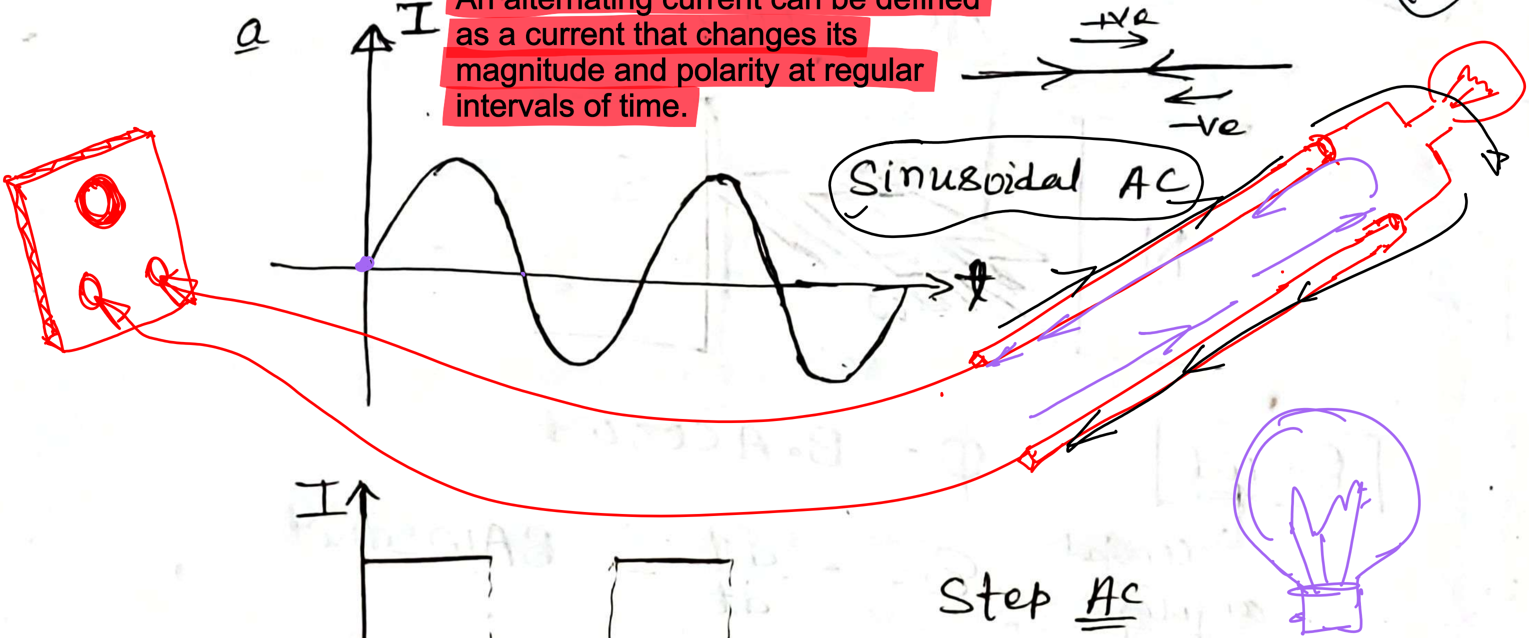
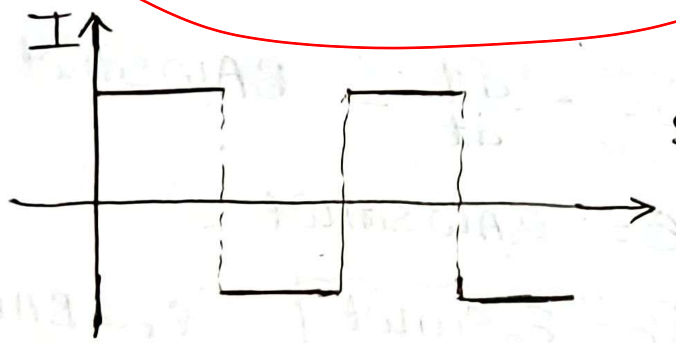
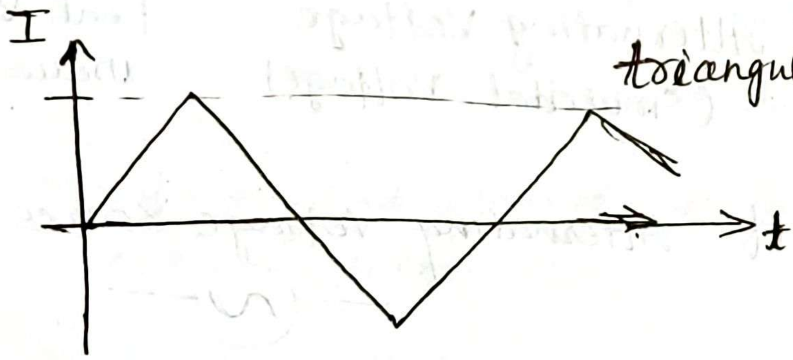
Alternating Currents (A.C)

An alternating current can be defined as a current that changes its magnitude and polarity at regular intervals of time.

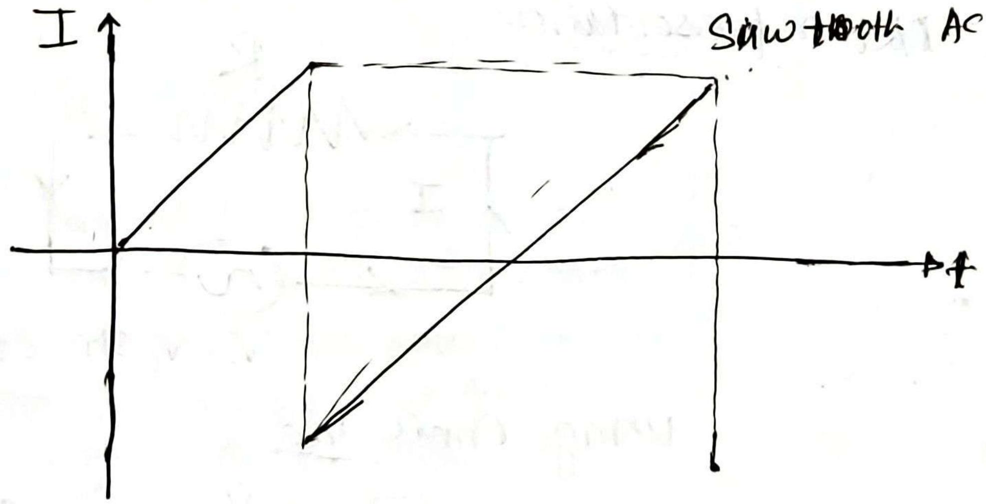




Step AC



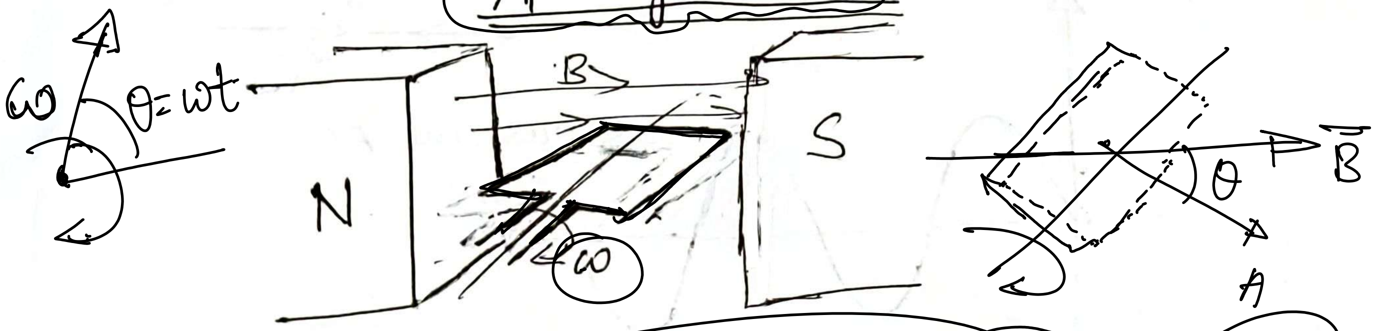
triangular AC



Saw tooth AC

Current which continuously (periodically) changes it's magnitude and direction called AC

Ac generator



$BA\cos\theta$

$\boxed{\theta = \omega t}$ → constant angular velocity

$\phi = NB \cdot A\cos\omega t$

$\mathcal{E} = -\frac{d\phi}{dt} = BA\omega\sin\omega t$

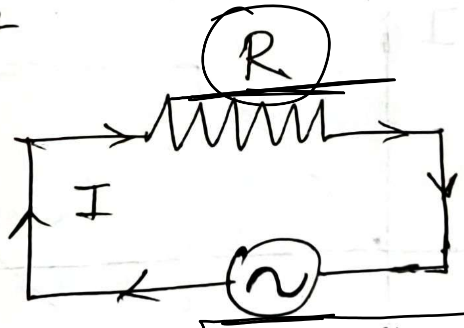
$\mathcal{E} = NBA\omega\sin\omega t$

or, $\boxed{\mathcal{E} = \mathcal{E}_0 \sin\omega t}$ $\quad$ $\mathcal{E}_0 = NBA\omega$

Alternating Voltage (Sinusoidal Voltage) $\quad$ Peak value of induced emf

Symbol of Alternating voltage Source

In ckt representation



$\boxed{V = V_0 \sin\omega t}$

Using Ohm's law

$I = \frac{V}{R} = \frac{V_0}{R}\sin\omega t$

$\boxed{I = I_0 \sin\omega t}$

$\boxed{I_0 = \frac{V_0}{R}}$

$V_0 \rightarrow$ Peak voltage (Max^m)

$I_0 \rightarrow$ Peak current (Max^m)

$I_0 \rightarrow$ Peak current or Maximum current

$V_0 \rightarrow$ Peak Voltage or maximum Voltage

$\omega \rightarrow$ Anguelar frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$\omega t \rightarrow$ Phase angle or phase of AC

In general eqn of Alternating voltage

$$V = V_0 \sin(\omega t + \phi)$$

ϕ is Inital phase (at $t=0$)

eqn of Alternating current

$$I = I_0 \sin(\omega t + \phi)$$

Ques $V = 10\sin\left(3t + \frac{\pi}{4}\right)$ volt

$R = 0.1\Omega$

$I = ?$

$I_0 = ?$

$\omega = ?$

$\phi = ?$

Note ✓ voltage and current in this case changes but "ω" doesn't change

✓ In house hold supply - $f = 50Hz$ so $\omega = 2\pi f = 100\pi$

220V

Average Value

$$I_{Avg} = \frac{\text{Total charge}}{\text{Total time}}$$

→ I_{av}

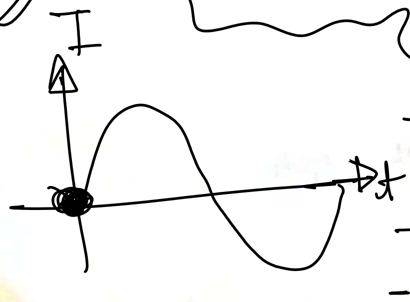
* Avg. current for a given time interval is defined as "the ratio of the total charge passed through the ckt during that time interval to the time taken"

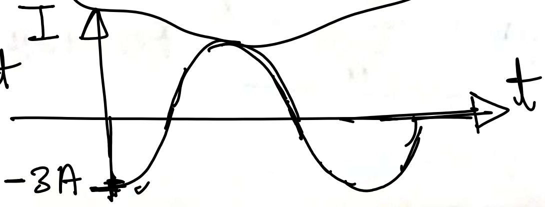
$$I_{av} = \frac{q}{t}$$

$V = V_0 \sin \omega t$

$I = I_0 \sin \omega t$

$I_0 = \frac{V_0}{R}$





$t=0$ $\omega \times 0 = 0$

$I = I_0 \sin(\omega t + \phi)$

① $I = I_0 \sin\phi$ (at $t=0$)

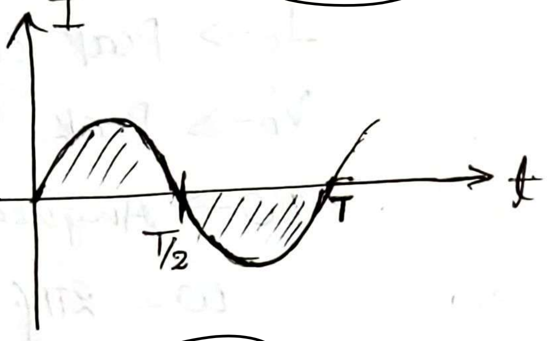
$I = \frac{dq}{dt} \Rightarrow dq = I\,dt$ $\quad I_0 = 3A \quad \phi = -\pi/2 \quad I = -3A$

$q = \int I\,dt$

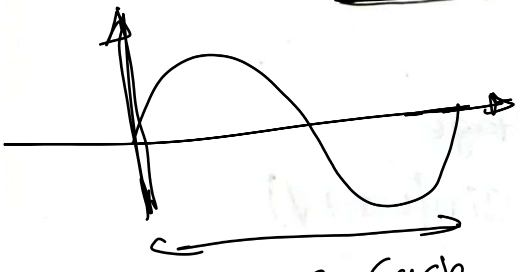
Mathematically

$$I_{av} = \frac{\int_0^t I\,dt}{t}$$

Total charge betn 0 to t time



Over a full cycle $t = T$



$$\boxed{I_{av} = \frac{\int_0^T I_0 \sin \omega t \, dt}{T}}$$

$$\frac{-I_0}{T\omega}\left[\cos\omega T - \cos 0\right]$$

$$I_{av} = \frac{I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^T = 0$$

$$\Rightarrow \frac{-I_0}{T\omega}\left[\cos \frac{2\pi}{T}\times T - \cos 0\right]$$

$$0$$

→ So, we calculate I_{av} over a Half cycle ie for $t = T/2$

V.V.I

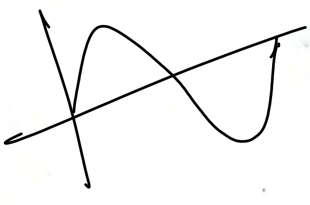
$$I_{av} = \frac{\int_0^{T/2} I_0 \sin\omega t\, dt}{T/2}$$

or, $$I_{av} = \frac{2I_0}{T}\int_0^{T/2} \sin\omega t\, dt$$

or, $$I_{av} = \frac{2I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

or, $$I_{av} = \frac{-2I_0}{\omega T}\left[\cos\frac{2\pi}{T}\times\frac{T}{2} - \cos\frac{2\pi}{T}\times 0\right] \quad \text{as, } \omega = \frac{2\pi}{T}$$

or, $$I_{av} = \frac{-2I_0}{\left(\frac{2\pi}{T}\right)\times T}\times(-2)$$



or, $$\boxed{I_{av} = \frac{2I_0}{\pi}}$$ For half cycle

Similarly

$$\boxed{V_{av} = \frac{2V_0}{\pi}}$$ For half cycle

& $\boxed{V_{av} = 0}$ For full cycle

→ The I_{av} & V_{av} can't be used to calculate Power calculation in ckt

RMS current and RMS Voltage

RMS (Root mean square)

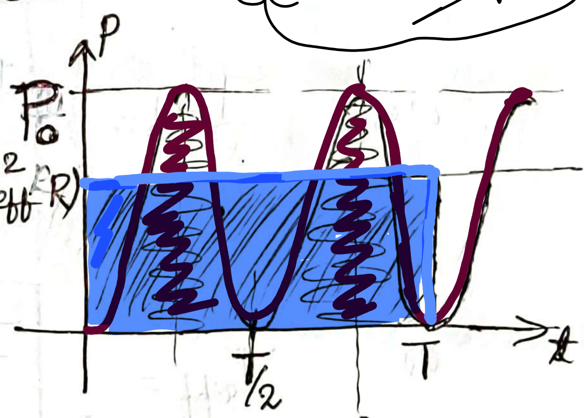
RMS ⇒ Root mean square

as $P = I^2R$

or, $P = I_0^2 R \sin^2 \omega t$ $(I_{eff}^2 R)$

or, $\boxed{P = P_0 \sin^2 \omega t}$

$\boxed{P_0 = I_0^2 R}$



Effective or RMS current is a steady (DC) current which gives us same amount of heat in one cycle (time period T) as that given by AC

Energy $= \int P\, dt$ → time, Power

If RMS current = I_{RMS} or I_{eff}

then Mathematically

$$\underbrace{I_{eff}^2 RT}_{\text{Heat generated in T time by } I_{eff}} = \underbrace{\int_0^T I^2 R\, dt}_{\text{Heat generated in 'T' time by AC}}$$

Using this result

$$I_{eff}^2 \cancel{R} T = \int_0^T I^2 \cancel{R}\, dt$$

I^2

$$I_{eff}^2 T = \int_0^T I_0^2 \sin^2 \omega t \, dt$$

$$\boxed{I_{eff} = \sqrt{\frac{\int_0^T I_0^2 \sin^2 \omega t \, dt}{T}}}$$

→ (mean) avg of square of I

or, $\boxed{I_{eff} = \sqrt{\frac{\int_0^T I^2 dt}{T}}}$

RMS

$$\text{or,} \quad I_{eff} = \sqrt{\frac{I_0^2 \int_0^T \frac{(1-\cos 2\omega t)}{2} dt}{T}}$$

$$\text{or,} \quad I_{eff} = \sqrt{\frac{I_0^2 \left[\frac{1}{2}\int_0^T dt - \left\{\int_0^T \frac{\cos 2\omega t}{2} dt\right\}\right]}{T}}$$

$$\text{or,} \quad I_{eff} = \sqrt{\frac{I_0^2 \times \cancel{T}/2}{\cancel{T}}}$$

→ Peak current

$$\text{or,} \quad \boxed{I_{eff} = \frac{I_0}{\sqrt{2}}}$$

$I_{RMS} =$

$I_0 = I_{max}$

Simillarly

$$\boxed{V_{eff} = \frac{V_0}{\sqrt{2}}}$$

$V_0 = V_{max}$

All AC appliances are rated with RMS values of voltage and current (alternating)

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151

Ex

A 60 W, 220 V. bulb

$\rightarrow$ RMS Voltage

$\rightarrow$ Resistance

to find ① Peak Voltage
(ii) RMS current
(iii) Resistance

P V, R

$$\frac{V^2}{R} = P$$

Sol^n

$$V_{rms} = 220V$$

Note

$$\boxed{\begin{aligned} P &= \frac{V_{rms}^2}{R} \\ P &= I_{rms}^2 R \\ P &= V_{rms} \cdot I_{rms} \end{aligned}}$$

→ Formulae valid ~~only~~ iff only resistances are in ckt (i.e no capacitor, no inductor)

So ① V_0 = Peak voltage

$$V_{RMS} = \frac{V_0}{\sqrt{2}} \Rightarrow V_0 = \sqrt{2}\, V_{RMS}$$

or, $\boxed{V_0 = \sqrt{2} \times 220\ V}$

(ii) RMS current

$$I_{RMS} = \frac{P}{V_{RMS}} = \frac{60}{220}$$

$$I_{RMS} = \frac{3}{11} A$$

$$I_0 = \sqrt{2}\, I_{rms} = \frac{3\sqrt{2}}{11} A$$

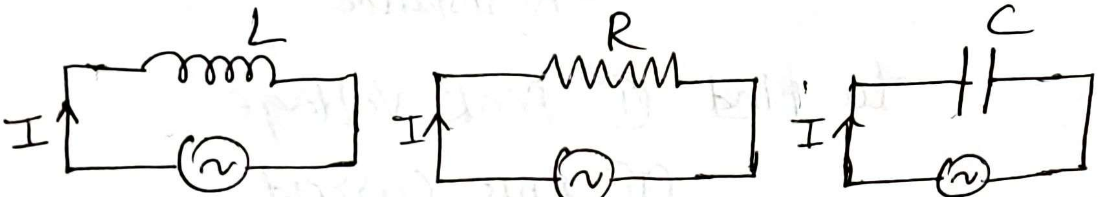
(iii) Resistance

$$R = \frac{V_{rms}^2}{P}$$

$$= \frac{220 \times 220}{60}$$

$$R = \frac{2420}{3}\ \Omega$$

AC Circuits



$V = V_0 \sin\omega t$ $V = V_0 \sin\omega t$ $V = V_0 \sin\omega t$

two SHM oscillation can be added by using phasor diagram

$$y = A\sin(\omega t + \phi)$$

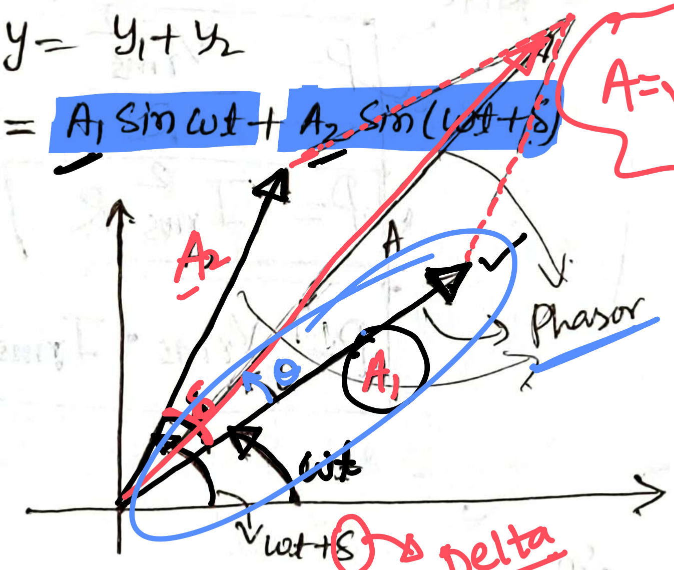
$$V = V_0 \sin(\omega t + \phi)$$

as $y = y_1 + y_2$

$y = A_1 \sin\omega t + A_2 \sin(\omega t + \delta)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$

Phasor diagram method.



δ → Delta

$$\tan\theta = \frac{A_2 \sin\delta}{A_1 + A_2\cos\delta}$$

$y = A\sin(\omega t + \theta)$

$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}}$$

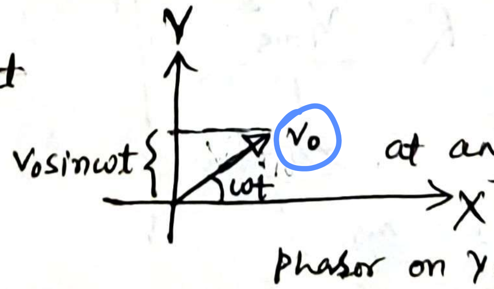
Q. $3\sin\omega t + 4\sin(\omega t + 60°)$

$\sqrt{37}\sin(\omega t + 14.8°)$

Since here Voltages and currents are Sinusoidal, these can't be added simply. We have to use a convinient method which ... using which we can add two Alternating (sinusoidal) quantities taking the effect of their phase and magnitude at the same time.

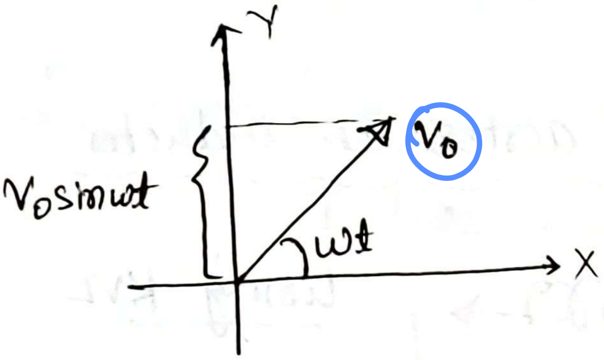
Phasor representation

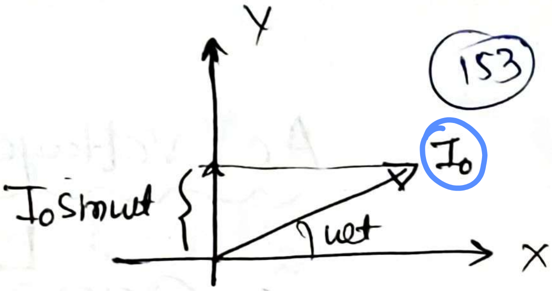
$V = V_0 \sin\omega t$



at any time 't'

projection of Phasor on Y axis give 'V' at a time 't'

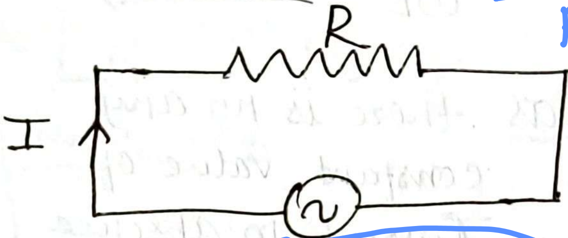




<u>Note</u> Voltage and current are not vectors but only this method we add these quantities as <u>vectors</u>

<u>AC voltage across a resistance</u>

<u>Pure</u> <u>Resistive</u> <u>circuit</u>



$I = \frac{V}{R}$

$V = V_0 \sin\omega t$

or, $\boxed{I = \frac{V_0}{R} \sin\omega t}$

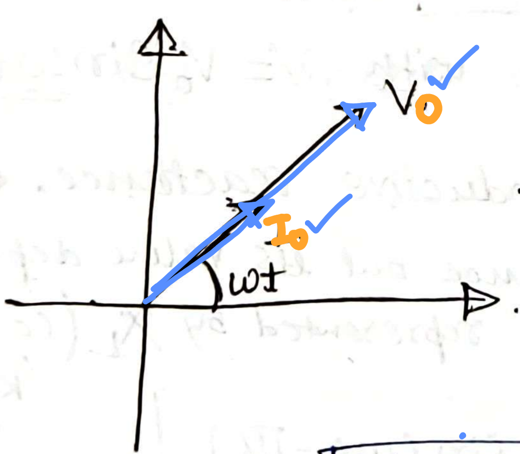
or $\boxed{I = I_0 \sin\omega t}$

$\frac{V_0}{R}$

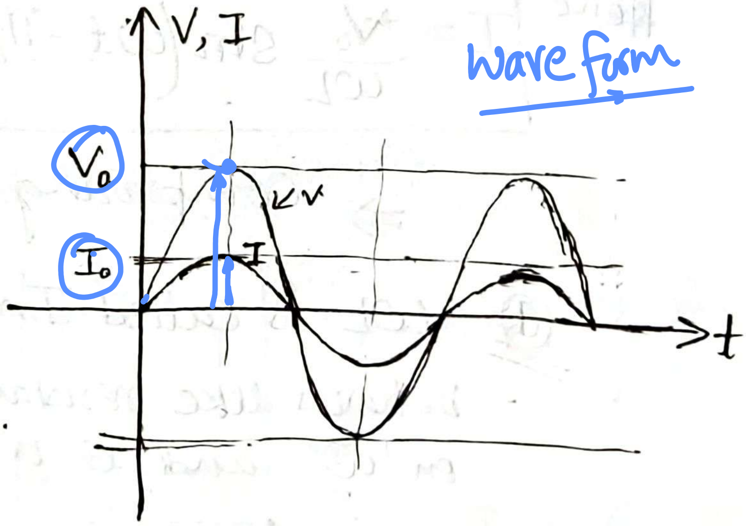
Here (a) V & I are in <u>phase</u>

(b) $I_0 = \frac{V_0}{R}$

<u>Phasor deagram</u>



<u>Wave form</u>



$\boxed{V_0 = I_0 R}$

$\boxed{V = IR}$. For AC resisutive (purely) <u>ckt</u>

Ac voltage across an inductor

Using KVL

$$-L\frac{dI}{dt} + V_0 \sin\omega t = 0$$

$$\Rightarrow L\frac{dI}{dt} = V_0 \sin\omega t$$

$$\Rightarrow dI = \frac{V_0}{L}\sin\omega t\, dt$$

$$\Rightarrow I = \int \frac{V_0}{L}\sin\omega t\, dt$$

$$\Rightarrow I = +\frac{V_0}{L}\left(\frac{-\cos\omega t}{\omega}\right) + c$$

$$\Rightarrow \boxed{I = \frac{-V_0}{\omega L}\cos\omega t + c}$$

or, $I = \frac{V_0}{\omega L}\sin(\omega t - \pi/2) + c$

as there is no any constant value of current in absence of voltage. so I will also oscillate about (up & down) '0' value, so $c = 0$

$-\cos\theta$

$\sin(\theta - \pi/2)$

$\sin(\pi/2 - \theta) = \cos\theta$

hence $\boxed{I = \frac{V_0}{\omega L}\sin(\omega t - \pi/2)}$

$\Rightarrow$ Comparing with $V = V_0 \sin\omega t$

(a) ωL is called Inductive reactance. it behaves like resistance but it's value depends on 'ω' and it is represented by X_L (called 'R_{eff}' also)

SI unit $= \Omega$

so $\boxed{I = \frac{V_0}{X_L}\sin(\omega t - \pi/2)}$

$$\boxed{V_0 = I_0 X_L}$$

$$\boxed{V_{rms} = I_{rms} X_L}$$

~~Purely~~ Purely Inductive AC Ckt

$$\boxed{X_L = \omega L}$$

$\omega = 2\pi f = \frac{2\pi}{T}$

$$\boxed{I = I_0 \sin(\omega t - \pi/2)}$$

$$\boxed{I_0 = V_0/X_L}$$

$X_L = 2\pi f L = \frac{2\pi}{T} L$

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② Comparing with $V = V_0 \sin \omega t$

I lags behind V by $\pi/2$

For DC source

$f = 0$

$\to \omega = 0$

$X_L = 0$

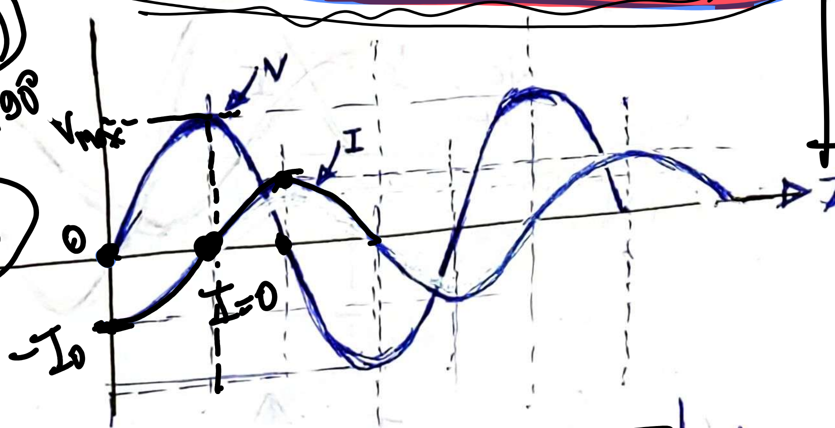
For DC inductor acts as short circuit

If $V = V_0 \sin 0° = 0$

$I = I_0 \sin(-90°)$

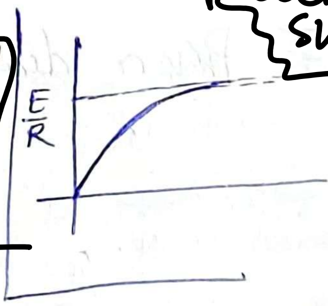
$I = -I_0 \sin 90°$

$I = -I_0$



For dc $\omega = 0$ so $\boxed{X_L = \omega L = 0}$

For dc No X_L exist



$V = 10 \sin(3t + 60°)$

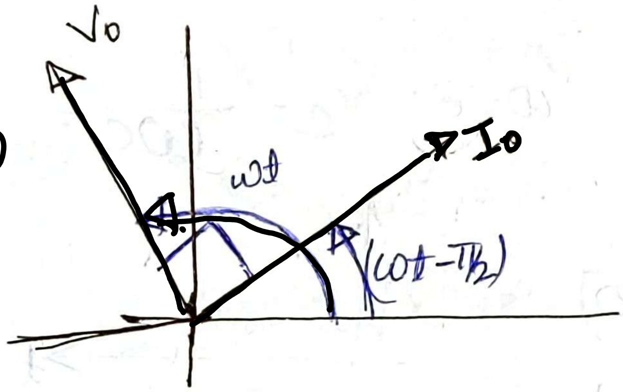
$L = 10\,mH$

$X_L =$

$I_0 = ?$

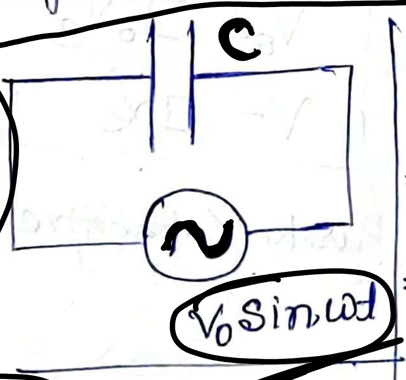
$V_0 = 0$

$\frac{10}{10 \times 10^{-3} \times 3}$



$I = \frac{1000}{3} \sin(3t - 30°)$

AC Voltage across capacitor



$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$

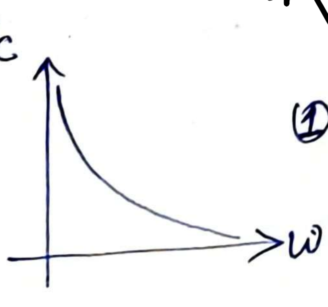
$q = CV = CV_0 \sin \omega t$

$\Rightarrow \frac{dq}{dt} = CV_0 \omega \cos \omega t$

$\Rightarrow I = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \sin(\omega t + \pi/2)$

or $I = \frac{V_0}{X_C} \sin(\omega t + \pi/2)$

$X_C = \frac{1}{\omega C}$



① $X_C = \frac{1}{\omega C}$

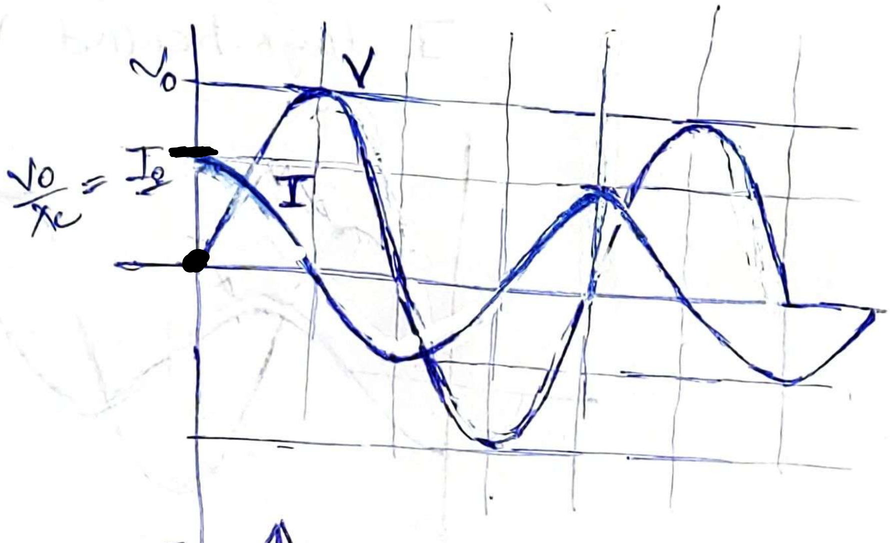
effective resistance

(capacitive Reactance)

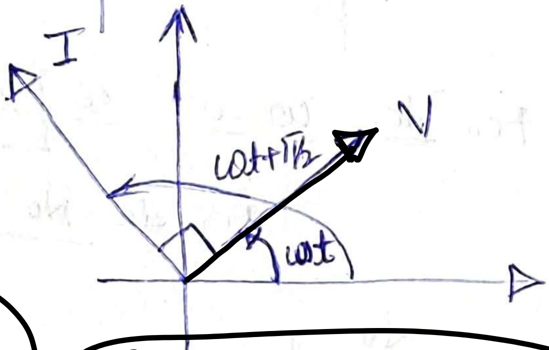
2. V lags behind I by $\pi/2$

$V = V_0 \sin 0^\circ = 0$

$I = I_0 \sin(\omega 90^\circ) = I_0$



3. Phasor diagram

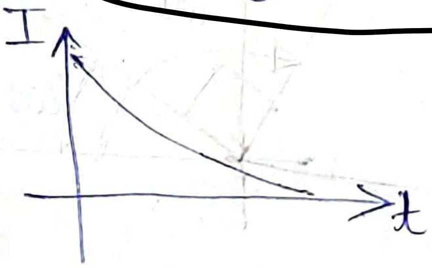


4. For DC $f = 0$, $\omega = 0$ → $X_C = \frac{1}{\omega C} \rightarrow \infty$ → open circuit

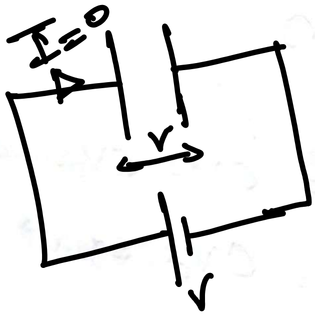
For DC ✓

$\boxed{X_C = \infty}$

Open ckt.



For DC circuit Capacitor acts as open circuit

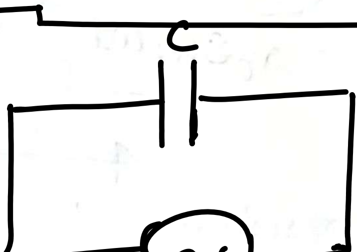


$$\boxed{\begin{aligned} V_0 &= I_0 X_C \\ V_{rms} &= I_{rms} X_C \end{aligned}}$$

Purely Capacitive AC Ckt

Ques

$V_{RMS} = \frac{V_0}{\sqrt{2}}$



$V = 15 \sin(\omega t - 30^\circ)$

$C = 3 \mu F$ (10^{-6})

$\omega = 100$

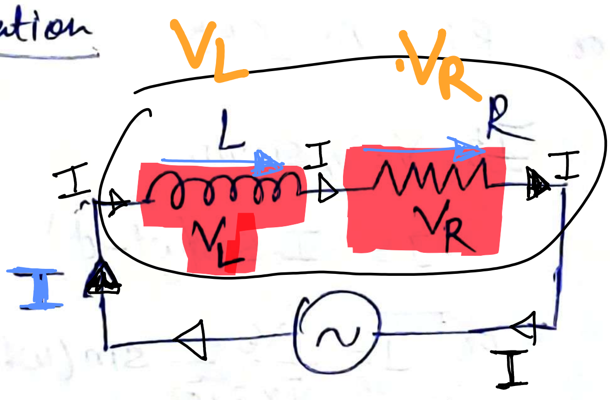
$X_C = \frac{1}{100 \times 3 \times 10^{-6}}$

$I_0 =$

$I = I_0$

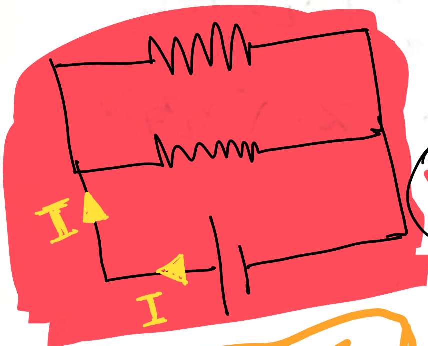
Series combination

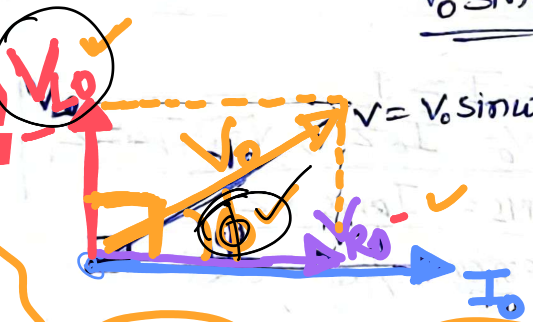
① L-R ckt



in series

$V_0 \sin\omega t$





V_{L0} = Max^m voltage across inductor

V_{R0} = max^m voltage across resistor

V_0 = Max^m supplied voltage

$\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$

$\sqrt{V_{L0}^2 + V_{R_0}^2 + 2V_{L0}V_{R_0}\cos 90°}$ also

$V_{L0} = I_0 X_L$; $V_{R_0} = I_0 R$

$$V_0 = \sqrt{V_{L0}^2 + V_{R0}^2}$$

$$V_0 = \sqrt{I_0^2 X_L^2 + I_0^2 R^2}$$

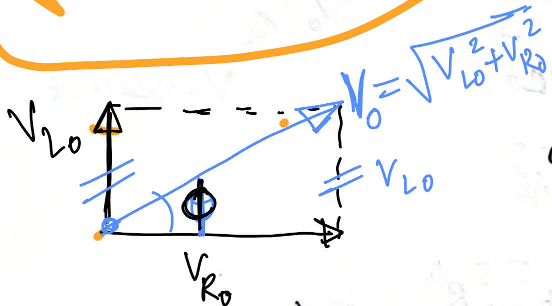
$$V_0 = I_0 \sqrt{X_L^2 + R^2}$$

or, $\boxed{V_0 = I_0 Z}$

$Z \rightarrow$ Impedance of ckt

$$Z = \sqrt{X_L^2 + R^2}$$

SI unit - Ω



Now

$$\tan\phi = \frac{V_{L0}}{V_{R0}} = \frac{\cancel{I_0} X_L}{\cancel{I_0} R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

also $\cos\phi = \dfrac{V_{R0}}{V_0} = \dfrac{\cancel{I_0} R}{\cancel{I_0} Z}$

Power factor

$$\boxed{\phi = \cos^{-1}(R/Z)}$$

I can be written as

$$I = I_0 \sin(\omega t - \phi)$$

because I is lagging behind V by angle ϕ

$$\boxed{I_0 = \frac{V_0}{Z}} = \frac{V_0}{\sqrt{X_L^2 + R^2}}$$

So, For RL AC ckt

If $V = V_0 \sin\omega t$

$I = I_0 \sin(\omega t - \phi)$

or, $\boxed{I = \dfrac{V_0}{\sqrt{X_L^2 + R^2}} \sin(\omega t - \phi)}$

$I_{RMS} = \dfrac{V_{RMS}}{Z}$

$X_L = \omega L = 2\pi f L$

~~V_0 ... I~~

$\boxed{V_0 = I_0 Z \; ; \; \sqrt{X_L^2 + R^2} = Z}$

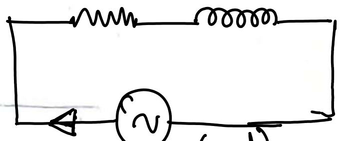
$\boxed{V_{RMS} = I_{RMS} \times Z}$

$\tan\phi = \dfrac{X_L}{R} = \dfrac{\omega L}{L}$

$\boxed{X_L = \omega L}$

$Z = \sqrt{X_L^2 + R^2}$

Ques $R = 4\Omega$ $L = 3mH$

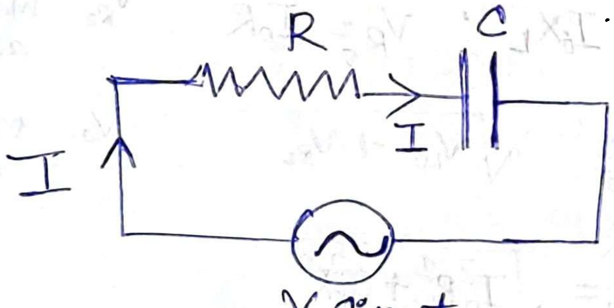


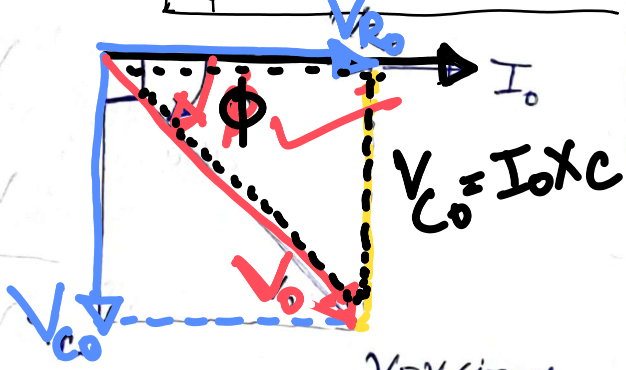
$V = 20 \sin(100t)$

$X_L = ?$ $Z =$

$\phi =$ $I_0 =$

R-C Series AC Ckt





$V_{C_0} = I_0 X_C$

$V_{R_0} = I_0 R$

$V_0 = \sqrt{V_{C_0}^2 + V_{R_0}^2}$

$V_{R_0} \rightarrow V_{max}$ across 'R'

$V_{C_0} \rightarrow V_{max}$ across 'C'

$V_0 \rightarrow$ 'Max^m supplied Voltage

$V_0 = \sqrt{I_0^2 X_C^2 + I_0^2 R^2}$

$V_0 = I_0 \sqrt{X_C^2 + R^2}$

or, $\boxed{V_0 = I_0 Z}$; $\boxed{Z = \sqrt{X_C^2 + R^2}}$

Impedance

$\tan\phi = \dfrac{V_{C_0}}{V_R} = \dfrac{\cancel{I_0} X_C}{\cancel{I_0} R} = \dfrac{X_C}{R} = \dfrac{1}{\omega R C}$

$\boxed{\tan\phi = \dfrac{1}{\omega R C}}$ $\boxed{\cos\phi = \dfrac{V_R}{V} = \dfrac{R}{Z}}$

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So For RC AC CKt

If $V = V_0 \sin\omega t$ supplied Voltage

$I = I_0 \sin(\omega t + \phi)$ (Forward)

$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{X_C^2 + R^2}}$ } Since I is leading by an angle ϕ from supplied Voltage

Also

$V_0 = I_0 Z$

$Z = \sqrt{X_C^2 + R^2}$

$\boxed{V_{RMS} = I_{RMS} \times Z}$

$X_C = \frac{1}{\omega C}$; $\boxed{\tan\phi = \frac{V_{C0}}{V_{R0}} = \frac{1}{\omega RC}}$

$\boxed{\cos\phi = \frac{R}{Z}}$

Ex

$\frac{V_{L0} = \cancel{I_0} X_L}{V_{R0} = \cancel{I_0} R}$

If given 100 V dc → 1 A current

If given 100 V, 50Hz AC → 0.5A current

Find a) Impedance of coil

b) Reactance, Resistance → R = ?

c) Instanteneous current in AC

if $V = V_0 \sin\omega t$ Supply voltage

$I = ?$

$\phi = ?$

Solution

In dc, $X_L = 0$

only R

so $100 = R \times I$

or, $R = \frac{100}{1} = 100\Omega$

$V = IZ$

In AC $X_L = \omega L$ & effective resistance $Z = \sqrt{X_L^2 + R^2}$

Given for AC

$V_{rms} = 100V$

$I_{rms} = 0.5A$

$I_{rms} = \frac{V_{rms}}{Z} \Rightarrow 0.5 = \frac{100}{Z} \Rightarrow Z = 200\Omega$

Now

$Z = \sqrt{X_L^2 + R^2} = \sqrt{(\omega L)^2 + R^2}$ $\quad V_0 = ?$

$\Rightarrow 200 = \sqrt{[(50 \times 2\pi) \times L]^2 + 100^2}$

$I_0 = \frac{V_0}{Z}$

$\Rightarrow 200^2 = (\underbrace{100\pi L}_{X_L})^2 + 100^2 \Big\} \to X_L = \sqrt{200^2 - 100^2}$

$\Rightarrow \cancel{200^2 - 100^2 = X_L}$

$\Rightarrow 100\sqrt{3} = X_L$

$\Rightarrow X_L = \omega L = 2 \times \pi \times 50 \times L = 100\sqrt{3}$

$$\boxed{L = \frac{\sqrt{3}}{\pi} H}$$

$X_L = \omega L$

$X_L = 2\pi f L$

$\phi = ?$

$I_0 = ?$

Now Instantenous current

$V_{RMS} = 100V = \frac{V_0}{\sqrt{2}}$

$V_0 = 100\sqrt{2}$ V

$V = 100$ V

$\frac{V_0}{\sqrt{2}} = V_{RMS} \Rightarrow V_0 = 100\sqrt{2}$

$I_{RMS} = 0.\cancel{5} = \frac{I_0}{\sqrt{2}}$

$I_0 = \sqrt{2}\, I_{RMS}$ ... or $I_0 = \frac{V_0}{Z} = \frac{1}{\sqrt{2}}$

Now using $I_0 = \frac{V_0}{Z}$ or $I = I_0 \sin(\omega t - \phi)$

as $Z = 100\Omega$

$V_0 = 100\sqrt{2}$ V

$$\boxed{I = \frac{1}{\sqrt{2}} \sin(\underbrace{100\pi t}_{\omega} - \pi/3)}$$

$\omega = 2\pi f$

$\tan\phi = \frac{V_L}{V_R} = \frac{\omega L}{R}$

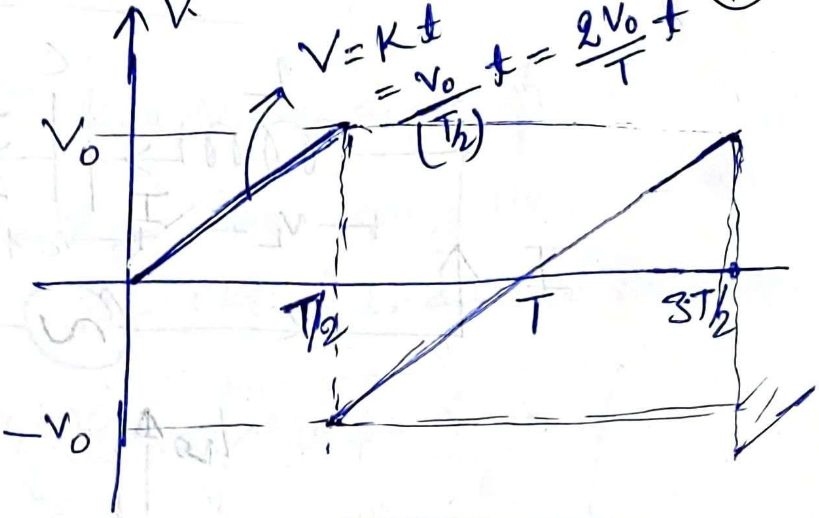
$\tan\phi = \frac{X_L}{R} = \frac{100\sqrt{3}}{100}$

$\phi = \pi/3$

Example

Find V_{RMS}

$$V = Kt = \frac{V_0}{(T/2)}\,t = \frac{2V_0}{T}\,t$$



$$V_{rms} = \sqrt{\frac{\int_0^{T/2} V_0^2\,dt}{T/2}}$$

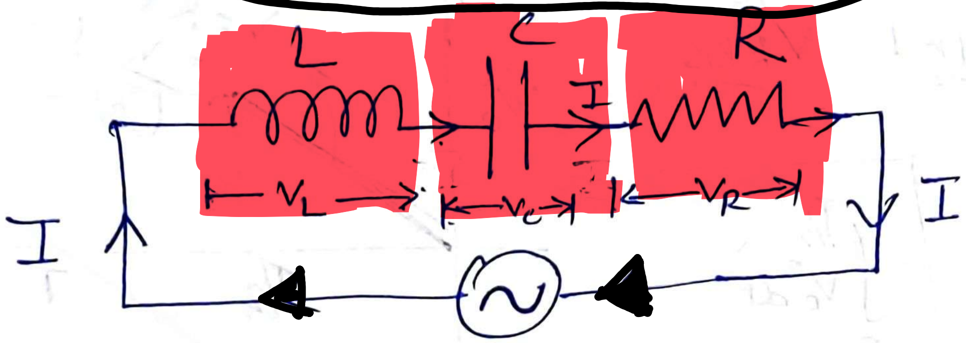
$$V_{rms} = \sqrt{\frac{\int_0^{T/2} \left(\frac{2V_0}{T}\right)^2 t^2\,dt}{T/2}} \Rightarrow V_{RMS} = \sqrt{\frac{\frac{4V_0^2}{T^2}\int_0^{T/2} t^2\,dt}{T/2}}$$

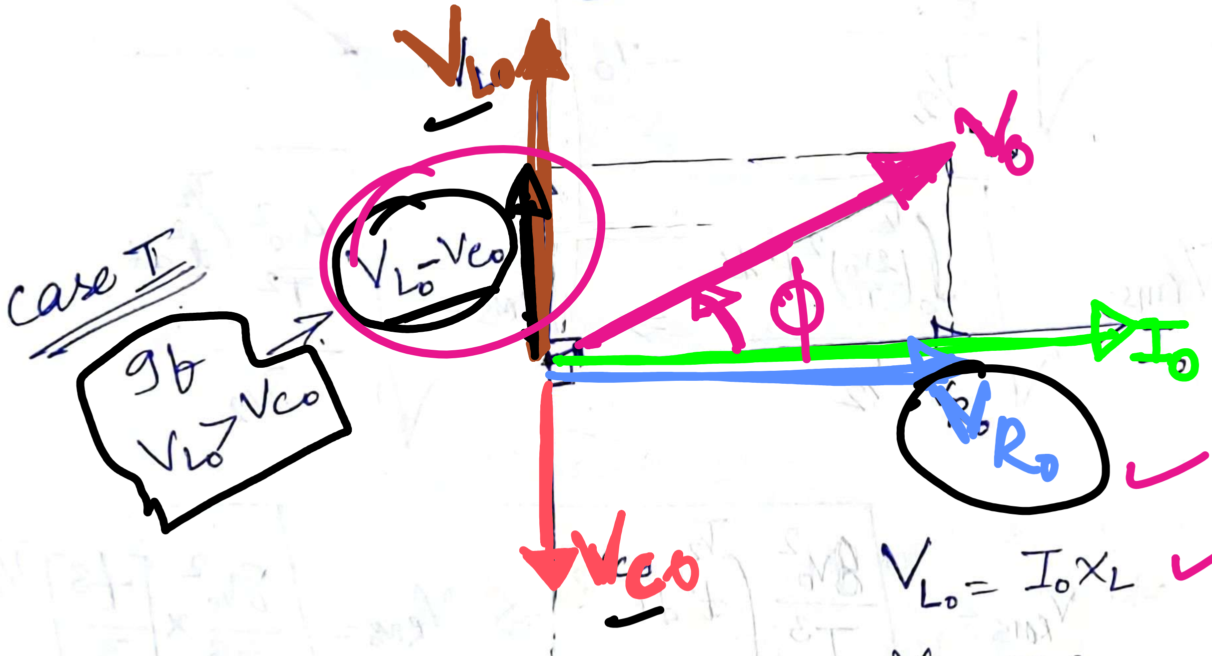
$$\Rightarrow V_{RMS} = \sqrt{\frac{8V_0^2}{T^3}\int_0^{T/2} t^2\,dt} \Rightarrow V_{RMS} = \sqrt{\frac{8V_0^2}{T^3}\times\left[\frac{t^3}{3}\right]_0^{T/2}}$$

$$V_{RMS} = \sqrt{\frac{\cancel{8}V_0^2}{\cancel{T^3}}\times\frac{\cancel{T^3}}{\cancel{8}}\times\frac{1}{3}}$$

$$\boxed{V_{RMS} = \frac{V_0}{\sqrt{3}}}$$

Series LCR ckt





$V_{L0} = I_0 X_L$

$V_{R0} = I_0 R$

$V_{C0} = I_0 X_C$

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2}$$

or, $$V_0 = \sqrt{I_0 R^2 + (I_0 X_L - I_0 X_C)^2}$$

or, $$\boxed{V_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}}$$

$V_0 = I_0 Z$

or, $\boxed{I_0 Z = V_0}$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Impedance

also

$\boxed{I_{RMS} = \frac{V_{RMS}}{Z}}$

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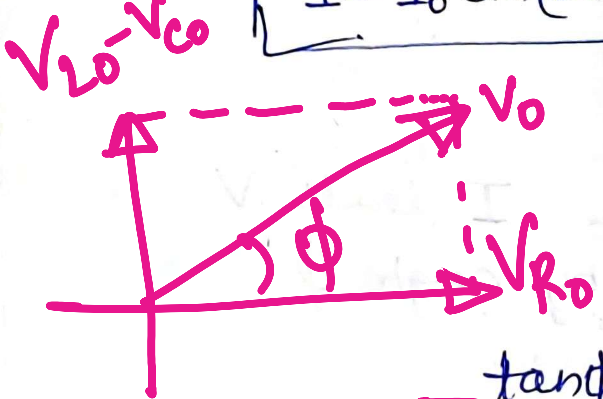
$$\boxed{I = I_0 \sin(\omega t - \phi)}$$



I lags V by angle ϕ

Ckt is ~~more~~ acting like RL-ckt (Inductive behaviour)

$$\tan\phi = \frac{V_{L0}-V_{C0}}{V_{R0}} \Rightarrow \boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

$\tan\phi =$

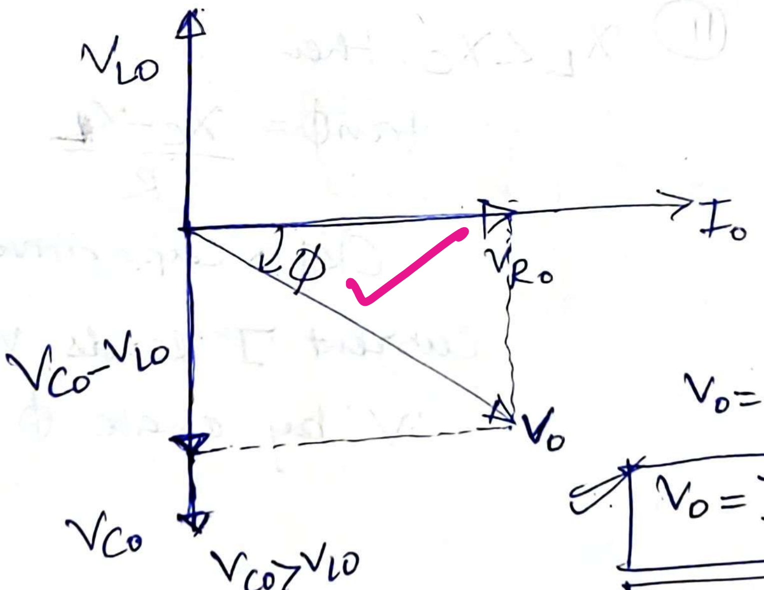
So

- $I_0 = \frac{V_0}{Z}$; $Z = \sqrt{R^2+(X_L-X_C)^2}$
- also $I_{RMS} = \frac{V_{RMS}}{Z}$
- If $V = V_0 \sin\omega t$
- $\boxed{I = \frac{V_0}{Z}\sin(\omega t - \phi)}$

$$\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$$

Case II

$V_{C0} > V_{L0}$



$$V_0 = \sqrt{V_{R0}^2 + (V_{C0}-V_{L0})^2}$$

$$\boxed{V_0 = I_0\sqrt{R^2+(X_C-X_L)^2}}$$

or, $\boxed{V_0 = I_0 Z}$

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

$$I = \frac{V_0}{Z}\sin(\omega t + \phi)$$

because I leads V by angle ϕ

$$\boxed{\tan\phi = \frac{X_C - X_L}{R}}$$

→ overall behaviour of ckt is capacitive

So

If

(i) $X_L > X_C$ then

$$\tan\phi = \frac{X_L - X_C}{R}$$

ckt → inductive

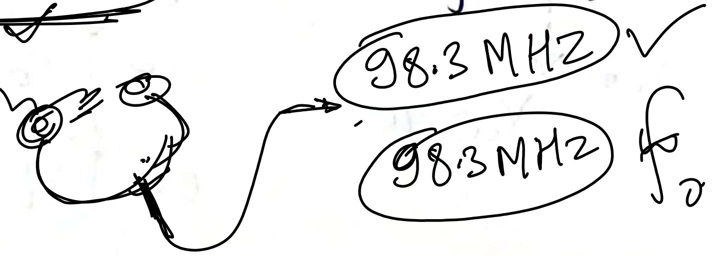
current I lags by ϕ w.r.t voltage 'V' (supplied

(ii) $X_L < X_C$, then

$$\tan\phi = \frac{X_C - X_L}{R}$$

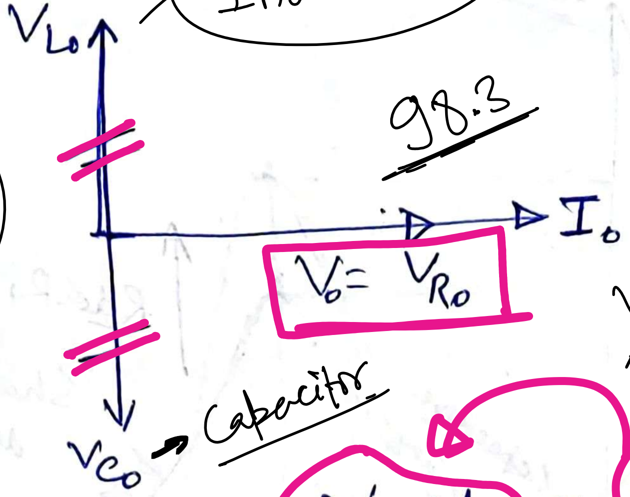
ckt → capacitive

current I leads voltage (supply 'V' by angle ϕ



Case-3

Resonance in L-C-R series AC CKt



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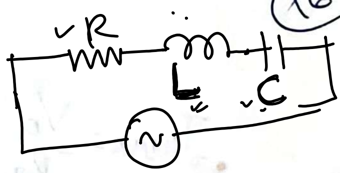


$\boxed{V_0 = V_{R_0}}$

$V_0 = \sqrt{I_0R^2 + (I_0X_L - I_0X_C)^2}$

$V_0 = I_0R$

$V_0 = V_{R_0}$ as $\boxed{V_0 = \sqrt{V_{R_0}^2 + \cancel{(V_L - V_C)}^2}}$

When

→ (V) Voltage and current (I) in series L-C-R AC CKt are in same Phase, this ~~cond~~ situation is called resonance

When $V_{Lo} = V_{Co}$ then $\boxed{V_{R_0} = V_0 \quad (V_R)_{rms} = V_{rms}}$

$\Rightarrow \cancel{I_0}X_L = \cancel{I_0}X_C$

$\Rightarrow \boxed{X_L = X_C}$

$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 L = \frac{1}{C}$

$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ ← Resonant Angular Frequency

$\omega_0 = 2\pi f_0$

Resonant Frequency → $\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$

$f_{circuit} = f_{current}$

* CKt behaviour → Purely resistive

* current at resonance

$\boxed{I_0 = \frac{V_0}{Z} = \frac{V_0}{R}}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\boxed{Z = R}$

$\boxed{I_{Rms} = \frac{V_{Rms}}{R}}$

$\uparrow I_0 = \frac{V_0}{Z \downarrow}$

* Instanteneous current

$\boxed{I = \frac{V_0}{R} \sin \omega t}$

$\because \tan\phi = \frac{X_L - X_C}{R} = 0$, $X_L - X_C \to 0$

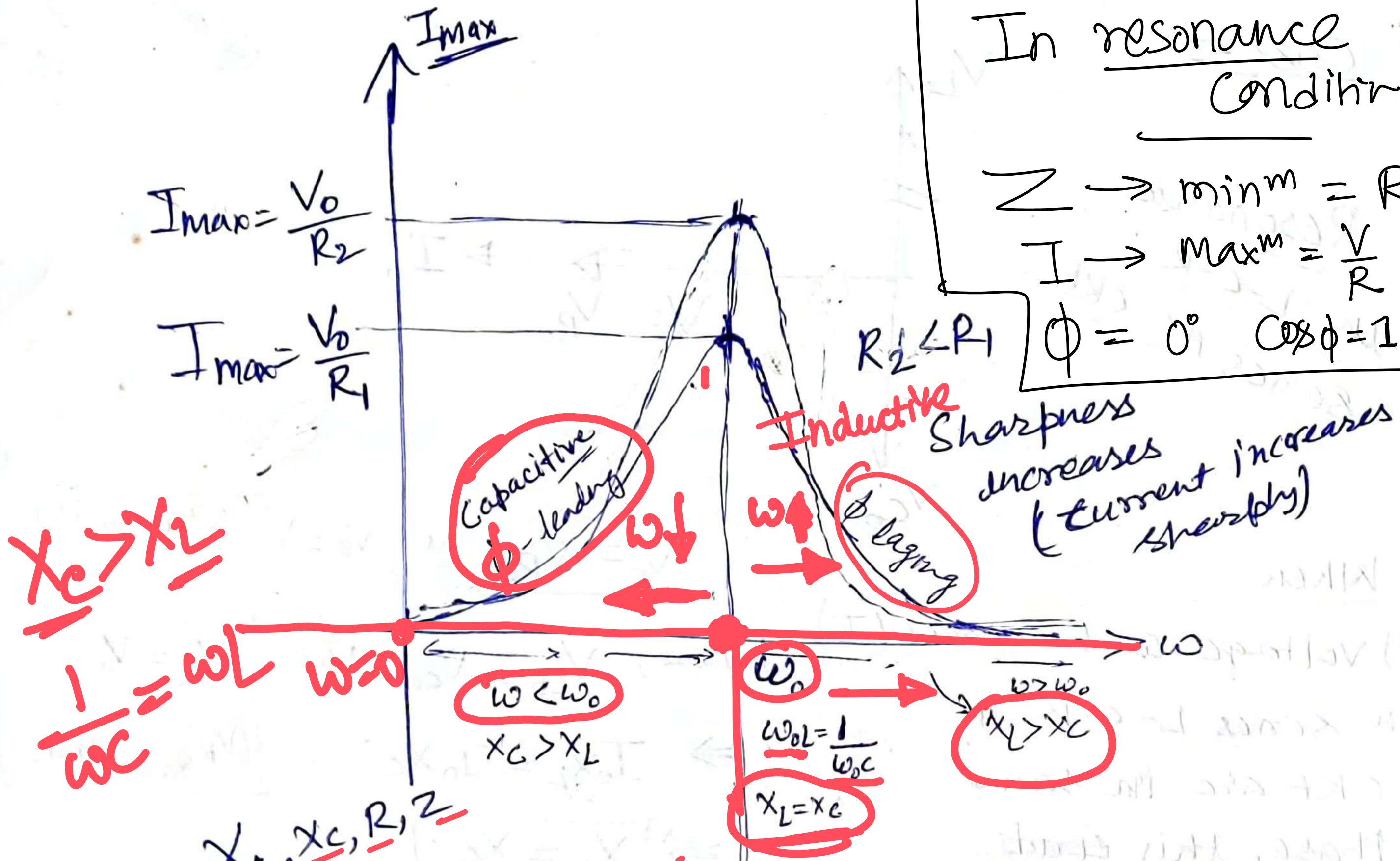
$\Rightarrow \phi = 0$

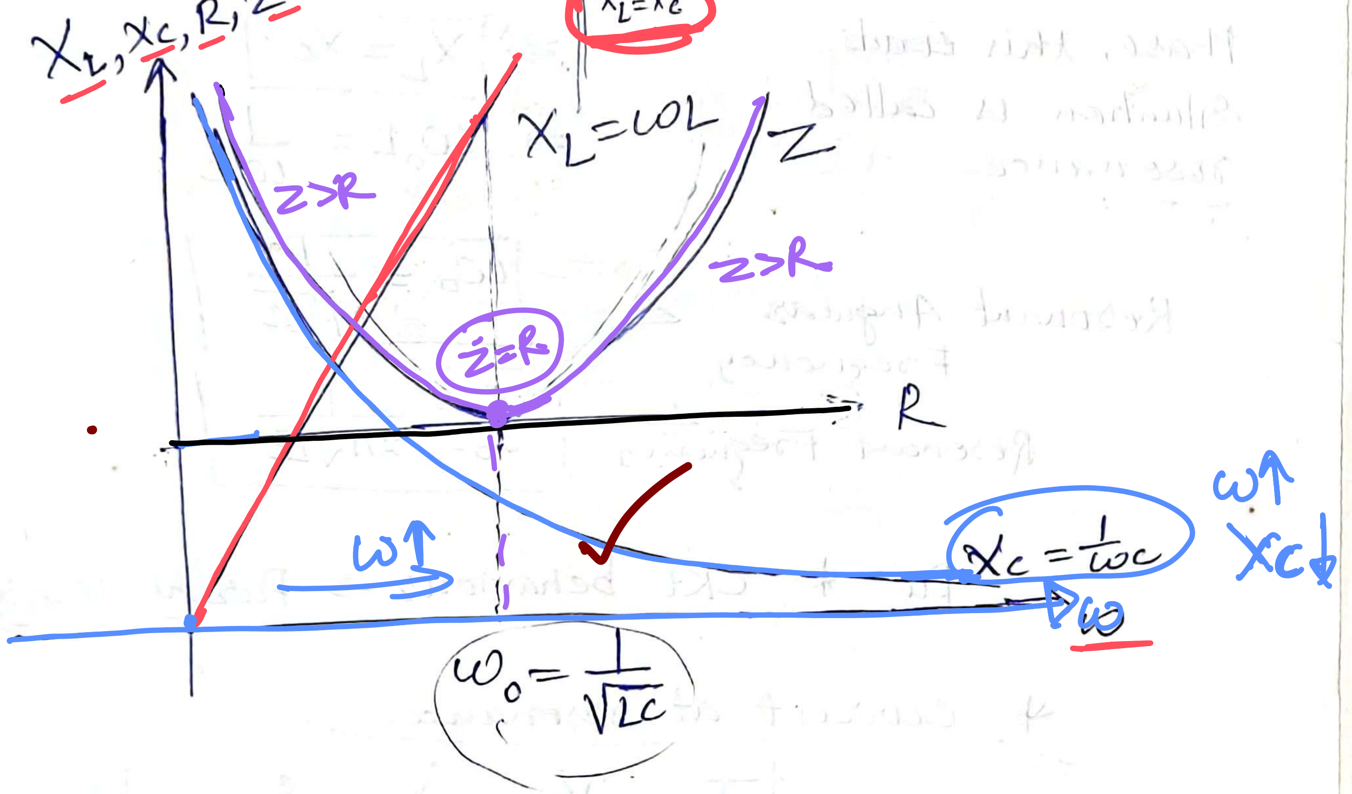
In resonance condition

$Z \rightarrow \min^m = R$

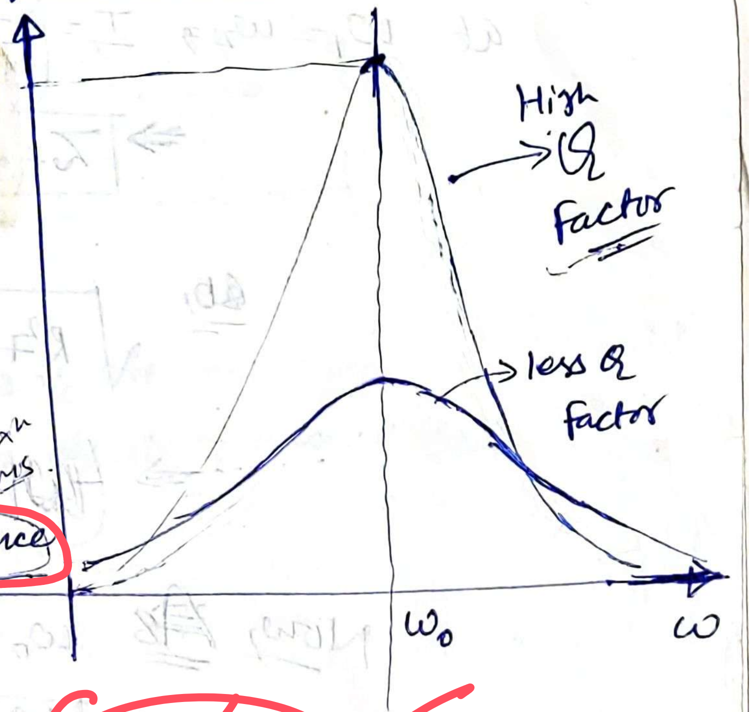
$I \rightarrow \max^m = \frac{V}{R}$

$\phi = 0^\circ \quad \cos\phi = 1$





Sharpness of resonance



Q-Factor defines Sharpness of Resonance.

Mathematically

Either max^n or RMS

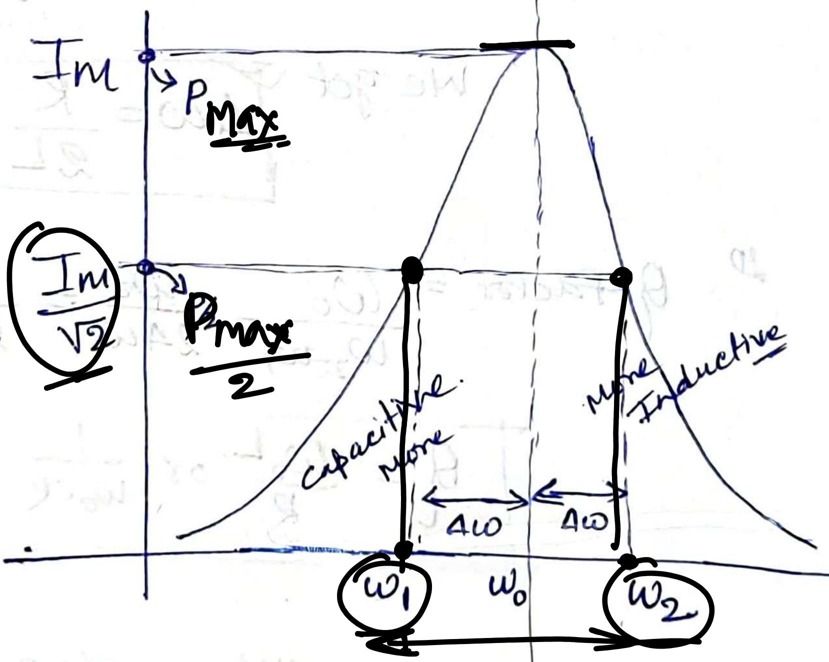
$$Q = \frac{V_C \text{ or } V_L \text{ at resonance}}{V_R}$$

$$Q = \frac{\cancel{I} X_C}{\cancel{I} R} \quad \text{or} \quad Q = \frac{\cancel{I} X_L}{\cancel{I} R}$$

$$\boxed{Q = \frac{X_C}{R} = \frac{1}{\omega RC}} \quad \text{also} \quad \boxed{Q = \frac{\omega L}{R} = \frac{X_L}{R}}$$

Half Power Frequencies

Frequencies (f_1 & f_2) or, ω_1 & ω_2 at which L-R-C ckt dissipates Half Power of the power in the resonance cond^n.



ω_1, ω_2 = half Power frequency

$$\boxed{Q = \frac{\omega_0}{2\Delta\omega}} \quad \text{or} \quad \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$$

$\omega_2 - \omega_1 \rightarrow$ Band width at ω_1 [redacted] $\omega_2 \Rightarrow I_0 = \left(\dfrac{I_{max}}{\sqrt{2}}\right)$ → $\frac{V_0}{R}$ → Resonant current

$\frac{V}{2}$

or $\Rightarrow \boxed{Z = \sqrt{2}R}$ ⇒ at $\omega_1 = \omega_2$

so, $\sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{2}R$

$\Rightarrow \left(\omega L - 1/\omega C\right) = R^2$ — ①

Now, As $\omega_0 - \omega_1 = \omega_2 - \omega_0 = \Delta\omega$

$\Rightarrow \boxed{\omega_2 = \omega_0 + \Delta\omega}$ also $\boxed{\omega_1 = \omega_0 - \Delta\omega}$

so eqn ① will be satisfied at $\omega = \omega_2$ & ω_1 also

then Putting $\omega = \omega_0 + \Delta\omega = \omega_2$

or $\omega = \omega_0 - \Delta\omega = \omega_1$

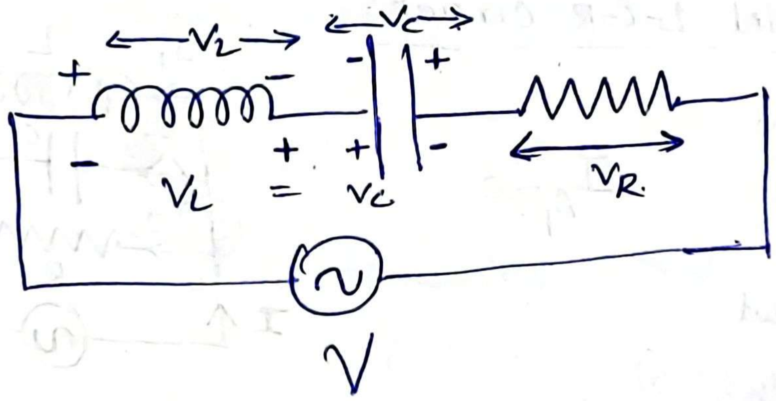
& $\boxed{\omega_2 - \omega_1 = 2\Delta\omega}$

We get $\boxed{\Delta\omega = \dfrac{R}{2L}}$

So Q-Factor $= \dfrac{\omega_0}{\omega_2 - \omega_1} = \dfrac{\omega_0}{2\Delta\omega} = \dfrac{\omega_0}{2 \times R/2L}$

$$\boxed{Q = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR}}$$ { as $\omega_0 L = \frac{1}{\omega_0 C}$ at resonance

✓ L-C-R- Series resonance ckt is also called frequency selector ckt



at resonance V_L & V_C are not zero

only compensate effects of each other

~~Parallel LCR Ckt~~

Power developed in AC circuit

$$P = VI$$

in AC — Alternating voltage

$$V = V_0 \sin\omega t$$
$$I = I_0 \sin(\omega t + \phi) \; ; \; I_0 = \frac{V_0}{Z}$$

$$P = VI = V_0 I_0 \sin\omega t \cdot \sin(\omega t + \phi)$$

Avg. power ⇒ $P_{av} = \dfrac{\int_0^T P\,dt}{T} = \dfrac{V_0 I_0}{2T}\int 2\sin\omega t \cdot \sin(\omega t+\phi)\,dt$

$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

or, $P_{av} = \dfrac{V_0 I_0}{2T}\int_0^T \cos[\cancel{\omega t} - (\cancel{\omega t}+\phi)] - \cos(\omega t + \omega t + \phi)\,dt$

or, $P_{av} = \dfrac{V_0 I_0}{2T}\int_0^T \cos\phi - \cos(2\omega t + \phi)\,dt$

or, $P_{av} = \dfrac{V_0 I_0}{2T}\left[\int_0^T \cos\phi\,dt - \int_0^T \cos(2\omega t+\phi)\,dt\right]$

or, $P_{av} = \dfrac{V_0 I_0 \cos\phi}{2T}\left[\int_0^T dt\right] - \int_0^T \cos(2\omega t+\phi)\,dt$ → zero

or, $P_{av} = \dfrac{V_0 I_0 \cos\phi \cdot \cancel{T}}{2\cancel{T}} = \dfrac{V_0 I_0}{2}\cos\phi$

or, $$\boxed{P_{AV} = \frac{V_0 I_0}{2}\cos\phi}$$

Also we can write

$$P_{av} = \left(\frac{V_0}{\sqrt{2}}\right)\cdot\left(\frac{I_0}{\sqrt{2}}\right)\cos\phi$$

$$\boxed{P_{av} = V_{rms} \cdot I_{rms} \cos\phi}$$

Phase diff betwn V & I

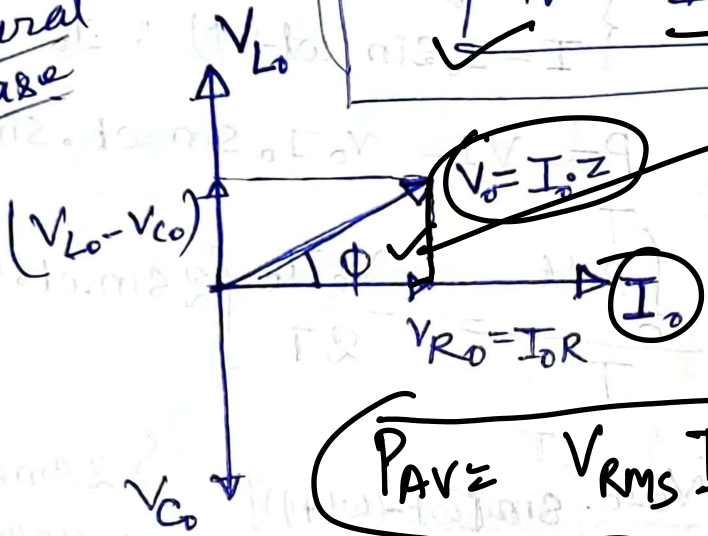
In case of pure resistance

$\phi = 0$; $P_{av} = V_{rms} I_{rms} \cos 0$

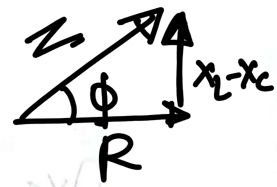
$$\Rightarrow \boxed{P_{av} = V_{rms} I_{rms}}$$

or $\boxed{P_{av} = I_{rms}^2 R}$ or $\boxed{P_{av} = \frac{V_{rms}^2}{R}}$

general case



$\cos\phi = \frac{R}{Z}$ → Power factor



$P_{av} = V_0 I_0 \left(\frac{R}{Z}\right)$

$$\boxed{P_{AV} = V_{RMS} I_{RMS} \cos\phi}$$

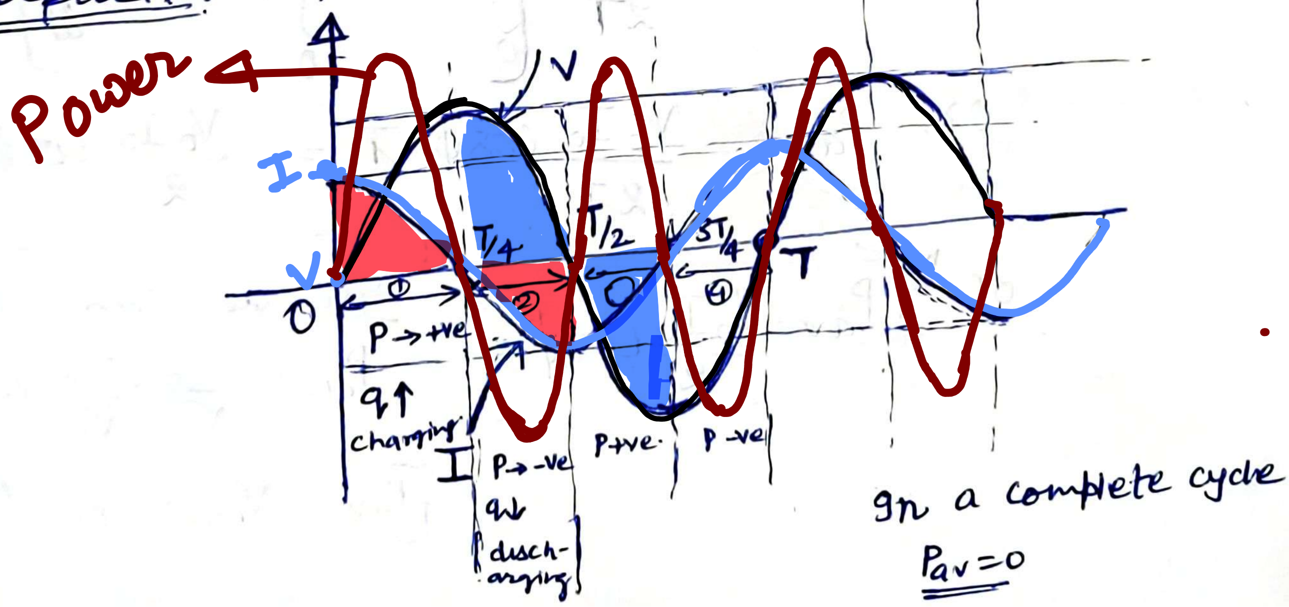
* For Pure Inductor & Pure capacitor

$\phi = 90°$ $\cos\phi = 0$

$$P_{av} = 0$$

Hence no power is consumed by Inductor & capacitor in a complete cycle.

* Capacitor → power consumed in AC ckt



In a complete cycle

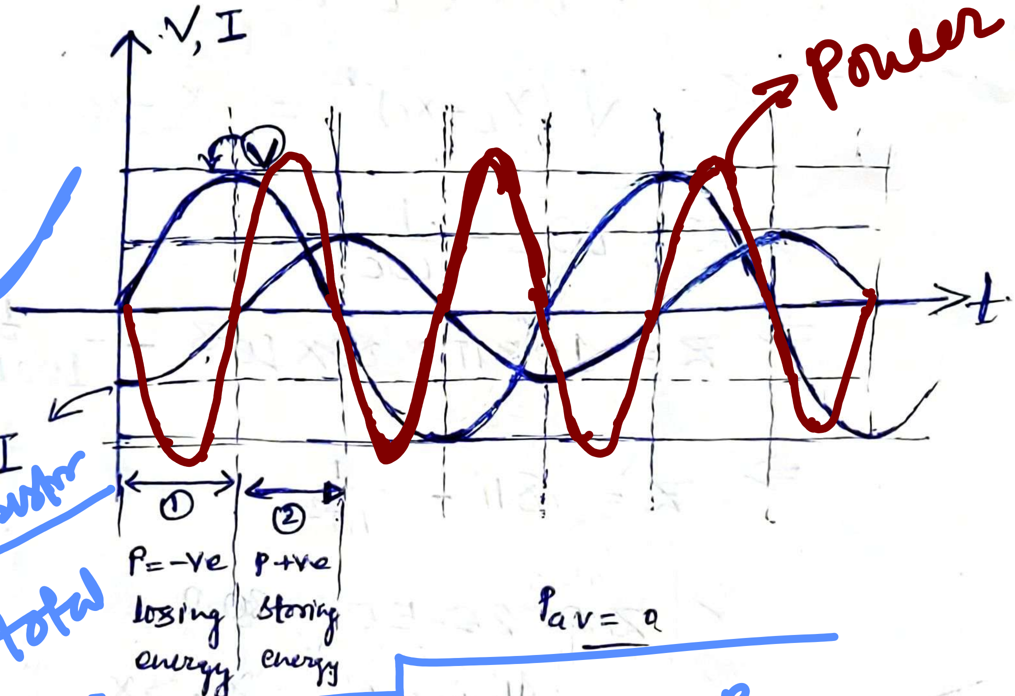
$P_{av} = 0$

Inductor Power consumed in AC ckt

Power factor

$\cos\phi = \frac{R}{Z}$

$\cos\phi = \frac{P_{Resistor}}{P_{total}}$



$P_{av} = 0$

* In resonance $Z = R$

$\boxed{\cos\phi = \frac{R}{R} = 1}$ → Power loss in resistor

General situation

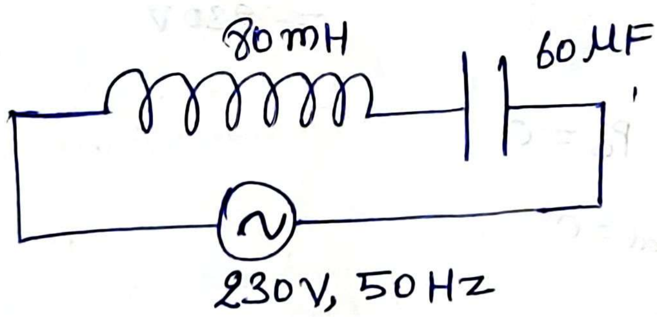
Power in LCR Ckt

$P_{av} = V_{rms} I_{rms} \cos\phi$

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\boxed{P_{av} = \frac{I_{rms} \cdot V_{rms} R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

Ex



① I_{rms}. ② P_L, P_c ③ P_{total} ④ V_L & V_c

$$Z = \sqrt{\cancel{R^2}^{\,0} + (X_L - X_C)^2}$$

$$\Rightarrow Z = \sqrt{(X_L - X_C)^2} = X_L - X_C$$

$$\Rightarrow Z = \omega L - \frac{1}{\omega C}$$

$$\Rightarrow Z = 100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times \cancel{10^{-6}}\, 10^{-3}}$$

$$\Rightarrow Z = 8\pi - \frac{1}{10^{3}\pi}$$

$$\Rightarrow Z \approx 25 - 55 = -30\ \Omega$$

It means $X_C > X_L$

Now For finding current We take the Z

$$\boxed{I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{30}}$$

$$I_{rms} = 7.6\,A$$

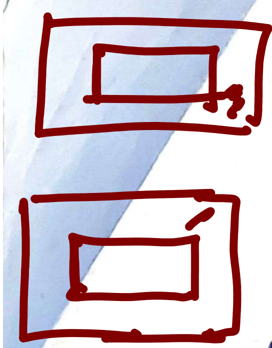
$$V_L = I.X_L \approx 207\ V$$

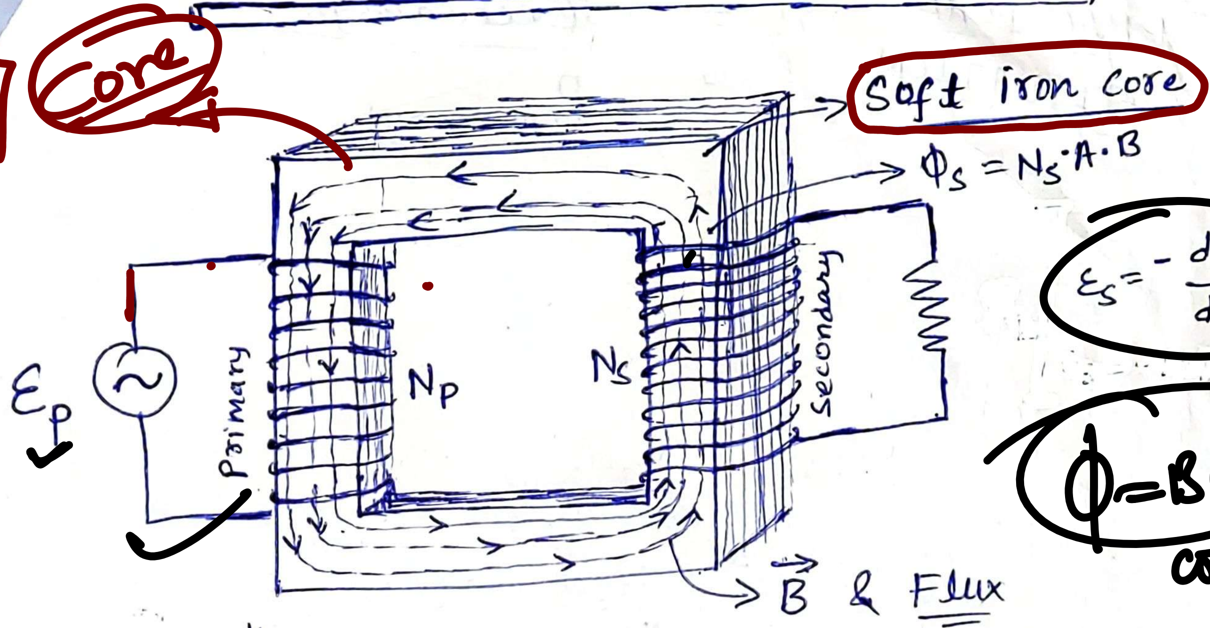
$$V_C = I X_C \approx 437\ V$$

$$V_{net} = 437 - 207$$
$$= 230\ V$$

* $P_L = P_C = 0$
* $P_{total} = 0$

TRANSFORMER





$\phi_s = N_s \cdot A \cdot B$

$\varepsilon_s = -\frac{d\phi_s}{dt}$

$\phi = BA$

$\cos 0° = 1$

Transformer is a device which transfers voltage (or current) from one circuit to another circuit without any actual contact between the circuits and without changing the supplied power and frequency

$$\varepsilon_p = \frac{-d\phi_p}{dt} = -N_p \cdot A\frac{dB}{dt}$$

$$\varepsilon_p = -N_p \cdot A\frac{dB}{dt}$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{\cancel{-}N_s \cancel{A} \cancel{\frac{dB}{dt}}}{\cancel{-}N_p \cancel{A} \cancel{\frac{dB}{dt}}}$$

$$\Rightarrow \boxed{\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}}$$

$\frac{N_s}{N_p} \rightarrow$ Turn ratio $= K$

* $K > 1 \Rightarrow \varepsilon_s > \varepsilon_p$
  Step up TF^r
* $K < 1 \Rightarrow \varepsilon_s < \varepsilon_p$
  Step down TF^r

In an ideal Transformer

Power at primary coil = Power at secondary coil

$$\Rightarrow \varepsilon_s I_s = \varepsilon_p I_p$$

$$\Rightarrow \boxed{\frac{I_s}{I_p} = \frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s}} \qquad \boxed{\frac{I_p}{I_s} = \frac{N_s}{N_p}}$$

Efficiency of Transformer (η)

PRACTICAL SITUATION

$$\eta = \frac{P_{(secondary)}}{P_{(primary)}}$$