2 16°} - 1\right]$$

$$= 48\,(1.224 - 1) = 10.76 \text{ mm} \quad \textbf{Ans.}$$

and addendum on wheel

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 28}{2}\left[\sqrt{1 + \frac{16}{28}\left(\frac{16}{28} + 2\right)\sin^2 16°} - 1\right]$$

$$= 84\,(1.054 - 1) = 4.56 \text{ mm} \quad \textbf{Ans.}$$

2. *Length of path of contact*

We know that the pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 28/2 = 84 \text{ mm}$$

and pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 16/2 = 48 \text{ mm}$$

$\therefore$ Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76 \text{ mm}$$

and addendum circle radius of pinion,

$$r_A = r + \text{Addendum of pinion} = 48 + 4.56 = 52.56 \text{ mm}$$

We know that the length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(94.76)^2 - (84)^2 \cos^2 16°} - 84 \sin 16°$$

$$= 49.6 - 23.15 = 26.45 \text{ mm}$$

and the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(52.56)^2 - (48)^2 \cos^2 16°} - 48 \sin 16°$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 26.45 + 11.94 = 38.39 \text{ mm} \textbf{ Ans.}$$

3. Maximum velocity of sliding of teeth on either side of pitch point

Let ω_2 = Angular speed of gear wheel.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t} = 1.75$ or $\omega_2 = \frac{\omega_1}{1.75} = \frac{25.136}{1.75} = 14.28$ rad/s

∴ Maximum velocity of sliding of teeth on the left side of pitch point *i.e.* at point K

$$= (\omega_1 + \omega_2)\, KP = (25.136 + 14.28)\, 26.45 = 1043 \text{ mm/s} \textbf{ Ans.}$$

and maximum velocity of sliding of teeth on the right side of pitch point *i.e.* at point L

$$= (\omega_1 + \omega_2)\, PL = (25.136 + 14.28)\, 11.94 = 471 \text{ mm/s} \textbf{ Ans.}$$

UNIT V - FRICTION IN MACHINE ELEMENTS

Friction

Friction is a measure of how hard it is to slide one object over another.

1. *Static friction*. It is the friction, experienced by a body, when at rest.
2. *Dynamic friction*. It is the friction, experienced by a body, when in motion. The dynamic friction is also called ***kinetic friction*** and is less than the static friction.
It is of the following three types:
(*a*) *Sliding friction*. It is the friction, experienced by a body, when it ***slides*** over another body.
(*b*) *Rolling friction*. It is the friction, experienced between the surfaces which have ***balls*** or ***rollers*** interposed between them.
(*c*) *Pivot friction.* It is the friction, experienced by a body, due to the ***motion of rotation*** as in case of foot step bearings.

The friction may further be classified as:

1. Friction between unlubricated surfaces, and
2. Friction between lubricated surfaces.

These are discussed in the following articles.

Laws of Static Friction

Following are the laws of static friction:

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

Coefficient of friction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = F/R_N$$

Consider that a body A of weight (W) is resting on a horizontal plane B, as shown in Fig.
If a horizontal force P is applied to the body, no relative motion will take place until the applied force P is equal to the force of friction F, acting opposite to the direction of motion. The magnitude of this force of friction is $F = \mu.W = \mu.R$

N, where R

N is the normal reaction.

In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces :

1. Weight of the body (W),
2. Applied horizontal force (P), and
3. Reaction (R) between the body A and the plane B.

The reaction R must, therefore, be equal and opposite to the resultant of W and P and will be inclined at an angle _ to the normal reaction R_N. This angle _ is known as the ***limiting angle of friction***.
It may be defined as the angle which the resultant reaction R makes with the normal reaction R_N.

From, $\tan\Phi = F/R$

Angle of Repose

Consider that a body A of weight (W) is resting on an inclined plane B, If the angle of inclination of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the angle of repose.

Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as **external threads**.

- But if the threads are cut on the internal surface of a hollow rod, these are known as **internal threads**.
- The screw threads are mainly of two types i.e. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V threads are used for the purpose of tightening pieces together.

1. Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.
2. Pitch. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw.
3. Lead. It is the distance, a screw thread advances axially in one turn.
4. Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of a thread).
5. Single-threaded screw. If the lead of a screw is equal to its pitch, it is known as single threaded screw.
6. Multi-threaded screw. If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw e.g. in a double threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

Lead = Pitch × Number of threads

Helix angle. It is the slope or inclination of the thread with the horizontal. Mathematically,

$$\tan \alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

$$= p/\pi d \qquad \text{...(In single-threaded screw)}$$

$$= n.p/\pi d \qquad \text{...(In multi-threaded screw)}$$

α = Helix angle,

p = Pitch of the screw,

d = Mean diameter of the screw, and

n = Number of threads in one lead.

1. ***An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor.***

Solution. Given : $W = 75$ kN $= 75 \times 10^3$ N ; $v = 300$ mm/min ; $p = 6$ mm ; $d_0 = 40$ mm $\mu = \tan \phi = 0.1$

We know that mean diameter of the screw,

$$d = d_0 - p/2 = 40 - 6/2 = 37 \text{ mm} = 0.037 \text{ m}$$

and

$$\tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

∴ Force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha . \tan \phi}\right]$$

$$= 75 \times 10^3 \left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right] = 11.43 \times 10^3 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 11.43 \times 10^3 \times 0.037/2 = 211.45 \text{ N-m}$$

We know that speed of the screw,

$$N = \frac{\text{Speed of the nut}}{\text{Pitch of the screw}} = \frac{300}{6} = 50 \text{ r.p.m.}$$

and angular speed, $\omega = 2\pi \times 50/60 = 5.24$ rad/s

∴ Power of the motor $= T.\omega = 211.45 \times 5.24 = 1108 \text{ W} = 1.108$ **kW Ans.**

2. The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.

Solution. Given : $d = 50$ mm ; $p = 12.5$ mm ; $\mu = \tan\phi = 0.13$; $W = 25$ kN $= 25 \times 10^3$ N

We know that, $$\tan\alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$$

and force required on the screw to raise the load,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 25 \times 10^3\left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13}\right] = 5305 \text{ N}$$

Torque required on the screw

We know that the torque required on the screw to raise the load,

$$T_1 = P \times d/2 = 5305 \times 50/2 = 132\,625 \text{ N-mm} \quad \textbf{Ans.}$$

Ratio of the torques required to raise and lower the load

We know that the force required on the screw to lower the load,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 25 \times 10^3\left[\frac{0.13 + 0.08}{1 + 0.13 \times 0.08}\right] = 1237 \text{ N}$$

and torque required to lower the load

$$T_2 = P \times d/2 = 1237 \times 50/2 = 30\,905 \text{ N-mm}$$

∴ Ratio of the torques required,

$$= T_1/T_2 = 132625/30925 = 4.3 \quad \textbf{Ans.}$$

Efficiency of the machine

We know that the efficiency,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\phi} = \frac{0.08(1 - 0.08 \times 0.13)}{0.08 + 0.13}$$

$$= 0.377 = 37.7\% \quad \textbf{Ans.}$$

Over Hauling and Self-Locking Screws

The torque required to lower the load

$$T = P \times \frac{d}{2} = W\tan(\phi - \alpha)\frac{d}{2}$$

In the above expression, if $\varphi < \alpha$, then torque required to lower the load will be negative. In other words, the load will start moving downward without the application of any torque. Such a condition is known as over hauling of screws. If however $\varphi > \alpha$, the torque required to lower the load will positive, indicating that an effort is applied to lower the load. Such a screw is known as self-locking screw. In other words, a screw will be self-locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle i.e. μ or $\tan\varphi > \tan\alpha$.

3. A load of 10 kN is raised by means of a screw jack, having a square threaded screw of 12 mm pitch and of mean diameter 50 mm. If a force of 100 N is applied at the end of a lever to raise the load, what should be the length of the lever used? Take coefficient of friction = 0.15. What is the mechanical advantage obtained? State whether the screw is self locking.

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{12}{\pi \times 50} = 0.0764$

∴ Effort required at the circumference of the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.0764 + 0.15}{1 - 0.0764 \times 0.15}\right] = 2290 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 2290 \times 50/2 = 57\ 250 \text{ N-mm} \qquad \ldots(i)$$

We know that torque applied at the end of the lever,

$$T = P_1 \times l = 100 \times l \text{ N-mm} \qquad \ldots(ii)$$

Equating equations (*i*) and (*ii*)

$$l = 57\ 250/100 = 572.5 \text{ mm}$$ **Ans.**

Mechanical advantage

We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{10 \times 10^3}{100} = 100$$ **Ans.**

Self locking of the screw

We know that efficiency of the screw jack,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\phi}$$

$$= \frac{0.0764(1 - 0.0764 \times 0.15)}{0.0764 + 0.15} = \frac{0.0755}{0.2264} = 0.3335 \text{ or } 33.35\%$$

Friction of Pivot and Collar Bearing

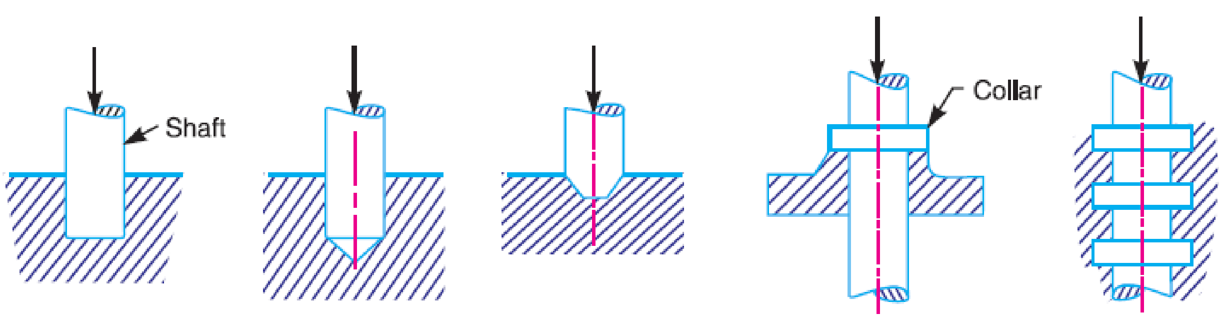
The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.

The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface

When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig. 10.16 (c).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii

of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution. Given : $W = 20$ kN $= 20 \times 10^3$ N ; $2\,\alpha = 120°$ or $\alpha = 60°$; $p_n = 0.3$ N/mm^2
$N = 200$ r.p.m. or $\omega = 2\,\pi \times 200/60 = 20.95$ rad/s ; $\mu = 0.1$

Outer and inner radii of the bearing surface

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm.

Since the external diameter is twice the internal diameter, therefore

$$r_1 = 2\,r_2$$

We know that intensity of normal pressure (p_n),

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$$\therefore \quad (r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3 \text{ or } r_2 = 84 \text{ mm} \quad \textbf{Ans.}$$

and $$r_1 = 2\,r_2 = 2 \times 84 = 168 \text{ mm} \quad \textbf{Ans.}$$

Power absorbed in friction

We know that total frictional torque (assuming uniform pressure),

$$T = \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \text{cosec } 60° = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2}\right] \text{N-mm}$$

$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

$\therefore$ Power absorbed in friction,

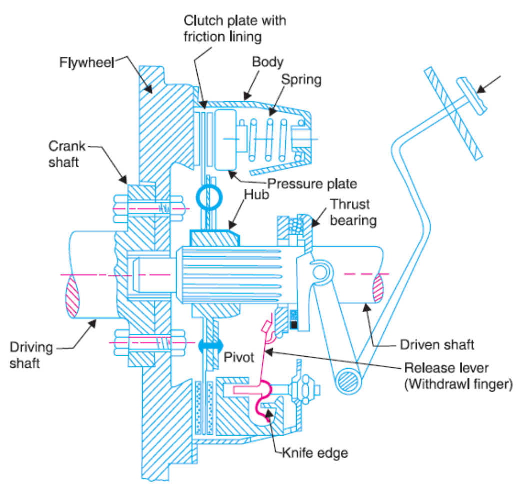
$$P = T.\omega = 301.76 \times 20.95 = 6322 \text{ W} = 6.322 \text{ kW} \quad \textbf{Ans.}$$

Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces

Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel.



1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu . p . r^2\, dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.\, p.r^2 .dr = 2\,\pi\mu\, p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\, p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (i),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. Considering uniform wear

n = Number of pairs of friction or contact surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \ldots\text{(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \ldots\text{(For uniform wear)}$$

$\therefore$ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

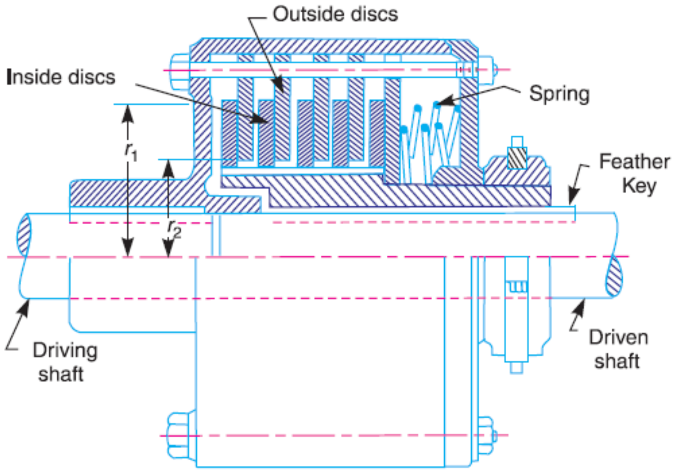
$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

Notes: 1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$T = n.\mu.W.R$$

Multiple Disc Clutch



Example 10.22. *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

$$\therefore \quad p_{max} = 4 \times 10^3/15\,710 = 0.2546 \text{ N/mm}^2$$ **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\,420\, p_{min}$$

$$\therefore \quad p_{min} = 4 \times 10^3/31\,420 = 0.1273 \text{ N/mm}^2$$ **Ans.**

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$ **Ans.**

Example 10.25. *A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The axial pressure is limited to 0.07 N/mm². If the coefficient of friction is 0.25, find 1. Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, and 2. Outer and inner radii of the clutch plate.*

Solution. Given : $P = 7.5$ kW $= 7.5 \times 10^3$ W ; $N = 900$ r.p.m or $\omega = 2\pi \times 900/60 = 94.26$ rad/s ; $p = 0.07$ N/mm² ; $\mu = 0.25$

1. ***Mean radius and face width of the friction lining***

Let R = Mean radius of the friction lining in mm, and

w = Face width of the friction lining in mm,

Ratio of mean radius to the face width,

$$R/w = 4 \qquad \text{...(Given)}$$

We know that the area of friction faces,

$$A = 2\pi R.w$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi R.w.p$$

We know that torque transmitted (considering uniform wear),

$$T = n.\mu.W.R = n.\mu\ (2\pi R.w.p)R$$

$$= n.\mu\left(2\pi R \times \frac{R}{4} \times p\right)R = \frac{\pi}{2} \times n.\mu.p.R^3 \qquad \text{...}(\because\ w = R/4)$$

$$= \frac{\pi}{2} \times 2 \times 0.25 \times 0.07\ R^3 = 0.055\ R^3 \text{ N-mm} \qquad \text{...}(i)$$

...($\because$ $n = 2$, for single plate clutch)

We also know that power transmitted (P),

$$7.5 \times 10^3 = T.\omega = T \times 94.26$$

$$\therefore \quad T = 7.5 \times 10^3/94.26 = 79.56 \text{ N-m} = 79.56 \times 10^3 \text{ N-mm} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$R^3 = 79.56 \times 10^3/0.055 = 1446.5 \times 10^3$ or $R = 113$ mm **Ans.**

and $w = R/4 = 113/4 = 28.25$mm **Ans.**

2. ***Outer and inner radii of the clutch plate***

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the width of the clutch plate is equal to the difference of the outer and inner radii, therefore

$$w = r_1 - r_2 = 28.25 \text{ mm} \qquad \text{...}(iii)$$

Also for uniform wear, the mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 113 = 226 \text{ mm} \qquad \text{...}(iv)$$

From equations (*iii*) and (*iv*),

$r_1 = 127.125$ mm ; and $r_2 = 98.875$ **Ans.**

Example 10.28. *A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm², find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction = 0.3.*

Solution. Given : $n_1 + n_2 = 5$; $n = 4$; $p = 0.127$ N/mm² ; $N = 500$ r.p.m. or $\omega = 2\pi \times 500/60 = 52.4$ rad/s ; $r_1 = 125$ mm ; $r_2 = 75$ mm ; $\mu = 0.3$

Since the intensity of pressure is maximum at the inner radius r_2, therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.127 \times 75 = 9.525 \text{ N/mm}$$

We know that axial force required to engage the clutch,

$$W = 2\pi C(r_1 - r_2) = 2\pi \times 9.525 (125 - 75) = 2990 \text{ N}$$

and mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2990 \times 0.1 = 358.8 \text{ N-m}$$

∴ Power transmitted,

$$P = T.\omega = 358.8 \times 52.4 = 18\,800 \text{ W} = 18.8 \text{ kW}$$ **Ans.**

Example 10.29. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform wear and coefficient of friction as 0.3, find the maximum axial intensity of pressure between the discs for transmitting 25 kW at 1575 r.p.m.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\pi \times 1575/60 = 165$ rad/s

Let T = Torque transmitted in N-m, and

W = Axial force on each friction surface.

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Number of pairs of friction surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

and mean radius of friction surfaces for uniform wear,

We know that torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3 \times W \times 0.09 = 0.108\, W$$

$$\therefore \quad W = 151.5/0.108 = 1403 \text{ N}$$

Let p = Maximum axial intensity of pressure.

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear

$$p.r_2 = C \quad \text{or} \quad C = p \times 60 = 60\,p \text{ N/mm}$$

We know that the axial force on each friction surface (W),

$$1403 = 2\pi.C(r_1 - r_2) = 2\pi \times 60\,p\,(120 - 60) = 22\,622\,p$$

$$\therefore \quad p = 1403/22\,622 = 0.062 \text{ N/mm}^2$$ **Ans.**

Belt, Rope Drives

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to insure uniform tension across the belt section.

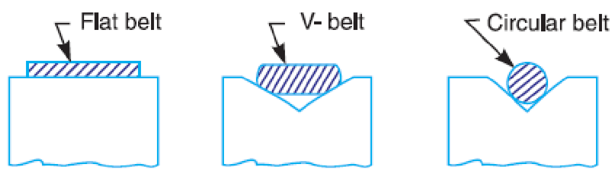
(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings

The belt drives are usually classified into the following three groups :

1. Light drives. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.
2. Medium drives. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.
3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators

Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

11.8. Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Fig. 11.7. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let d_1 = Diameter of the pulley 1,

N_1 = Speed of the pulley 1 in r.p.m.,

d_2, d_3, d_4, and N_2, N_3, N_4 = Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{...}(i)$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{...}(ii)$$

Multiplying equations (*i*) and (*ii*),

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{...}(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

$$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

Slip
The frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage

An engine, running at 150 rpm drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive

Solution. Given : $N_1 = 150$ r.p.m. ; $d_1 = 750$ mm ; $d_2 = 450$ mm ; $d_3 = 900$ mm ; $d_4 = 150$ mm

The arrangement of belt drive is shown in Fig. 11.10.

Let N_4 = Speed of the dynamo shaft .

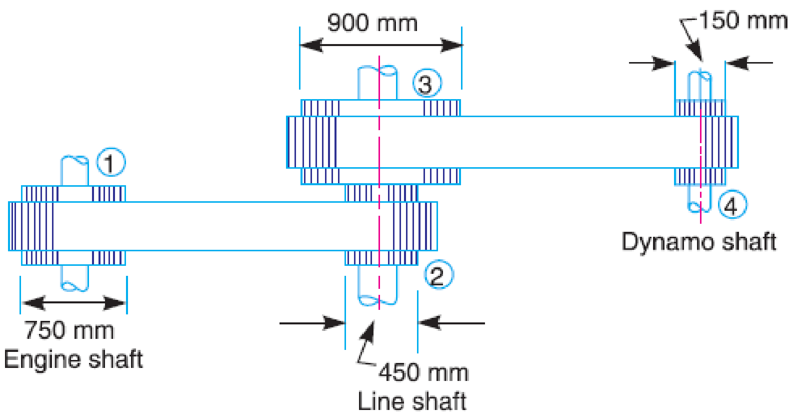


Fig. 11.10

1. ***When there is no slip***

We know that $\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$ or $\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$

$\therefore$ $N_4 = 150 \times 10 = 1500$ r.p.m. **Ans.**

2. ***When there is a slip of 2% at each drive***

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$\therefore$ $N_4 = 150 \times 9.6 = 1440$ r.p.m. **Ans.**

Creep of Belt
When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

A shaft which rotates at a constant speed of 160 r.p.m. is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80 and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for 1. a crossed belt, and 2. an open belt. Neglect belt thickness and slip.

1. *For a crossed belt*

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

or $$r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm} \quad \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \quad \text{or} \quad r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2r_3$$

We know that for a crossed belt drive,

$$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = 40 + 106.7 = 146.7 \text{ mm} \qquad ...(i)$$

$\therefore$ $$r_3 + 2\,r_3 = 146.7 \quad \text{or} \quad r_3 = 146.7/3 = 48.9 \text{ mm} \quad \textbf{Ans.}$$

and $$r_4 = 2\,r_3 = 2 \times 48.9 = 97.8 \text{ mm} \quad \textbf{Ans.}$$

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or} \quad r_6 = r_5 \times \frac{N_5}{N_6} = r_5 \times \frac{160}{100} = 1.6 r_5$$

From equation (*i*),

$$r_5 + 1.6\,r_5 = 146.7 \quad \text{or} \quad r_5 = 146.7/2.6 = 56.4 \text{ mm} \quad \textbf{Ans.}$$

and $$r_6 = 1.6\,r_5 = 1.6 \times 56.4 = 90.2 \text{ mm} \quad \textbf{Ans.}$$

2. *For an open belt*

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \quad \text{or} \quad r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm} \quad \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \quad \text{or} \quad r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2r_3$$

We know that length of belt for an open belt drive,

$$L = \pi(r_1 + r_2) + \frac{(r_2 - r_1)^2}{x} + 2x$$

$$= \pi(40 + 106.7) + \frac{(106.7 - 40)^2}{720} + 2 \times 720 = 1907 \text{ mm}$$

Since the length of the belt in an open belt drive is constant, therefore for pulleys 3 and 4, length of the belt (L),

$$= \pi(r_3 + 2r_3) + \frac{(2r_3 - r_3)^2}{720} + 2 \times 720$$

$$= 9.426\, r_3 + 0.0014\, (r_3)^2 + 1440$$

or $$0.0014\, (r_3)^2 + 9.426\, r_3 - 467 = 0$$

$$\therefore \quad r_3 = \frac{-9.426 \pm \sqrt{(9.426)^2 + 4 \times 0.0014 \times 46}}{2 \times 0.0014}$$

$$= \frac{-9.426 \pm 9.564}{0.0028} = 49.3 \text{ mm} \quad \textbf{Ans.}$$

and $$r_4 = 2\, r_3 = 2 \times 49.3 = 98.6 \text{ mm} \quad \textbf{Ans.}$$

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or}$$

$$r_6 = \frac{N_5}{N_6} \times r_5 = \frac{160}{100} \times r_5 = 1.6\, r_5$$

and length of the belt (L),

$$1907 = \pi(r_5 + r_6) + \frac{(r_6 - r_5)^2}{x} + 2x$$

$$= \pi(r_5 + 1.6r_5) + \frac{(1.6r_5 - r_5)^2}{720} + 2 \times 720$$

$$= 8.17\, r_5 + 0.0005\, (r_5)^2 + 1440$$

or $$0.0005\, (r_5)^2 + 8.17\, r_5 - 467 = 0$$

$$\therefore \quad r_5 = \frac{-8.17 \pm \sqrt{(8.17)^2 + 4 \times 0.0005 \times 467}}{2 \times 0.0005}$$

$$= \frac{-8.17 \pm 8.23}{0.001} = 60 \text{ mm} \quad \textbf{Ans.}$$

Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Solution. Given : $d_1 = 450$ mm $= 0.45$ m or $r_1 = 0.225$ m ; $d_2 = 200$ mm $= 0.2$ m or $r_2 = 0.1$ m ; $x = 1.95$ m ; $N_1 = 200$ r.p.m. ; $T_1 = 1$ kN $= 1000$ N ; $\mu = 0.25$

We know that speed of the belt,

$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

Length of the belt

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{m} \quad \textbf{Ans.}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \quad \text{or} \quad \alpha = 9.6°$$

$$\therefore \quad \theta = 180° + 2\,\alpha = 180° + 2 \times 9.6° = 199.2°$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad} \quad \textbf{Ans.}$$

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{...(Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\,v = (1000 - 419)\,4.714 = 2740 \text{ W} = 2.74 \text{ kW} \quad \textbf{Ans.}$$

Example 11.15. *An open belt running over two pulleys 240 mm and 600 mm diameter connects two parallel shafts 3 metres apart and transmits 4 kW from the smaller pulley that rotates at 300 r.p.m. Coefficient of friction between the belt and the pulley is 0.3 and the safe working tension is 10N per mm width. Determine :* **1.** *minimum width of the belt,* **2.** *initial belt tension, and* **3.** *length of the belt required.*

Solution. Given : d_2 = 240 mm = 0.24 m ; d_1 = 600 mm = 0.6 m ; x = 3 m ; P = 4 kW = 4000 W; N_2 = 300 r.p.m. ; μ = 0.3 ; T_1 = 10 N/mm width

1. *Minimum width of belt*

We know that velocity of the belt,

$$v = \frac{\pi d_2.N_2}{60} = \frac{\pi \times 0.24 \times 300}{60} = 3.77 \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

$\therefore$ Power transmitted (P),

$$4000 = (T_1 - T_2)\,v = (T_1 - T_2)\,3.77$$

or

$$T_1 - T_2 = 4000 / 3.77 = 1061 \text{ N} \qquad \text{...}(i)$$

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.6 - 0.24}{2\times 3} = 0.06 \quad \text{or} \quad \alpha = 3.44°$$

and angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180° - 2\times 3.44° = 173.12°$$

$$= 173.12 \times \pi / 180 = 3.022 \text{ rad}$$

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\times 3.022 = 0.9066$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.9066}{2.3} = 0.3942 \quad \text{or} \quad \frac{T_1}{T_2} = 2.478 \qquad ...(ii)$$

...(Taking antilog of 0.3942)

From equations (*i*) and (*ii*),

$$T_1 = 1779 \text{ N}, \text{ and } T_2 = 718 \text{ N}$$

Since the safe working tension is 10 N per mm width, therefore minimum width of the belt,

$$b = \frac{T_1}{10} = \frac{1779}{10} = 177.9 \text{ mm} \quad \textbf{Ans.}$$

2. *Initial belt tension*

We know that initial belt tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1779 + 718}{2} = 1248.5 \text{N} \quad \textbf{Ans.}$$

3. *Length of the belt required*

We know that length of the belt required,

$$L = \frac{\pi}{2}(d_1 - d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(0.6 + 0.24) + 2\times 3 + \frac{(0.6 - 0.24)^2}{4\times 3}$$

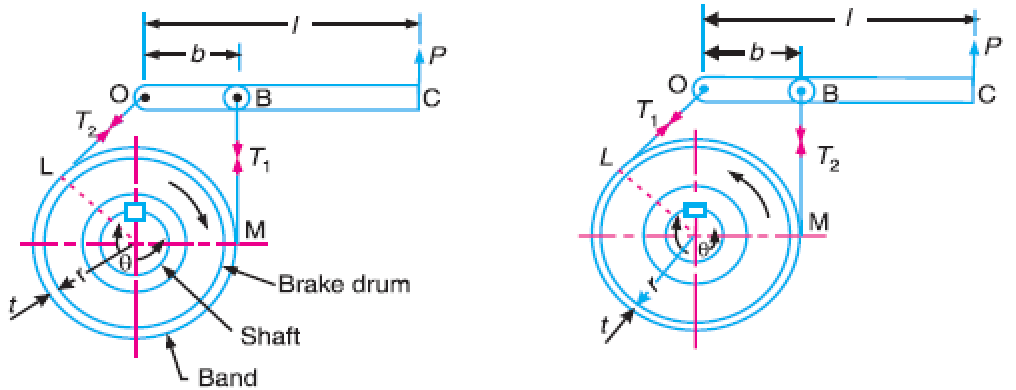
$$= 1.32 + 6 + 0.01 = 7.33 \text{ m} \quad \textbf{Ans.}$$

Friction in brakes- Band and Block brakes.

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc.

Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 19.11, is called a simple band brake in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Example 19.6. *A band brake acts on the 3/4th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force is applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the operating force when the drum rotates in the (a) anticlockwise direction, and (b) clockwise direction.*

Solution. Given : $d = 450$ mm or $r = 225$ mm $= 0.225$ m ; $T_B = 225$ N-m ; $b = OB = 100$ mm $= 0.1$ m ; $l = 500$ mm $= 0.5$ m ; $\mu = 0.25$

Let P = Operating force.

(a) Operating force when drum rotates in anticlockwise direction

Drums for band brakes.

The band brake is shown in Fig. 19.11. Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates anticlockwise, as shown in Fig. 19.11 (*b*), the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2.

We know that angle of wrap,

$$\theta = \frac{3}{4}\text{th of circumference} = \frac{3}{4} \times 360° = 270°$$

$$= 270 \times \pi / 180 = 4.713 \text{ rad}$$

and

$$2.3 \log\left(\frac{T_1}{T_1}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5123 \text{ or } \frac{T_1}{T_2} = 3.253 \quad \ldots (i)$$

. . . (Taking antilog of 0.5123)

We know that braking torque (T_B),

$$225 = (T_1 - T_2) r = (T_1 - T_2)\, 0.225$$

$$\therefore \quad T_1 - T_2 = 225 / 0.225 = 1000 \text{ N} \quad \ldots (ii)$$

From equations (*i*) and (*ii*), we have

$$T_1 = 1444 \text{ N; and } \quad T_2 = 444 \text{ N}$$

Now taking moments about the fulcrum O, we have

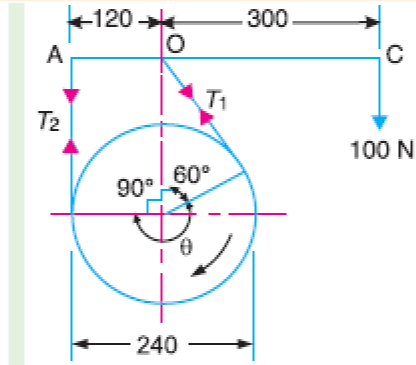
$$P \times l = T_2.b \quad \text{or} \quad P \times 0.5 = 444 \times 0.1 = 44.4$$

$$\therefore \quad P = 44.4 / 0.5 = 88.8 \text{ N} \quad \textbf{Ans.}$$

Example 19.7. *The simple band brake, as shown in Fig. 19.12, is applied to a shaft carrying a flywheel of mass 400 kg. The radius of gyration of the flywheel is 450 mm and runs at 300 r.p.m.*

If the coefficient of friction is 0.2 and the brake drum diameter is 240 mm, find :

1. *the torque applied due to a hand load of 100 N,*
2. *the number of turns of the wheel before it is brought to rest, and*
3. *the time required to bring it to rest, from the moment of the application of the brake.*



All dimensions in mm.

Fig. 19.12

Solution. Given : $m = 400$ kg ; $k = 450$ mm $= 0.45$ m ; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s ; $\mu = 0.2$; $d = 240$ mm $= 0.24$ m or $r = 0.12$ m

1. ***Torque applied due to hand load***

First of all, let us find the tensions in the tight and slack sides of the band *i.e.* T_1 and T_2 respectively.

From the geometry of the Fig. 19.12, angle of lap of the band on the drum,

$$\theta = 360° - 150° = 210° = 210 \times \frac{\pi}{180} = 3.666 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.2 \times 3.666 = 0.7332$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.7332}{2.3} = 0.3188 \qquad \text{or} \qquad \frac{T_1}{T_2} = 2.08 \qquad \ldots (i)$$

... (Taking antilog of 0.3188)

Taking moments about the fulcrum O,

$$T_2 \times 120 = 100 \times 300 = 30\,000 \qquad \text{or} \qquad T_2 = 30\,000/120 = 250 \text{ N}$$

$$\therefore \quad T_1 = 2.08 T_2 = 2.08 \times 250 = 520 \text{ N} \qquad \ldots [\text{From equation } (i)]$$

We know that torque applied,

$$T_B = (T_1 - T_2)\, r = (520 - 250)\, 0.12 = 32.4 \text{ N-m} \quad \textbf{Ans.}$$

2. ***Number of turns of the wheel before it is brought to rest***

Let n = Number of turns of the wheel before it is brought to rest.

We know that kinetic energy of rotation of the drum

$$= \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 = \frac{1}{2} \times 400 (0.45)^2 (31.42)^2 = 40\,000 \text{ N-m}$$

This energy is used to overcome the work done due to the braking torque (T_B).

$$\therefore \quad 40\,000 = T_B \times 2\pi n = 32.4 \times 2\pi n = 203.6\, n$$

or $$n = 40\,000 / 203.6 = 196.5 \quad \textbf{Ans.}$$

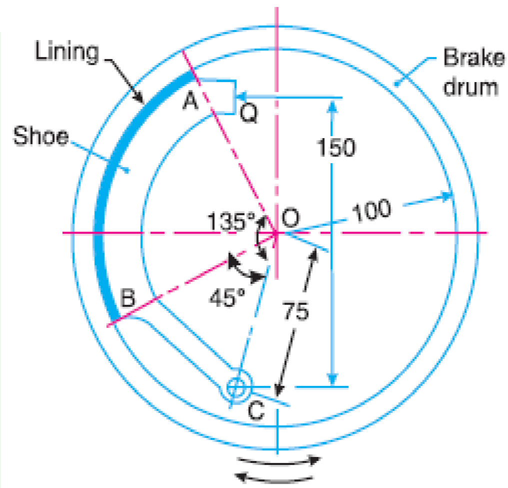
Time required to bring the wheel to rest

We know that the time required to bring the wheel to rest = n / N = 196.5 / 300 = 0.655 min = 39.3 s **Ans**

Example 19.13. *The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at 'C' is shown in Fig. 19.26. The distance 'CO' is 75 mm, O being the centre of the drum. The internal radius of the brake drum is 100 mm. The friction lining extends over an arc AB, such that the angle AOC is 135° and angle BOC is 45°. The brake is applied by means of a force at Q, perpendicular to the line CQ, the distance CQ being 150 mm.*

The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of ' θ ' with OC and may be taken as equal to $p_1 \sin \theta$, where p_1 is the maximum intensity of normal pressure.

The coefficient of friction may be taken as 0.4 and the braking torque required is 21 N-m. Calculate the force Q required to operate the brake when 1. The drum rotates clockwise, and 2. The drum rotates anticlockwise.



All dimensions in mm

Fig. 19.26

Solution. Given : $OC = 75$ mm ; $r = 100$ mm ; $\theta_2 = 135° = 135 \times \pi/180 = 2.356$ rad ; $\theta_1 = 45° = 45 \times \pi/180 = 0.786$ rad ; $l = 150$ mm ; $\mu = 0.4$; $T_B = 21$ N-m $= 21 \times 10^3$ N-mm

1. *Force 'Q' required to operate the brake when drum rotates clockwise*

We know that total braking torque due to shoe (T_B),

$$21 \times 10^3 = \mu . p_1 . b . r^2 (\cos\theta_1 - \cos\theta_2)$$

$$= 0.4 \times p_1 \times b (100)^2 \ (\cos 45° - \cos 135°) = 5656 \ p_1 . b$$

$$\therefore \qquad p_1 . b = 21 \times 10^3 / 5656 = 3.7$$

Total moment of normal forces about the fulcrum C,

$$M_N = \frac{1}{2} p_1 . b . r . OC \left[(\theta_2 - \theta_1) + \frac{1}{2} (\sin 2\theta_1 - \sin 2\theta_2) \right]$$

$$= \frac{1}{2} \times 3.7 \times 100 \times 75 \left[(2.356 - 0.786) + \frac{1}{2} (\sin 90° - \sin 270°) \right]$$

$$= 13\ 875\ (1.57 + 1) = 35\ 660 \text{ N-mm}$$

and total moment of friction force about the fulcrum C,

$$M_F = \mu . p_1 . b . r \left[r (\cos\theta_1 - \cos\theta_2) + \frac{OC}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]$$

$$= 0.4 \times 3.7 \times 100 \left[100 \ (\cos 45° - \cos 135°) + \frac{75}{4} (\cos 270° - \cos 90°) \right]$$

$$= 148 \times 141.4 = 20\ 930 \text{ N-mm}$$

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N + M_F = 35\,660 + 20\,930 = 56\,590$$

$$\therefore \quad Q = 56\,590 / 150 = 377 \text{ N}$$ **Ans.**

2. Force 'Q' required to operate the brake when drum rotates anticlockwise

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N - M_F = 35\,660 - 20\,930 = 14\,730$$

$$\therefore \quad Q = 14\,730/150 = 98.2 \text{ N}$$ **Ans.**