Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH(CHE)/SEM-7/CHE-701/2011-12

2011

MATHEMATICAL METHODS IN CHEMICAL ENGINEERING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If A is a n^{th} order square matrix then det ($5A$) is

      a) $5\,[\det(A)]^n$ b) $5\det(A)$

      c) $5^n\,[\det(A)]^n$ d) $5^n\det(A)$.

   ii) The solution of a system of n linear equations with n unknowns is unique if and only if

      a) det $A = 0$ b) det $A > 0$

      c) det $A < 0$ d) det $A \neq 0$.

iii) If the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$ is singular, then the value of λ is

a) 3 b) 5

c) 2 d) 4.

iv) If λ is an eigenvalue of A, then λ^4 is an eigenvalue of

a) A^4 b) A^3

c) A d) none of these.

v) I.F. of the differential equation $\frac{dy}{dx} - \frac{y}{x+1} = e^x (x+1)$ is equal to

a) $(x+1)$ b) $\frac{1}{x+1}$

c) x d) e^{x+1}.

vi) An equation of the form $y = px + f(p)$ is known as

a) Leibnitz equation b) Bernoulli's equation

c) Clairaut's equation d) none of these.

vii) Δ (forward difference operator) and E (shift operator) is related as

a) $E + \Delta = 1$ b) $\frac{E}{\Delta} = 1$

c) $E - 1 = \Delta$ d) $E\Delta = 1$.

viii) What is the P.I. of $\frac{dy}{dx} - 4y = e^{2x}$?

a) $-\frac{1}{2}e^{2x}$ b) $-e^{2x}$

c) $2e^{2x}$ d) e^{2x}.

ix) When $k > 0$ the Bessel function $J_k(0)$ is

a) 0 b) 1

c) ∞ d) $-\infty$.

x) If $X^T AX = \lambda$ where λ is a diagonal matrix comprising the eigenvalues of square matrix A, the matrix A is known as

a) diagonal matrix b) orthogonal matrix

c) orthonormal matrix d) self adjoint matrix.

xi) A boundary condition in which the outward normal derivative and the variable is specified on the surface is termed

a) Dirichlet boundary conditions

b) Neumann boundary conditions

c) Robin or mixed boundary conditions

d) Cauchy conditions.

xii) The differential equation

$$\frac{d^4y}{dx^4} + \left(\frac{d^2y}{dx^2}\right)^4 - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 5e^x \cos^2 x$$ is a

a) 2nd order 2nd degree linear differential equation

b) 4th order 1st degree nonlinear differential equation

c) 2nd order 4th degree linear differential equation

d) 4th order 4th degree linear differential equation.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following $3 \times 5 = 15$

2. Solve the following system of equations by matrix method :

$3x + y + 2z = 3$

$2x - 3y - z = -3$

$x + 2y + z = 4$

3. Solve : $xy - \frac{dy}{dx} = y^4 \exp\left(\frac{-3x^2}{2}\right)$

4. Solve : $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 4x^3 + 3x^2$

5. A rectangular block of metal is subject to temperature variation in all directions, but the surface at $x = 0$ is thermally insulated. Express this condition by an equation.

6. Find the thermal conductivity of propane at $1 \cdot 013 \times 10^4$ kN/m^2 and 99°C from the following data :

Temperature (°C)	Pressure (kN/ m^2)	Thermal Conductivity (W/m-K)
68	$9 \cdot 7981 \times 10^3$	0·0848
	$13 \cdot 324 \times 10^3$	0·0897
87	$9 \cdot 0078 \times 10^3$	0·0762
	$13 \cdot 335 \times 10^3$	0·0807
106	$9 \cdot 7981 \times 10^3$	0·0696
	$14 \cdot 277 \times 10^3$	0·0753
140	$9 \cdot 6563 \times 10^3$	0·0611
	$12 \cdot 463 \times 10^3$	0·0651

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. The figure below illustrates a distillation apparatus consisting of a boiler *B* with a constant level device *C*, fed with the condenser cooling water. The steam is condensed in *A* and collected in receiver *D*. Some of the latent heat of evaporation is returned to the boiler by preheating the feed. Denoting the condenser feed rate by *F* kg/s, and temperature by *T*°C, the excess water overflow rate by *W* kg/s and the distillation rate by *G* kg/s, calculate the value of *G*. From the expression of *G*, find the limiting value of *F* for which *G* becomes infinite and hence obtain the restriction on the value of *F*. Can the values of *G* and rate of collection of distillate *D* be different ? If so, obtain the complete solution for the rate of distillate collection.

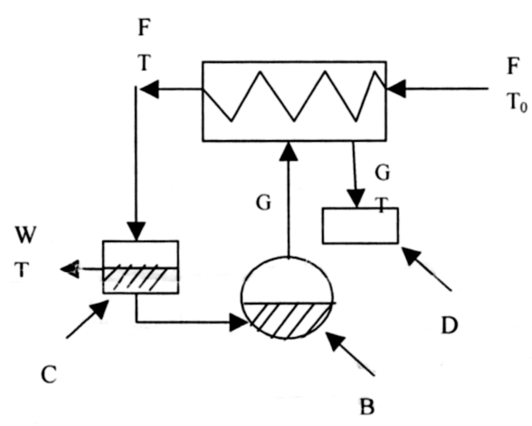


Fig. Water still with heat exchanger

8. An elevated horizontal cylindrical tank 1 m diameter and 2 m long is insulated with asbestos lagging of thickness 1 = 4 cm and is employed for a batch chemical process. Liquid at 100°C is charged into the tank and allowed to mature over 5 hrs. Calculate the final temperature of the liquid. The following data is available :

Liquid film coefficient of heat transfer is 140 W/m^2 °C

Thermal conductivity of asbestos (k) is 0·3 W/m °C

Surface coefficient of heat transfer by convection and radiation (h_2) is 10 W/m^2 °C.

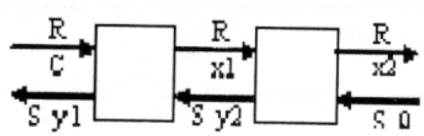
Density of liquid is 1000 kg/m^3

Heat capacity of liquid is 2500 J/kg °C

The atmospheric temperature is assumed to vary according to relation $t = 10 + 10 \cos(\pi\theta/24)$, where θ is time in hours.

The atmospheric temperature at the time of charging is 10°C. Heat loss through the supports and thermal capacity of the lagging can be neglected.

9\. a) The apparatus shown in figure is to be used for the continuous extraction of benzoic acid from toluene, using water as the extracting solvent. Here two stages are used. Each stage consists of two tanks, a mixer & a settler, with counter current flow through the stages. Find out what proportion of benzoic acid has passed into the solvent phase.



b) For the following reaction between A and B $aA + bB \rightarrow pP$, derive the following reaction to relate x, *i.e.* the moles of P formed as a function of time.

$$x = \frac{mnptk}{mbkt + p}$$

where, m and n are the moles of the reactants A and B taken initially such that, $\frac{m}{n} = \frac{a}{b}$ and k is the rate constant. Assume that the rate of formation of P is proportional to the product of the concentrations of A and B. 9 + 6

10. Two concentric cylindrical metallic shells are separated by a solid material. If the two metal surfaces are maintained at different constant temperature (Temperature at centre is T_0 and temperature at outermost cylinder is T_1), what is the steady state temperature distribution within the separating material at any radial distance r from the centre ?

11. G_{N+1} kg moles/h of a wet gas containing Y_{N+1} kg moles/mole of solute are fed into the base of plate absorption column where the solute is to be stripped from the gas by absorption in L_0 kg moles/h of lean oil which is fed into the top of the column. If the solute in the entering oil is X_0 kg moles/mole of lean oil and the solute in the exit gas is Y_1 kg moles of solute/moles of wet gas, show that the performance of the absorber can be expressed in terms of the absorption factor $A = (L_0 / KG_{N+1})$ and the number of ideal stages by the Kremser-Brown equation

$$\frac{Y_{N+1} - Y_1}{Y_{N+1} - Y_0} = \frac{A^{N+1} - A}{A^{N+1} - 1}$$

where K is the equilibrium constant.

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