

# FE I Sem. I (ReD) A.M. I

4/12/12

P4 (U) - Sem-I Oct-12-456

**Con. 8962-12.**

**(REVISED COURSE)**  
(3 Hours)

**KR-3357**

**[Total Marks : 80]**

- N.B. :** (1) Question No. 1 is **compulsory**.  
 (2) Attempt any **three** questions from the remaining **five**.  
 (3) **Figures to the right** indicate **full marks**.

1. (a) Prove that  $\frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{\dots}}}} = \cosh^2 x$  3

$$1 - \cosh^2 x$$

(b) If  $u = \log [\tan x + \tan y]$ , prove that, 3

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2$$

(c) If  $u = \frac{x+y}{1-xy}$ ,  $v = \tan^{-1} x + \tan^{-1} y$ , find  $\frac{\partial(u,v)}{\partial(x,y)}$  3

(d) Show that  $\log[1+\sin x] = x - \frac{x^2}{2} + \frac{x^3}{6} + \dots$  3

(e) Show that every square matrix can be uniquely expressed as  $P + iQ$ , where  $P$  and  $Q$  are Hermitian matrices. 4

(f) Find  $n^{\text{th}}$  order derivative of  $\frac{x^2+4}{(x-1)^2(2x+3)}$  4

2. (a) Show that the roots of the equation  $(x+1)^6 + (x-1)^6 = 0$  are given by 6

$$-i \cot \left[ \frac{(2k+1)\pi}{12} \right], k = 0, 1, 2, 3, 4, 5.$$

(b) Reduce the following matrix into normal form and find its rank 6

$$\begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$$

(c) State and prove the Euler's theorem for a homogeneous function in two variables. 8

Hence find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  if  $u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$

[TURN OVER]

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3. (a) Test for consistency and solve if consistent -

6

$$\begin{aligned}x_1 - 2x_2 + x_3 - x_4 &= 2 ; \\ x_1 + 2x_2 + 2x_4 &= 1 ; \\ 4x_2 - x_3 + 3x_4 &= -1\end{aligned}$$

- (b) Find all the stationary values of
- $x^3 + 3xy - 15x^2 - 15y^2 + 72x$
- .

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- (c) If
- $\tan\left(\frac{\pi}{4} + iv\right) = re^{i\theta}$
- , show that

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$$(i) r = 1 \quad (ii) \tan\theta = \sinh 2v \quad (iii) \tanh v = \tan \frac{\theta}{2}.$$

4. (a) If
- $x = u + e^{-v} \sin u$
- ,
- $y = v + e^{-u} \cos u$
- find
- $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$
- by using Jacobian.

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- (b) Considering only the principal value, if
- $(1 + i \tan \alpha)^{1+i \tan \beta}$
- is real, prove that its value is
- $(\sec \alpha)^{\sec^2 \beta}$
- .

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- (c) Solve the system of linear equation by Crout's method

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$$x - y + 2z = 2 ; \quad 3x + 2y - 3z = 2 ; \quad 4x - 4y + 2z = 2.$$

5. (a) Expand
- $\cos^7 \theta$
- in a series of cosines of multiples of
- $\theta$
- .

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$$(b) \text{ Evaluate } \lim_{x \rightarrow 0} \left[ \frac{1}{x^2} - \cot^2 x \right]$$

6

$$(c) \text{ If } y = (\sin^{-1} x)^2, \text{ obtain } y_n(0).$$

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6. (a) Show that the vectors are linearly dependent and find the relation between them :-

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$$X_1 = [1, 2, -1, 0], \quad X_2 = [1, 3, 1, 2], \quad X_3 = [4, 2, 1, 0], \quad X_4 = [6, 1, 0, 1]$$

$$(b) \text{ If } \frac{x^2}{1+u} + \frac{y^2}{2+u} + \frac{z^2}{3+u} = 1,$$

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Prove that,

$$\left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 = 2 \left[ x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right]$$

- (c) Fit a second degree parabolic curve to the following data :-

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x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9