No. ....

BT-1/DX : 8001

[ath-101 E: Mathematics-I (2004-05 onwards)
(Common for all Branches)

: 3 Hours Maximum Marks : 100

. Attempt FIVE questions in all, selecting at least ONE question from each Unit. All questions carry equal marks.

UNIT-I

a) Compute the value of Cos 32°, by using Taylor's series expansion, to four decimal places.

b) Find the asymptotes of:

$$(x+y)^2(x+2y+2)=(x+9y-2)$$

a) Show that the radius of curvature at the point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3}+y^{2/3}=a^{2/3}$ is $3a\sin\theta\cos\theta$.

b) Trace the curve: $r=2+3\cos\theta$

UNIT-II

a) If $v=\frac{1}{\sqrt{t}}.e^{-x^2/4a^2t}$, prove that

$$\frac{\partial v}{\partial t}=a^2\frac{\partial^2 v}{\partial x^2}$$

Contd.

Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates.

Q.4. a) Divide 24 into three parts such that the con product of the first, square of the second and the of the third may be maximum.

b) Expand $f(x, y) = x y^2 + \text{Cos } xy$ in powers of $(x-1)$ and $(y-\pi/2)$ upto the second degree te

UNIT-III

Q.5 a) Evaluate:

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx.$$

b) Find the area lying inside the cardioid $r = 1 + \text{Cos}\theta$ and outside the parabola $r(1+\text{Cos}\theta) = 1$

6 a) Find the volume of the solid bounded by the xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$.

b) Show that :

$$\int_0^{\pi/2} \sqrt{\text{Cot}\theta} \, . \, d\theta = \frac{1}{2} \sqrt{(1/4)} \sqrt{(3/4)}$$

UNIT-IV

In What direction from (3,1,-2) is the directional derivative of $\phi = x^2 y^2 z^4$ maximum? Find also the magnitude of this maximum.

If $\vec{V_1}$ and $\vec{V_2}$ be the vectors joining the fixed points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively to a variable point (x, y, z), prove that curl $(\vec{V_1} \times \vec{V_2}) = 2(\vec{V_1} - \vec{V_2})$

Apply Green's theorem to evaluate

$\int_C [(2x^2-y^2)\,dx + (x^2+y^2)\,dy]$, Where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2+y^2=a^2$

Verify Divergence theorem for

$\vec{F} = (x^2-yz)\hat{i} + (y^2-zx)\hat{j} + (z^2-xy)\hat{k}$

taken over the rectangular

parallelopiped $0 \leq x \leq a,\ 0 \leq y \leq b,\ 0 \leq z \leq c$.