

(3 Hours)

[ Total Marks : 100

- N.B.** (1) Question No. 1 is compulsory.  
(2) Attempt any four from remaining six questions.  
(3) Figures to the right indicate the full marks.

1. (a) Let  $X$  be a continuous random variable with probability distribution — 5

$$P(X) = \frac{x}{6} + K, \quad 0 \leq x \leq 3$$
$$= 0, \quad \text{otherwise}$$

Find  $K$  and  $P(1 \leq X \leq 3)$ .

- (b) A relation  $R$  is the set of integers is defined by  $xRy$  if and only if  $x < y + 1$ . 5  
Examine whether  $R$  is —

- (i) reflexive  
(ii) symmetric  
(iii) transitive.

- (c) Find the eigen values and eigen vectors corresponding to the following matrix — 5

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

- (d) Find Laurent's series for — 5

$$f(z) = (z - 3) \sin\left(\frac{1}{z+2}\right) \text{ about } z = -2.$$

2. (a) Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five? 7

(b) Evaluate  $\int_0^{2\pi} \frac{\cos 2\theta}{(5 + 4 \cos \theta)} d\theta$ . 7

- (c) Show that the set of matrices  $m = \begin{bmatrix} a & b \\ -5b & a \end{bmatrix}$ ,  $a, b \in \mathbb{Z}$  form an integral domain. 6

Is it a field?

3. (a) Evaluate  $\oint_C \tan z \, dz$  where  $C$  7

(i) is the circle  $|z| = 2$

(ii) is the circle  $|z| = 1$ .

- (b) Is the following function injective, surjective? 7

$$f: \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = 2x^2 + 5x - 3.$$

- (c) Fit a Binomial distribution to the following data:— 6

<b>X</b>	0	1	2	3	4
<b>Frequency</b>	12	66	109	59	10

4. (a) If  $X$  is a normal variate with mean 10 and standard deviation 4, find — 7

(i)  $P(|X - 14| < 1)$

(ii)  $P(5 \leq X \leq 18)$

(iii)  $P(X \leq 12)$ .

(b) Let  $(G, *)$  be a group. Prove that  $G$  is an Abelian group if and only if 7

$$(a * b)^2 = a^2 * b^2,$$

where  $a^2$  stands for  $a * a$ .

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- (c) Using Poisson distribution find the approximate value of 6  
 ${}^{300}C_2 (0.02)^2 (0.98)^{298} + {}^{300}C_3 (0.02)^3 (0.98)^{297}$ .
5. (a) Show that the matrix  $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$  is similar to a diagonal matrix. Also 7  
 find the transforming matrix and the diagonal matrix.
- (b) A die was thrown 132 times and the following frequencies were observed : 7  

<b>No. obtained</b>	: 1	2	3	4	5	6	Total
<b>Frequency</b>	: 15	20	25	15	29	28	132

 Test the hypothesis that the die is unbiased.
- (c) If C is the circle  $|z|=1$ , using the integral  $\oint_C \frac{e^{kz}}{z} dz$ , where K is real, 6  
 show that  $\int_0^\pi e^{k \cos \theta} \cos(k \sin \theta) d\theta = \pi$ .
6. (a) Let  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$  and R be the relation 'is divisible by'. Obtain 7  
 the relation matrix and draw the Hasse diagram.
- (b) A certain injection administered to 12 patients resulted in the following changes 7  
 of blood pressure,  
 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4.  
 Can it be concluded that the injection will be in general accompanied by an increase in blood pressure ?
- (c) If  $X_1$  has mean 5 and variance 5,  $X_2$  has mean -2 and variance 3. If  $X_1$  and  $X_2$  6  
 are independent random variables, find —  
 (i)  $E(X_1 + X_2)$ ,  $V(X_1 + X_2)$   
 (ii)  $E(2X_1 + 3X_2 - 5)$ ,  $V(2X_1 + 3X_2 - 5)$ .
7. (a) A random variable X has the following probability distribution :— 7  

<b>X</b>	: -2	3	1
<b>P(X = x)</b>	: 1/3	1/2	1/6

 Find (i) Moment generating function  
 (ii) First two raw moments  
 (iii) First two central moments.
- (b) Verify Cayley-Hamilton theorem for the matrix A and hence find  $A^{-1}$  where 7  

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$
.
- (c) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24. 6  
 Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance ?