CENTROID & MOMENT OF INERTIA

SYLLABUS

- Centroid – Definition, Moment of an area about an axis, centroid of geometrical figures such as squares, rectangles, triangles, circles, semicircles & quarter circles, centroid of composite figures.
- Moment of Inertia – Definition, Parallel axis & Perpendicular axis Theorems. M.I. of plane lamina & different engineering sections.

CENTRE OF GRAVITY

The point, through which the whole weight of the body acts, irrespective of its position, is known as centre of gravity (briefly written as C.G.). It may be noted that everybody has one and only one centre of gravity.

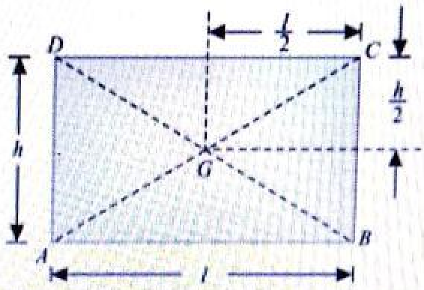
CENTROID

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as centroid. The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

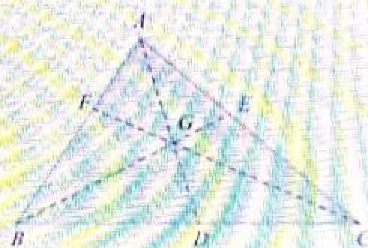
CENTRE OF GRAVITY BY GEOMETRICAL CONSIDERATIONS:

The centre of gravity of simple figures may be found out from the geometry of the figure as given below.

1. The centre of gravity of uniform rod is at its middle point.
2. The centre of gravity of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle as shown in Fig.



3. The centre of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig.

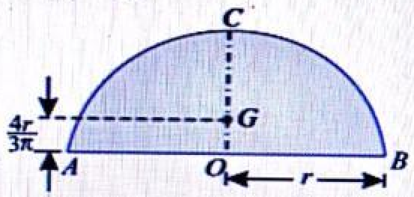


4. The centre of gravity of a trapezium with parallel sides a and b is at a distance of

$$\frac{h}{3} \times \left(\frac{b + 2a}{b + a} \right)$$

measured form the side b as shown in Fig.

5. The centre of gravity of a semicircle is at a distance of 4r/3 π from its base measured along the vertical radius as shown in Fig.

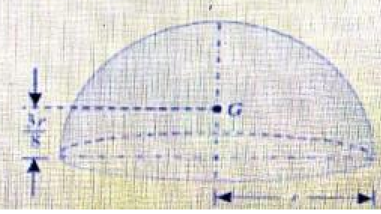


6. The centre of gravity of a circular sector making semi-vertical angle α is at a distance of $\frac{2r}{3}\frac{\sin\alpha}{\alpha}$

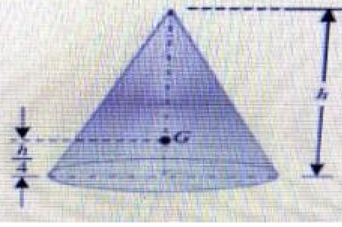
7. The centre of gravity of a cube is at a distance of *l*/ 2 from every face (where *l* is the length of each side).

8. The centre of gravity of a sphere is at a distance of *d* /2 from every point (where *d* is the diameter of the sphere).

9. The centre of gravity of a hemisphere is at a distance of 3*r*/ 8 from its base, measured along the vertical radius as shown in Fig.



10. The centre of gravity of right circular solid cone is at a distance of *h* /4 from its base, measured along the vertical axis as shown in Fig.



AXIS OF REFERENCE

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference). The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{y}$ and the left line of the figure for calculating $\bar{x}$.

CENTRE OF GRAVITY OF PLANE FIGURES

Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity with respect to some axis of reference, then

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots}{a_1 + a_2 + a_3}$$

and

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

where a_1, a_2, a_3....... etc., are the areas into which the whole figure is divided x_1, x_2, x_3 ..... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on X-X axis with respect to same axis of reference.

y_1, y_2, y_3 .... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on Y-Y axis with respect to same axis of the reference.

CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about *X-X* axis or *Y-Y* axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified, as we have only to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

EXAMPLE

Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section

Solution.

As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles ABCH and DEFG as shown in Fig

Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and

$$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and

$$y_2 = \frac{120}{2} = 60 \text{ mm}$$

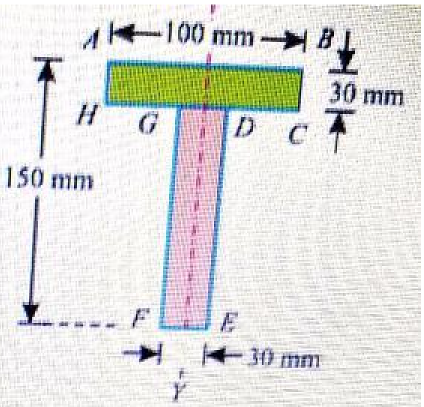


Fig. 6.10.

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE

Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm

Solution.

As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles ABFJ, EGKJ and CDHK as shown in Fig.

Let the face AC be the axis of reference.

(*i*) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(*iii*) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$

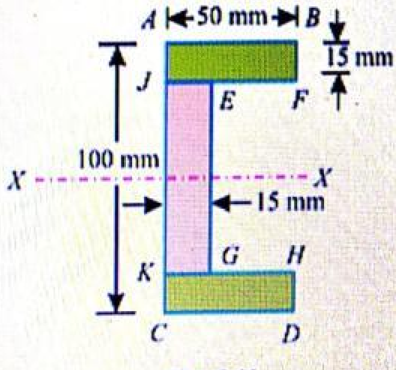


Fig. 6.11.

We know that distance between the centre of gravity of the section and left face of the section AC,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE

An I-section has the following dimensions in mm units : Bottom flange = 300 × 100 Top flange = 150 × 50, Web = 300 × 50. Determine mathematically the position of centre of gravity of the section.

Solution:-

Solution. As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig. 6.12.

Let bottom of the bottom flange be the axis of reference.

(*i*) *Bottom flange*

$$a_1 = 300 \times 100 = 30\ 000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Web*

$$a_2 = 300 \times 50 = 15\ 000 \text{ mm}^2$$

and $$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

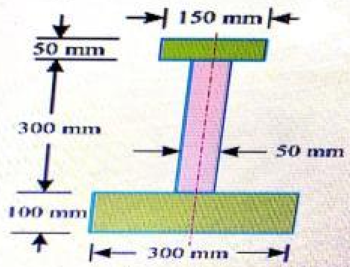


Fig. 6.12.

(*iii*) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\ 000 \times 50) + (15\ 000 \times 250) + (7500 \times 425)}{30\ 000 + 15\ 000 + 7500} = 160.7 \text{ mm} \quad \textbf{Ans.}$$

CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about *X-X* axis or *Y-Y* axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$

Example

Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.

Solution

As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in Fig. 6.13.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(*i*) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

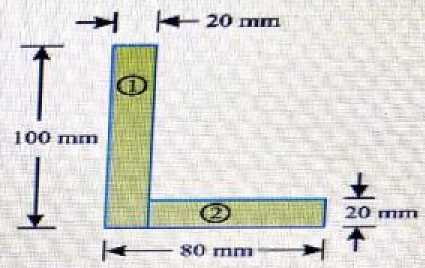


Fig. 6.13.

We know that distance between centre of gravity of the section and left face,

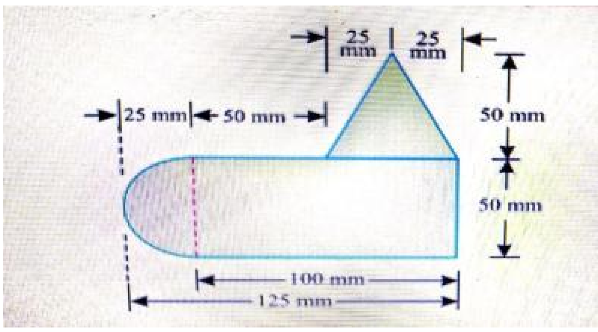
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm} \quad \textbf{Ans.}$$

Example

A uniform lamina shown in Fig. consists of a rectangle, a circle and a triangle. Determine the centre of gravity of the lamina. All dimensions are in mm.



Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.

(*i*) *Rectangular portion*

$$a_1 = 100 \times 50 = 5000 \text{ mm}^2$$

$$x_1 = 25 + \frac{100}{2} = 75 \text{ mm}$$

and $$y_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Semicircular portion*

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982 \text{ mm}^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$$

and $$y_2 = \frac{50}{2} = 25 \text{ mm}$$

(*iii*) *Triangular portion*

$$a_3 = \frac{50 \times 50}{2} = 1250 \text{ mm}^2$$

$$x_3 = 25 + 50 + 25 = 100 \text{ mm}$$

and $$y_3 = 50 + \frac{50}{3} = 66.7 \text{ mm}$$

We know that distance between centre of gravity of the section and left edge of the circular portion,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250}$$

$$= 71.1 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face of the rectangular portion,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250} \text{ mm}$$

$$= 32.2 \text{ mm} \quad \textbf{Ans.}$$

MOMENT OF INERTIA

The moment of a force (P) about a point, is the product of the force and perpendicular distance (x) between the point and the line of action of the force (i.e. P.x). This moment is also called first moment of force. If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force i.e. P.x (x) = Px^2, then this quantity is called moment of the moment of a force or second moment of force or moment of inertia (briefly written as M.I.)

MOMENT OF INERTIA OF A PLANE AREA

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

Let $a_1, a_2, a_3, \ldots$ = Areas of small elements, and

$r_1, r_2, r_3, \ldots$ = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area.

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots$$

$$= \Sigma a r^2$$

UNITS OF MOMENT OF INERTIA

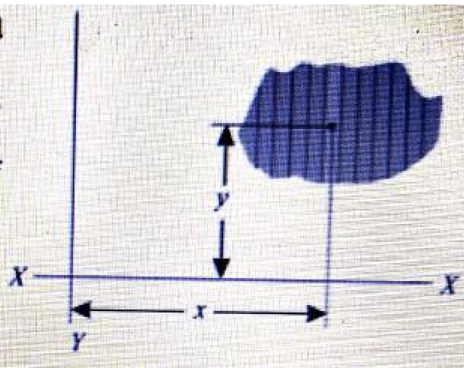
As a matter of fact the units of moment of inertia of a plane area depend upon the units of the area and the length. e.g.

1. If area is in m^2 and the length is also in m, the moment of inertia is expressed in m^4
2. If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4

MOMENT OF INERTIA BY INTEGRATION

The moment of inertia of an area may also be found out by the method of integration as discussed below:

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig Let us divide the whole area into a no. of strips. Consider one of these strips.



Moment of inertia by integration.

Let dA = Area of the strip

x = Distance of the centre of gravity of the strip on X-X axis and

y = Distance of the centre of gravity of the strip on Y-Y axis.

We know that the moment of inertia of the strip about Y-Y axis

$$= dA \, . \, x^2$$

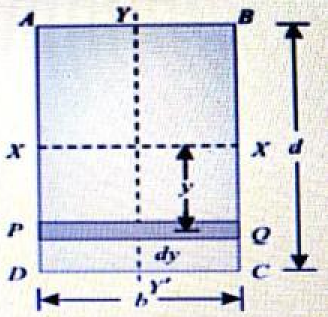
Now the moment of inertia of the whole area may be found out by integrating above equation. *i.e.*,

$$I_{YY} = \Sigma \, dA \, . \, x^2$$

Similarly $$I_{XX} = \Sigma \, dA \, . \, y^2$$

MOMENT OF INERTIA OF A RECTANGULAR SECTION:

Consider a rectangular section *ABCD* as shown in Fig. whose moment of inertia is required to be found out.



Rectangular section

Let b = Width of the section and

d = Depth of the section.

Now consider a strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure

$\therefore$ Area of the strip

$$= b.dy$$

We know that moment of inertia of the strip about X-X axis.

$$= \text{Area} \times y^2 = (b.\,dy)\,y^2 = b.\,y^2.\,dy$$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\frac{d}{2}$ to $+\frac{d}{2}$.

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b\int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \,.\, dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $$I_{YY} = \frac{db^3}{12}$$

MOMENT OF INERTIA OF A HOLLOW RECTANGULAR SECTION:

Consider a hollow rectangular section, in which *ABCD* is the main section and *EFGH* is the cut out section as shown in Fig

Let b = **Breadth of the outer rectangle.**

d = **Depth of the outer rectangle and**

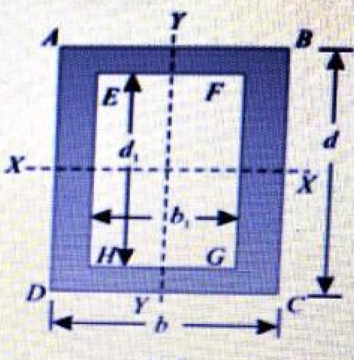
b_1, d_1 = **Corresponding values for the cut out rectangle.**

We know that the moment of inertia, of the outer rectangle *ABCD* about *X-X* axis

$$= \frac{bd^3}{12} \quad \text{...(i)}$$

and moment of inertia of the cut out rectangle *EFGH* about *X-X* axis

$$= \frac{b_1 d_1^3}{12} \quad \text{...(ii)}$$



Hollow rectangular section

∴ M.I. of the hollow rectangular section about *X-X* axis.

$$I_{XX} = \text{M.I. of rectangle } ABCD - \text{M.I. of rectangle } EFGH$$

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$

THEOREM OF PERPENDICULAR AXIS:

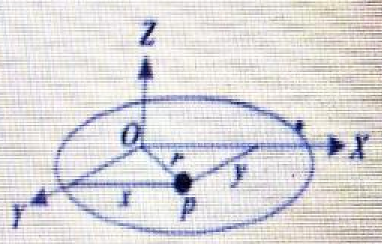
It states, *If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:*

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof :

Consider a small lamina (*P*) of area *da* having co-ordinates as *x* and *y* along *OX* and *OY* two mutually perpendicular axes on a plane section as shown in Fig.

Now consider a plane *OZ* perpendicular to *OX* and *OY*. Let (*r*) be the distance of the lamina (*P*) from *Z-Z* axis such that *OP* = *r*.



Theorem of perpendicular axis

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina *P* about *X-X* axis,

$$I_{XX} = da \cdot y^2 \qquad [\because I = \text{Area} \times (\text{Distance})^2]$$

Similarly, $I_{YY} = da \cdot x^2$

and $I_{ZZ} = da \cdot r^2 = da\,(x^2 + y^2) \qquad (\because r^2 = x^2 + y^2)$

$$= da \cdot x^2 + da \cdot y^2 = I_{YY} + I_{XX}$$

MOMENT OF INERTIA OF A CIRCULAR SECTION:

Consider a circle *ABCD* of radius (r) with centre *O* and *X-X'* and *Y-Y'* be two axes of reference through *O* as shown in Fig.

Now consider an elementary ring of radius x and thickness dx. Therefore area of the ring,

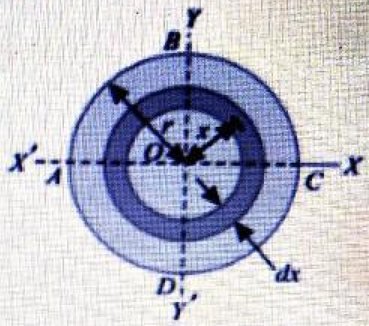
$$da = 2\pi x . dx$$

and moment of inertia of ring, about *X-X* axis or *Y-Y* axis

$$= \text{Area} \times (\text{Distance})^2$$

$$= 2\pi x . dx \times x^2$$

$$= 2\pi x^3 . dx$$



Circular section.

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to r.

$$\therefore \quad I_{ZZ} = \int_0^r 2\pi x^3 . dx = 2\pi \int_0^r x^3 . dx$$

$$I_{ZZ} = 2\pi \left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad \left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

$$\therefore I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$

MOMENT OF INERTIA OF A HOLLOW CIRCULAR SECTION:

Consider a hollow circular section as shown in Fig. whose moment of inertia is required to be found out.

Let $\quad D$ = **Diameter of the main circle, and**

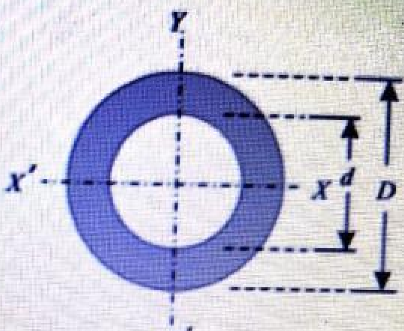
$\quad d$ = **Diameter of the cut out circle.**

We know that the moment of inertia of the main circle about *X-X* axis

$$= \frac{\pi}{64}(D)^4$$

and moment of inertia of the cut-out circle about *X-X* axis

$$= \frac{\pi}{64}(d)^4$$



Hollow circular section.

$\therefore$ Moment of inertia of the hollow circular section about *X-X* axis.

I_{XX} = Moment of inertia of main circle – Moment of inertia of cut out circle.

$$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(D^4 - d^4)$$

Similarly, $\quad I_{YY} = \frac{\pi}{64}(D^4 - d^4)$

THEOREM OF PARALLEL AXIS:

It states, ***If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:***

$$I_{AB} = I_G + ah^2$$

where I_{AB} = Moment of inertia of the area about an axis AB,

I_G = Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

h = Distance between centre of gravity of the section and axis AB.

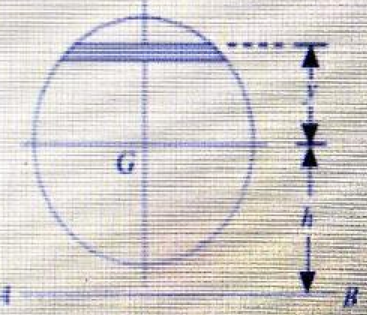
Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in Fig.

Let δa = Area of the strip

y = Distance of the strip from the centre of gravity the section and

h = Distance between centre of gravity of the section and the axis AB



Theorem of parallel axis

We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section

$$= \delta a . y^2$$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \Sigma\, \delta a . y^2$$

$\therefore$ Moment of inertia of the section about the axis AB,

$$I_{AB} = \Sigma\, \delta a\,(h + y)^2 = \Sigma\, \delta a\,(h^2 + y^2 + 2\,h\,y)$$

$$= (\Sigma\, h^2 . \delta a) + (\Sigma\, y^2 . \delta a) + (\Sigma\, 2\,h\,y . \delta a)$$

$$= a\,h^2 + I_G + 0$$

It may be noted that $\Sigma\, h^2 . \delta a = a\,h^2$ and $\Sigma\, y^2 . \delta a = I_G$ [as per equation (*i*) above] and $\Sigma\, \delta a . y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to $a.\bar{y}$, where $\bar{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.

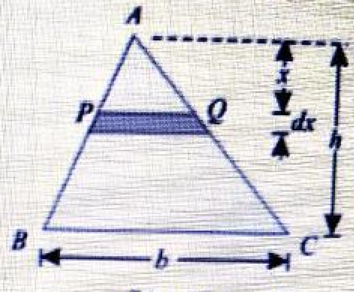
MOMENT OF INERTIA OF A TRIANGULAR SECTION:

Consider a triangular section *ABC* whose moment of inertia is required to be found out.

Let b = Base of the triangular section and

h = Height of the triangular section.

Now consider a small strip *PQ* of thickness dx at a distance of x from the vertex *A* as shown in Fig. From the geometry of the figure, we find that the two triangles *APQ* and *ABC* are similar. Therefore



Triangular section

$$\frac{PQ}{BC} = \frac{x}{h} \quad \text{or} \quad PQ = \frac{BC.x}{h} = \frac{bx}{h} \qquad (\because BC = \text{base} = b)$$

We know that area of the strip *PQ*

$$= \frac{bx}{h}\ dx$$

and moment of inertia of the strip about the base *BC*

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h}\, dx\, (h-x)^2 = \frac{bx}{h}(h-x)^2\, dx$$

Now moment of inertia of the whole triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to h

$$I_{BC} = \int_0^h \frac{b\,x}{h}(h-x)^2\, dx$$

$$= \frac{b}{h}\int_0^h x\,(h^2 + x^2 - 2hx)\, dx$$

$$= \frac{b}{h}\int_0^h (xh^2 + x^3 - 2hx^2)\, dx$$

$$= \frac{b}{h}\left[\frac{x^2h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3}\right]_0^h = \frac{bh^3}{12}$$

We know that distance between centre of gravity of the triangular section and base *BC*.

$$d = \frac{h}{3}$$

∴ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to X-X axis.

$$I_G = I_{BC} - ad^2 \qquad (\because I_{XX} = I_G + a\,h^2)$$

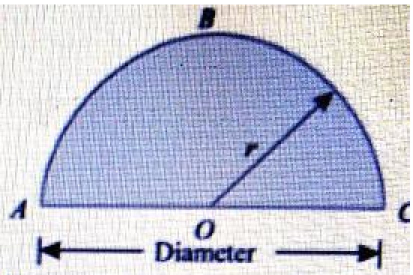
$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

MOMENT OF INERTIA OF A SEMICIRCULAR SECTION:

Consider a semicircular section *ABC* whose moment of inertia is required to be found out as shown in Fig. .....

Let r = Radius of the semicircle.

We know that moment of inertia of the semicircular section about the base *AC* is equal to half the moment of inertia of the circular section about *AC*. Therefore moment of inertia of the semicircular section *ABC* about the base *AC*.



Semicircular section *ABC*.

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393\ r^4$$

We also know that area of semicircular section.

$$a = \frac{1}{2} \times \pi r^2 \ \frac{\pi r^2}{2}$$

and distance between centre of gravity of the section and the base *AC*.

$$h = \frac{4r}{3\pi}$$

∴ Moment of inertia of the section through its centre of gravity and parallel to x-x axis.

$$I_G = I_{AC} - ah^2 = \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2\right]$$

$$= \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{8}{9\pi} \times (r)^4\right] = 0.11\ r^4$$

Note. The moment of inertia about y-y axis will be the same as that about the base *AC* *i.e.*, $0.393\ r^4$

MOMENT OF INERTIA OF A COMPOSITE SECTION

The moment of inertia of a composite section may be found out by the following steps :

1. First of all, split up the given section into plane areas (*i.e.*, rectangular, triangular, circular etc., and find the centre of gravity of the section).
2. Find the moments of inertia of these areas about their respective centres of gravity.
3. Now transfer these moment of inertia about the required axis (*AB*) by the Theorem of Parallel Axis, *i.e.*,

$$I_{AB} = I_G + ah^2$$

where I_G = Moment of inertia of a section about its centre of gravity and parallel to the axis.

a = Area of the section.

h = Distance between the required axis and centre of gravity of the section.

4. The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

EXAMPLE

Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.

Solution:-

Given: Width of the section (b) = 30 mm and

Depth of the section (d) = 40 mm.

We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times (40)^3}{12} = 160 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly

$$I_{YY} = \frac{db^3}{12} = \frac{40 \times (30)^3}{12} = 90 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

EXAMPLE

Find the moment of inertia of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.

Solution. Given: External breadth (b) = 60 mm; External depth (d) = 80 mm ; Internal breadth (b_1) = 30 mm and internal depth (d_1) = 40 mm.

We know that moment of inertia of hollow rectangular section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{60\,(80)^3}{12} - \frac{30\,(40)^3}{12} = 2400 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly,

$$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12} = \frac{80\,(60)^3}{12} - \frac{40\,(30)^3}{12} = 1350 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

EXAMPLE

Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its centre.

Solution.

Given: Diameter (d) = 50 mm We know that moment of inertia of the circular section about an axis passing through its centre.

$$I_{XX} = \frac{\pi}{64}(d)^4 = \frac{\pi}{64} \times (50)^4 = 307 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

EXAMPLE

Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.

First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz.*, 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

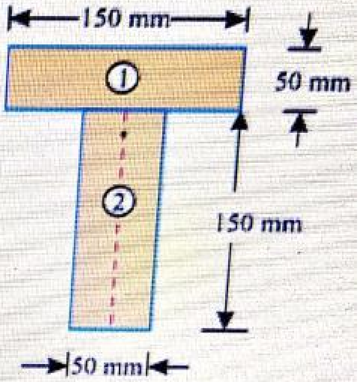


Fig. 7.14.

(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

Moment of inertia about X-X axis

We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis.

$$I_{G1} = \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 175 - 125 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *X-X* axis

$$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and *X-X* axis,

$$h_2 = 125 - 75 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about *X-X* axis

$$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2] = 32.8125 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6 \text{ mm}^4$$ **Ans.**

Moment of inertia about Y-Y axis

We know that M.I. of rectangle (1) about *Y-Y* axis

$$= \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and moment of inertia of rectangle (2) about *Y-Y* axis,

$$= \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *Y-Y* axis,

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4$$ **Ans.**

EXAMPLE

An I-section is made up of three rectangles as shown in Fig. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.

Solution. First of all, let us find out centre of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into three rectangles 1, 2 and 3 as shown in Fig. 7.15. Let bottom face of the bottom flange be the axis of reference.

(*i*) *Rectangle* 1

$$a_1 = 60 \times 20 = 1200 \text{ mm}$$

and $$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

(*iii*) *Rectangle* 3

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_3 = \frac{20}{2} = 10 \text{ mm}$$

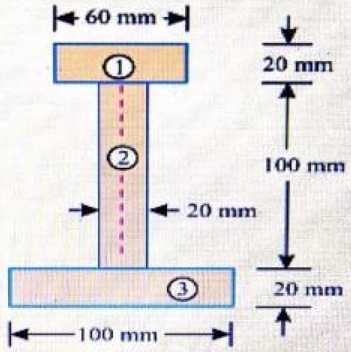


Fig. 7.15.

We know that the distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$
$$= 60.8 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis,

$$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786 \times 10^3 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis,

$$= I_{G2} + a_2 h_2^2 = (1666.7 \times 10^3) + [2000 \times (9.2)^2] = 1836 \times 10^3 \text{ mm}^4$$

Now moment of inertia of rectangle (3) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (3) and X-X axis,

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

∴ Moment of inertia of rectangle (3) about X-X axis,

$$= I_{G3} + a_3 h_3^2 = (66.7 \times 10^3) + [2000 \times (50.8)^2] = 5228 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12\,850 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

DYNAMICS

SYLLABUS

- Kinematics & Kinetics, Principles of Dynamics, Newton's Laws of Motion, Motion of Particle acted upon by a constant force, Equations of motion, DeAlembert's Principle.
- Work, Power, Energy & its Engineering Applications, Kinetic & Potential energy & its application.
- Momentum & impulse, conservation of energy & linear momentum, collision of elastic bodies, and Coefficient of Restitution.

KINETICS

It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

KINEMATICS

It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

PRINCIPLE OF DYNAMICS

- A body can possess acceleration only when some force is applied on it. Or in other words, if no force is applied on the body, then there will be no acceleration, and the body will continue to move with the existing uniform velocity.
- The force applied on a body is proportional to the product of the mass of the body and the acceleration produced in it.

RIGID BODY

A rigid body consists of a system of innumerable particles. If the positions of its various particles remain fixed, relative to one another (or in other words, distance between any two of its particles remain constant), it is called a solid body. In actual practice, all the solid bodies are not perfectly rigid bodies. However, they are regarded as such, since all the solid bodies behave more or less like rigid bodies.

NEWTON'S LAWS OF MOTION:

1. **Newton's First Law of Motion** states, "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

2. **Newton's Second Law of Motion** states, "The rate of change of momentum is directly proportional to the impressed force, and takes place in the same direction, in which the force acts."

$$F = ma = \text{Mass} \times \text{Acceleration}$$

3. **Newton's Third Law of Motion** states, "To every action, there is always an equal and opposite reaction."

NEWTON'S FIRST LAW OF MOTION

It states "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force." It is also called the law of inertia, and consists of the following two parts:

1. A body at rest continues in the same state, unless acted upon by some external force.
2. A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compelled by some external force to change its state.

The effect of inertia is of the following two types :

1. A body at rest has a tendency to remain at rest. It is called inertia of rest.
2. A body in uniform motion in a straight line has a tendency to preserve its motion. It is called inertia of motion.

NEWTON'S SECOND LAW OF MOTION

It states, "The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts."

Now consider a body moving in a straight line. Let its velocity be changed while moving.

Let m = Mass of a body,
u = Initial velocity of the body,
v = Final velocity of the body,
a = Constant acceleration,
t = Time, in seconds required to change the velocity from u to v, and
F = Force required to change velocity from u to v in t seconds.
Initial momentum = mu and
Final momentum = mv

∴ Rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma \qquad \left[\because \frac{v-u}{t} = a\right]$$

According to Newton's Second Law of Motion, the rate of change of momentum is directly proportional to the impressed force.

$\therefore F \propto ma = k\,ma$

Where k is a constant of proportionality.
The unit of force adopted is such that it produces unit acceleration to a unit mass.
F = ma = Mass × Acceleration.
In S.I. system of units, the unit of force is called newton briefly written as N. A Newton may be defined as the force while acting upon a mass of 1 kg, produces an acceleration of 1 m/s2 in the direction of which it acts. It is also called the Law of dynamics and consists of the following two parts:

1. A body can possess acceleration only when some force is applied on it. Or in other words, if no force is applied on the body, then there will be no acceleration, and the body will continue to move with the existing uniform velocity.
2. The force applied on a body is proportional to the product of the mass of the body and the acceleration produced in it.

EXAMPLE

A body of mass 7.5 kg is moving with a velocity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.

Solution. Given: Mass of body = 7.5 kg ; Velocity (u) = 1.2 m/s ; Force (F) = 15 N and time (t) = 2 s.

We know that acceleration of the body

$$a = \frac{F}{m} = \frac{15}{7.5} = 2 \text{ m/s}^2$$

∴ Velocity of the body after 2 seconds

$$v = u + at = 1.2 + (2 \times 2) = 5.2 \text{ m/s} \quad \textbf{Ans.}$$

EXAMPLE

A vehicle, of mass 500 kg, is moving with a velocity of 25 m/s. A force of 200 N acts on it for 2 minutes. Find the velocity of the vehicle : (1) when the force acts in the direction of motion, and (2) when the force acts in the opposite direction of the motion.

Solution. Given : Mass of vehicle (m) = 500 kg ; Initial velocity (u) = 25 m/s ; Force (F) = 200 N and time (t) = 2 min = 120 s

1. *Velocity of vehicle when the force acts in the dirction of motion*

We know that acceleration of the vehicle,

$$a = \frac{F}{m} = \frac{200}{500} = 0.4 \text{ m/s}^2$$

∴ Velocity of the vehicle after 120 seconds

$$v_1 = u + at = 25 + (0{\cdot}4 \times 120) = 73 \text{ m/s} \quad \textbf{Ans.}$$

2. *Velocity of the vehicle when the force acts in the opposite direction of motion.*

We know that velcoity of the vehicle in this case after 120 seconds, (when $a = -0.4$ m/s^2),

$$v_2 = u + at = 25 + (-0.4 \times 120) = -23 \text{ m/s} \quad \textbf{Ans.}$$

Minus sign means that the vehicle is moving in the reverse direction or in other words opposite to the direction in which the vehicle was moving before the force was made to act.

EXAMPLE

A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a velocity of 15 m/s. How long the body will take to stop?

Solution. Given: Retarding force (F) = 50 N ; Mass of the body (m) = 20 kg ; Initial velocity (u) = 15 m/s and final velocity (v) = 0 (because it stops)

Let t = Time taken by the body to stop.

We know that retardation of the body

$$a = \frac{F}{m} = \frac{50}{20} = 2.5 \text{ m/s}^2$$

and final velocity of the body,

$$0 = u + at = 15 - 2.5\,t \quad \text{...(Minus sign due to retardation)}$$

$$\therefore \quad t = \frac{15}{2.5} = 6 \text{ s} \quad \textbf{Ans.}$$

EXAMPLE

A car of mass 2.5 tonnes moves on a level road under the action of 1 kN propelling force. Find the time taken by the car to increase its velocity from 36 km. p.h. to 54 km.p.h.

Solution. Given : Mass of the car (m) = 2.5 t ; Propelling force (F) = 1 kN ; Initial velocity (u) = 36 km.p.h. = 10 m/s and final velocity (v) = 54 km.p.h. = 15 m/s

Let t = Time taken by the car to increase its speed.

We know that acceleration of the car,

$$a = \frac{F}{m} = \frac{1}{2.5} = 0.4 \text{ m/s}^2$$

and final velocity of the car (v),

$$15 = u + at = 10 + 0.4\,t$$

$$t = \frac{15 - 10}{0.4} = \frac{5}{0.4} = 12.5 \text{ s} \quad \textbf{Ans.}$$

NEWTON’S THIRD LAW OF MOTION

It states “To every action, there is always an equal and opposite reaction.”

By action is meant the force, which a body exerts on another and the reaction means the equal and opposite force, which the second body exerts on the first. This law, therefore, states that a force always occurs in pair. Each pair consisting of two equal opposite forces.

According to Newton’s Third Law of Motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.
Let M = Mass of the gun,
V = Velocity of the gun with which it recoils,
m = mass of the bullet, and
v = Velocity of the bullet after explosion.
∴ Momentum of the bullet after explosion = mv ... (i)
And momentum of the gun = MV ... (ii)
Equating the equations (i) and (ii),
MV = mv
This relation is known as Law of Conservation of Momentum

EXAMPLE

A machine gun of mass 25 kg fires a bullet of mass 30 gram with a velocity of 250 m/s. Find the velocity with which the machine gun will recoil.

Solution. Given : Mass of the machine gun (M) = 25 kg ; Mass of the bullet (m) = 30 g = 0.03 kg and velocity of firing (v) = 250 m/s.

Let V = Velocity with which the machine gun will recoil.

We know that $MV = mv$

$$25 \times v = 0.03 \times 250 = 7.5$$

$$\therefore \quad v = \frac{7.5}{25} = 0.3 \text{ m/s} \quad \textbf{Ans.}$$

D’ALEMBERT’S PRINCIPLE

It states, “If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics.”

We know that force acting on a body. P = ma ... (i)
where m = mass of the body, and a = Acceleration of the body
The equation (i) may also be written as: P – ma = 0 ... (ii)

It may be noted that equation (i) is the equation of dynamics whereas the equation (ii) is the equation of statics. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as D' Alembert’s principle.

EQUATIONS OF MOTION

Let u = Initial velocity,
v = Final velocity,
t = Time (in seconds) taken by the particle to change its velocity from u to v.
a = Uniform positive acceleration, and
s = Distance travelled in t seconds.

Since in t seconds, the velocity of the particle has increased steadily from (u) to (v) at the rate of a, therefore total increase in velocity

$$= at$$

$$\therefore \quad v = u + at \qquad ...(i)$$

and average velocity $= \left(\dfrac{u+v}{2}\right)$

We know that distance travelled by the particle,

$$s = \text{Average velocity} \times \text{Time}$$

$$= \left(\frac{u+v}{2}\right) \times t \qquad ...(ii)$$

Substituting the value of v from equation (i),

$$s = \left(\frac{u+u+at}{2}\right) \times t = ut + \frac{1}{2}at^2 \qquad ...(iii)$$

From equation (i), $(i.e.\ v = u + at)$ we find that

$$t = \frac{v-u}{a}$$

Now substituting this vlaue of t in equation (ii),

$$s = \left(\frac{u+v}{2}\right) \times \left(\frac{v-u}{a}\right) = \frac{v^2 - u^2}{2a}$$

or $\quad 2as = v^2 - u^2$

$\therefore \quad v^2 = u^2 + 2as$

Example 17.1. *A car starting from rest is accelerated at the rate of 0.4 m/s^2. Find the distance covered by the car in 20 seconds.*

Solution. Given : Initial velocity (u) = 0 (because, it starts from rest) ; Acceleration (a) = 0.4 m/s^2 and time taken (t) = 20 s

We know that the distance covered by the car,

$$s = ut + \frac{1}{2}at^2 = (0 \times 20) + \frac{1}{2} \times 0.4 \times (20)^2 \text{ m} = 80 \text{ m} \quad \textbf{Ans.}$$

Example 17.2. *A train travelling at 27 km.p.h is accelerated at the rate of 0.5 m/s^2. What is the distance travelled by the train in 12 seconds ?*

Solution. Given : Initial velocity (u) = 27 km.p.h. = 7.5 m/s ; Accceleration (a) = 0.5 m/s^2 and time taken (t) = 12 s.

We know that distance travelled by the train,

$$s = ut + \frac{1}{2}at^2 = (7.5 \times 12) + \frac{1}{2} \times 0.5 \times (12)^2 \text{ m}$$

$$= 90 + 36 = 126 \text{ m} \quad \textbf{Ans.}$$

Example 17.3. *A scooter starts from rest and moves with a constant acceleration of 1.2 m/s^2. Determine its velocity, after it has travelled for 60 meters.*

Solution. Given : Initial velocity (u) = 0 (because it starts from rest) Acceleration (a) = 1.2 m/s^2 and distance travelled (s) = 60 m.

Let v = Final velocity of the scooter.

We know that $v^2 = u^2 + 2as = (0)^2 + 2 \times 1.2 \times 60 = 144$

$$v = 12 \text{ m/s} = \frac{12 \times 3600}{1000} = 43.2 \text{ km.p.h.} \quad \textbf{Ans.}$$

Example 17.5. *A motor car takes 10 seconds to cover 30 meters and12 seconds to cover 42 meters. Find the uniform acceleration of the car and its velocity at the end of 15 seconds.*

Solution. Given : When t = 10 seconds, s = 30 m and when t = 12 seconds, s = 42 m.

Uniform acceleration

Let u = Initial velocity of the car, and

a = Uniform acceleration.

We know that the distance travelled by the car in 10 seconds,

$$30 = ut + \frac{1}{2}at^2 = u \times 10 + \frac{1}{2} \times a(10)^2 = 10u + 50a$$

Multiplying the above equation by 6,

$$180 = 60u + 300a \quad ...(i)$$

Similarly, distance travelled by the car in 12 seconds,

$$42 = u \times 12 + \frac{1}{2} \times a(12)^2 = 12u + 72a$$

Mulitiplying the above equation by 5,

$$210 = 60u + 360a \quad ...(ii)$$

Subtracting equation (i) from (ii),

$$30 = 60a \quad \text{or} \quad a = \frac{30}{60} = 0.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

Velocity at the end of 15 seconds

Substituting the value of a in equation (i)

$$180 = 60u + (300 \times 0.5) = 60u + 150$$

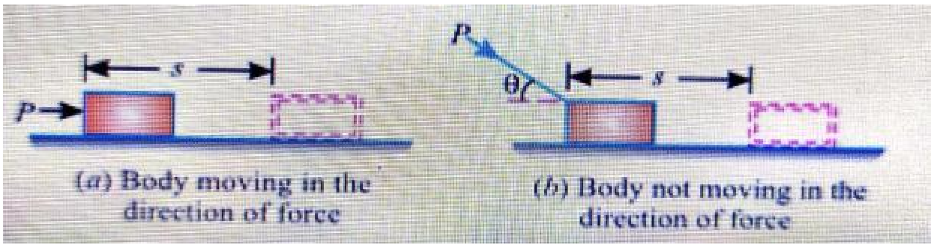
$$\therefore \quad u = \frac{(180 - 150)}{60} = 0.5 \text{ m/s}$$

We know that the velocity of the car after 15 seconds,

$$v = u + at = 0.5 + (0.5 \times 15) = 8 \text{ m/s} \quad \textbf{Ans.}$$

WORK

Whenever a force acts on a body, and the body undergoes some displacement, then work is said to be done. e.g., if a force P, acting on a body, causes it to move through a distance s as shown in Fig.(a).



Then work done by the force P = Force × Distance = P × s
Sometimes, the force P does not act in the direction of motion of the body, or in other words, the body does not move in the direction of the force as shown in Fig.(b).
Then work done by the force P = Component of the force in the direction of motion × Distance = P cos θ × s

UNITS OF WORK:

The units of work (or work done) are:

1. One N-m: It is the work done by a force of 1 N, when it displaces the body through 1 m. It is called joule (briefly written as J), Mathematically. 1 joule = 1 N-m

2. One kN-m: It is the work done by a force of 1 kN, when it displaces the body through 1 m. It is also called kilojoule (briefly written as kJ). Mathematically. 1 kilo-joule = 1 kN-m

POWER

The power may be defined as the rate of doing work. It is thus the measure of performance of engines.

UNITS OF POWER

In S.I. units, the unit of power is watt (briefly written as W) which is equal to 1 N-m/s or 1 J/s. Generally, a bigger unit of power (kW) is used.

ENERGY

The energy may be defined as the capacity to do work. It exists in many forms i.e., mechanical, electrical chemical, heat, light etc.

UNITS OF ENERGY:

Since the energy of a body is measured by the work it can do, therefore the units of energy will be the same as those of the work.

POTENTIAL ENERGY

It is the energy possessed by a body, for doing work, by virtue of its position. e.g.,

1. A body, raised to some height above the ground level, possesses some potential energy, because it can do some work by falling on the earth's surface.

2. Compressed air also possesses potential energy because it can do some work in expanding, to the volume it would occupy at atmospheric pressure.

3. A compressed spring also possesses potential energy; because it can do some work in recovering to its original shape.

Now consider a body of mass (m) raised through a height (h) above the datum level. We know that work done in raising the body = Weight × Distance = (mg) h = mgh This work (equal to m.g.h) is stored in the body as potential energy.

KINETIC ENERGY: It is the energy, possessed by a body, for doing work by virtue of its mass and velocity of motion.

Now consider a body, which has been brought to rest by a uniform retardation due to the applied force.

Let m = Mass of the body
u = Initial velocity of the body
P = Force applied on the body to bring it to rest,
a = Constant retardation, and
s = Distance travelled by the body before coming to rest.
Since the body is brought to rest, therefore its final velocity, v = 0
and work done, W = Force × Distance = P × s ...(i)

Now substituting value of $(P = m.a)$ in equation (i),

$$W = ma \times s = mas \quad ...(ii)$$

We know that $v^2 = u^2 - 2\,as$ ...(Minus sign due to retardation)

$\therefore$ $2as = u^2$ ...($\because v = 0$)

or $$as = \frac{u^2}{2}$$

Now substituting the value of $(a.s)$ in equation (ii) and replacing work done with kinetic energy,

$$\text{KE} = \frac{mu^2}{2}$$

Cor. In most of the cases, the initial velocity is taken as v (instead of u), therefore kinetic energy,

$$\text{KE} = \frac{mv^2}{2}$$

LAW OF CONSERVATION OF ENERGY

It states "The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which the energy can exist."

From the above statement, it is clear, that no machine can either create or destroy energy, though it can only transform from one form into another. We know that the output of a machine is always less than the input of the machine. This is due to the reason that a part of the input is utilized in overcoming friction of the machine. This does not mean that this part of energy, which is used in overcoming the friction, has been destroyed. But it reappears in the form of heat energy at the bearings and other rubbing surfaces of the machine, though it is not available to us for useful work.

IMPULSE AND MOMENTUM

- Impulse is the change of momentum of an object when the object is acted upon by a force for an interval of time.
  Impulse = Force X time
- Momentum is the quantity of motion of a moving body, measured as a product of its mass and velocity.
  Momentum = mass x velocity

PHENOMENON OF COLLISION

Whenever two elastic bodies collide with each other, the phenomenon of collision takes place as given below:

1. The bodies, immediately after collision, come momentarily to rest.

2. The two bodies tend to compress each other, so long as they are compressed to the maximum value.

3. The two bodies attempt to regain its original shape due to their elasticity. This process of regaining the original shape is called restitution.

The time taken by the two bodies in compression, after the instant of collision, is called the time of compression and time for which restitution takes place is called the time of restitution. The sum of the two times of collision and restitution is called time of collision, period of collision, or period of impact.

LAW OF CONSERVATION OF MOMENTUM

It states, "The total momentum of two bodies remains constant after their collision or any other mutual action."

Mathematically $m_1u_1 + m_2u_2 = m_1 v_1 + m_2 v_2$

Where; m_1 = Mass of the first body,
u_1 = Initial velocity of the first body,
v_1 = Final velocity of the first body, and
m_2, u_2, v_2 = Corresponding values for the second body.

NEWTON'S LAW OF COLLISION OF ELASTIC BODIES

It states, "When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

Mathematically, $(v2 - v1) = e\ (u1 - u2)$

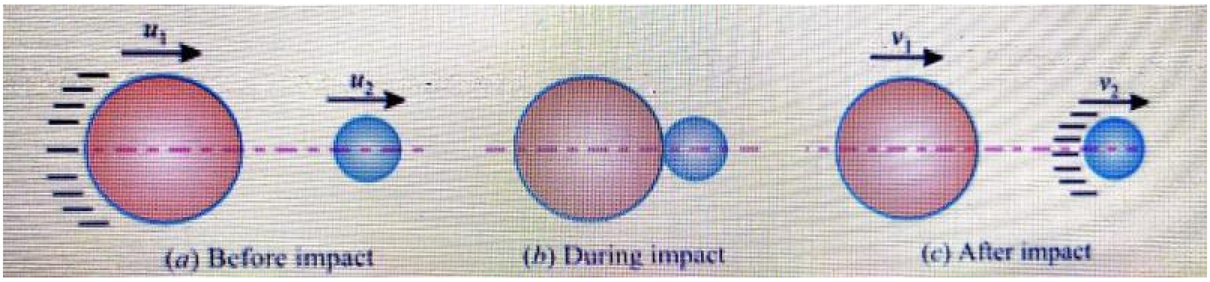
Where v1 = Final velocity of the first body

u1 = Initial velocity of the first body

v2, u2 = Corresponding values for the second body, and

e = Constant of proportionality.

COEFFICIENT OF RESTITUTION:



Consider two bodies A and B having a direct impact as shown in Fig. (a).

Let

u1 = Initial velocity of the first body,

v1 = Final velocity of the first body, and

u2, v2 = Corresponding values for the second body.

The impact will take place only if u1 is greater than u2. Therefore, the velocity of approach will be equal to (u1 – u2). After impact, the separation of the two bodies will take place, only if v2 is greater than v1. Therefore the velocity of separation will be equal to (v2 – v1).

Now as per Newton's Law of Collision of Elastic Bodies:

Velocity of separation = e × Velocity of approach

$(v2 - v1) = e\ (u1 - u2)$

Where e is a constant of proportionality, and is called the coefficient of restitution.

Its value lies between 0 and 1. It may be noted that if e = 0, the two bodies are inelastic. But if e = 1, the two bodies are perfectly elastic.

NOTE:

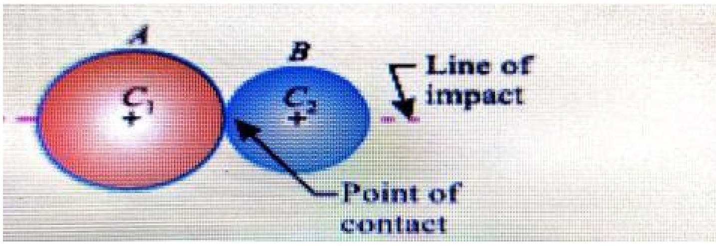
If the two bodies are moving in the same direction, before or after impact, then the velocity of approach or separation is the difference of their velocities. But if the two bodies are moving in the opposite directions, then the velocity of approach or separation is the algebraic sum of their velocities.

TYPES OF COLLISIONS

When two bodies collide with one another, they are said to have an impact.
Following are the two types of impacts.

1. Direct impact, and
2. Indirect (or oblique) impact.

DIRECT COLLISION OF TWO BODIES:



The line of impact, of the two colliding bodies, is the line joining the centres of these bodies and passes through the point of contact or point of collision as shown in Fig. If the two bodies, before impact, are moving along the line of impact, the collision is called as direct impact as shown in Fig.

Now; $m_1u_1 + m_2u_2 = m_1 v_1 + m_2 v_2$

NOTES

1. Since the velocity of a body is a vector quantity, therefore its direction should always be kept in view while solving the examples.
2. If velocity of a body is taken as + ve in one direction, then the velocity in opposite direction should be taken as – ve.
3. If one of the bodies is initially at rest, then such a collision is also called impact.

EXAMPLE

A ball of mass 1 kg moving with a velocity of 2 m/s impinges directly on a ball of mass 2 kg at rest. The first ball, after impinging, comes to rest. Find the velocity of the second ball after the impact and the coefficient of restitution.

Solution. Given : Mass of first ball (m_1) = 1 kg ; Initial velocity of first ball (u_1) = 2 m/s ; Mass of second ball (m_2) = 2 kg ; Initial velocity of second ball (u_2) = 0 (because it is at rest) and final velocity of first ball after impact (v_1) = 0 (because, it comes to rest)

Velocity of the second ball after impact.

Let v_2 = Velocity of the second ball after impact.

We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(1 \times 2) + (2 \times 0) = (1 \times 0) + (2 \times v_2)$$

$\therefore$ $2 = 2v_2$

or $v_2 = 1$ m/s **Ans.**

Coefficient of restitution

Let e = Coefficient of restitution.

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2)$$

$$(1 - 0) = e\,(2 - 0)$$

or $e = \frac{1}{2} = 0.5$ **Ans.**