Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/MCA/SEM-4/MM-401/2011

2011

OPERATIONS RESEARCH & OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) What is the method used to solve an LPP involving artificial variables ?

      a) Dominance method

      b) Charnes-Big M method

      c) VAM

      d) None of these.

   ii) The optimality condition for minimization LPP in the simplex method is

      a) $z_j - c_j \geq 0, \forall_j$ b) $z_j - c_j > 0, \forall_j$

      c) $z_j - c_j < 0, \forall_j$ d) $z_j - c_j \leq 0, \forall_j$.

iii) The maximization problem in the primal becomes .... problem in its dual.

a) minimization b) maximization

c) max-min d) min-max.

iv) For a travelling salesman problem who has visited n cities, the number of possible routes are

a) $n\,!$ b) $(n+1)\,!$

c) $(n-1)\,!$ d) $n-1$.

v) The basic feasible solution of the system of equations

$x_1 + x_2 + x_3 = 8$, $3x_1 + 2x_2 = 18$ are

a) no basic solution b) (2, 6, 0), (6, 0, 2)

c) (1, 7, 0), (7, 1, 0) d) none of these.

vi) In an assignment problem, the minimum number of lines covering all zeros in the reduced cost matrix of order n can be

a) at least n b) $n + 1$

c) $n - 1$ d) at most n.

vii) Which of the following is not a method to obtain the basic feasible solution in transportation problem ?

a) VAM

b) Least cost method

c) North-West corner method

d) MODI method.

viii) A two-person zero-sum game is said to be fair if

a) both the players have equal number of strategies

b) the game has a saddle point

c) the game does not have a saddle point

d) the value of the game is zero.

ix) In an ($M/M/1$) : (∞/FIFO) model, the average number of customers $E(n)$ =

a) ρ^n b) $\dfrac{\rho}{1+\rho}$

c) $\dfrac{\rho^2}{1+\rho}$ d) none of these.

Here the symbols have their usual meanings.

x) Number of basic variables in a balanced transportation problem is

a) mn b) $m+n-1$

c) $m+n$ d) $(m-1)(n-1)$.

xi) In PERT analysis, the variance of a job having optimistic time 5, pessimistic time 17 and most likely time 8, is

a) 3   b) 4

c) 7   d) none of these.

xii) CPM is

a) probabilistic   b) deterministic

c) event oriented   d) all of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that the vectors (1, 1, 1), (1, 1, 0) and (1, 0, 0) form a basis in E^3. Prove also that the vector (1, 3, 1) can replace any of three vectors of the basis to form a new basis.

3. Prove that the number of basic variables in a transportation problem is $(m+n-1)$.

4. If the arrival rate is λ and service rate is μ, then prove that the expected queue length is $\dfrac{\lambda^2}{\mu(\mu-\lambda)}$.

5. Find any two basic feasible solutions of the following set of equations.

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1,\ x_2,\ x_3,\ x_4 \geq 0.$$

6. Solve the game whose pay-off matrix is given by

$$\begin{pmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{pmatrix}$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) A firm manufactures products *A* and *B* and sells them at a profit of Rs. 2 and Rs. 3 respectively. Each product is processed on two machines I and II. Type *A* requires 1 minute processing on machine I and 2 minutes in machine II. Type *B* takes 1 minute in both machines. Machine I is available for not more than 6 hours and 40 minutes, while machine II is available not more than 10 hours in any working day. Formulate this LPP mathematically and solve by graphical method.

b) Find the dual of the following LPP :

Minimize $Z = x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1,\ x_1,\ x_2 \geq 0.$ 12 + 3

8. a) Solve the following LPP :

Maximize $Z = x_1 + x_2 + x_3$

Subject to $3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + 2x_3 \leq 2$

and $x_1,\ x_2,\ x_3 \geq 0.$

b) Solve the following transportation problem by VAM and find out the optimal solution :

	D_1	D_2	D_3	D_4	a_i
O_1	3	8	7	4	30
O_2	5	2	9	5	50
O_3	4	3	6	2	80
b_i	20	60	55	40	

7 + 8

9. Optimistic, most likely and pessimistic times of a project is given below :

	Estimated Duration (in weeks)		
Activity	Optimistic	Most likely	Pessimistic
1 — 2	1	1	7
1 — 3	1	4	7
1 — 4	2	2	8
2 — 5	1	1	1
3 — 5	2	5	14
4 — 6	2	5	8
5 — 6	3	6	15

a) Draw the project network.

b) Find the expected duration and variance of each activity.

c) Calculate the early and late occurrence for each event and the expected project length.

d) Calculate the variance and standard deviations of project length.

What is the probability that the project will be completed

i) 4 weeks earlier than expected ?

ii) not more than 4 weeks later than expected ?

e) If the project due date is 19 weeks, what is the probability of meeting the due date ?

[Given : Φ (1·33) = 0·4082 and Φ (0·666) = 0·2514]

4 + 2 + 2 + (2 + 2) + 3

10. a) Find the dual of the following LPP and hence solve it :

Maximize $Z = 3x_1 - 2x_2$

Subject to $x_1 \le 4$

$x_2 \le 6$

$x_1 + x_2 \le 5$

$-x_2 \le -1,\ x_1,\ x_2 \ge 0.$

b) Show that in a pure birth process with mean arrival rate λ, the probability that there will be n arrivals in time t is

$P_n(t) = e^{-\lambda t} (\lambda t)^n / n!$ 7 + 8

11. a) Use dynamic programming to solve the problem :

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

Subject to $y_1 + y_2 + y_3 \ge 15$ and $y_1,\ y_2,\ y_3 \ge 0.$

b) Establish EOQ model with uniform production rate, known demand, lead time zero and not shortage. Find the optimum order quantity and the optimum cost.

8 + 7