USN

06MAT11

First Semester B.E. Degree Examination, June-July 2009
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note:1. ***Answer any Five full questions, choosing at least two from each part.***
2. Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.
3. Answer to the objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. i) The n^{th} derivative of $\frac{1}{(ax+b)^2}$ is

(A) $\frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$ (B) $\frac{(-1)^n n+1! a^n}{(ax+b)^{n+2}}$ (C) $\frac{n+1! a^n}{(ax+b)^n}$ (D) $\frac{n! a^n}{(ax+b)^{n+1}}$

ii) If $y^2 = f(x)$, a polynomial of degrees 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ equals

(A) $f'''(x)+f''(x)$ (B) $f(x)f''(x)$ (C) $f(x)f'''(x)$ (D) $f'''(x)f(x)$

iii) The Pedal equation in polar coordinate system

(A) $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$ (B) $|\phi_1-\phi_2|$ (C) $\tan\phi - r\frac{d\theta}{dr}$ (D) $\cot\phi = r\frac{dr}{d\theta}$

iv) The curve $r=\frac{a}{1+\cos\theta}$ intersect orthogonally with the following curve

(A) $r=\frac{b}{1-\cos\theta}$ (B) $r=\frac{b}{1-\sin\theta}$ (C) $r=\frac{c}{1+\sin\theta}$ (D) $r=\frac{d}{1+\cos^2\theta}$ **(04 Marks)**

b. Find the nth derivative of $y = \cosh x \sin x$. **(04 Marks)**

c. If $y = \left[x+\sqrt{x^2+1}\right]^m$ prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ **(06 Marks)**

d. Show that the pairs of curves $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$ intersect orthogonally. **(06 Marks)**

2 a. i) If $f(x,y)=\frac{1}{x^3}+\frac{1}{y^3}+\frac{1}{x^3+y^3}$, then $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$ is

(A) 0 (B) 3f (C) 9 (D) – 3f

ii) If $u = f(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$

(A) 2 (B) 0 (C) 1 (D) $x + y + z$

iii) If an error of 1% is made in measuring its base and height, the percentage error in the area of a triangle is

(A) 0.2% (B) 1% (C) 2% (D) 0.1%

iv) In polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$ then $\partial(x,y)/\partial(r,\theta)$ is equal to

(A) r^3 (B) r^2 (C) r (D) - r **(04 Marks)**

b. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$. **(04 Marks)**

c. If $u = x^2-y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$ then determine the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$. **(06 Marks)**

d. Two sides of a triangle are 10cm & 12cm respectively, the angle between them is measured as 15° with an error of 15 mins. Find the error in the calculated length of the third side of the triangle due to error in the angle. **(06 Marks)**

3 a. i) The value of the definite integral $\int_{-1}^{+1} |x|\, dx$ is equal to

(A) 0 (B) 1 (C) $\pi/2$ (D) $\pi/4$

ii) The asymptote for the curve $x^3 + y^3 = 3axy$ is equal to

(A) $x + y + a = 0$ (B) $x - y - a = 0$ (C) No asymptotes (D) $x + y - a = 0$

iii) If $I_n = \int_0^{\pi/4} \cot^n \theta\, d\theta$, then $n(I_{n-1} + I_{n+1})$ is equal to

(A) 0 (B) 1 (C) 3 (D) None of these.

iv) The value of the definite integral $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}} dx$ is equal to

(A) 4/15 (B) $2\pi/15$ (C) 2/15 (D) 15/2 **(04 Marks)**

b. Obtain the reduction formula for $\int \tan^n x\, dx$. **(04 Marks)**

c. Evaluate $\int_0^{\pi} x \sin^2 x \cos^4 x\, dx$. **(06 Marks)**

d. Trace the curve $y^2(a-x) = x^3$, $a > 0$. **(06 Marks)**

4 a. i) The volume generated by the parabola $y^2 = 4ax$ when revolved about the y-axis between $y = 0$ & $y = 2a$ is

(A) $\frac{2\pi a^3}{5}$ (B) $\frac{32\pi a^5}{5a^2}$ (C) $\frac{5\pi a^2}{3}$ (D) $\frac{10\pi^2 a^3}{5}$

ii) The entire length of the cardioid $r = 5(1 + \cos\theta)$ is

(A) 40 (B) 30 (C) 20 (D) 5

iii) If $x = x(t)$, $y = y(t)$ then ds/dt is equal to

(A) $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ (B) $\sqrt{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2}$ (C) $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ (D) None of these

iv) $\frac{d}{d\alpha}\left[\int_a^b f(x,\alpha)dx\right]$ is equal to

(A) $\int_a^b \frac{d}{d\alpha} f(x,\alpha)dx$ (B) $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$ (C) $\int_b^a \frac{\partial}{\partial\alpha} f(x,\alpha)dx$ (D) 0 **(04 Marks)**

b. Find $ds/d\theta$ and ds/dr for the curve $r = a(1 - \cos\theta)$. **(04 Marks)**

c. Find the surface area of the solid generated by revolving the cycloid $x = a(t + \sin t)$ $y = a(1 + \cos t)$ **(06 Marks)**

d. Given that $\int_0^{\pi} \frac{dx}{\alpha - \cos x} = \frac{\pi}{\sqrt{\alpha^2 - 1}}$, hence evaluate $\int_0^{\pi} \frac{dx}{(\alpha - \cos x)^2}$ **(06 Marks)**

PART – B

5 a. i) The solution of the differential equation $\frac{dy}{dx} = xe^{y-x^2}$

(A) $2e^{-y} + e^{-x^2} = c$ (B) $e^{-y} - e^{-x^2} = c$ (C) $e^{y-x^2} = c$ (D) $e^{y+x^2} - c = 0$

ii) The integrating factor of the differential equation $\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$

(A) e^{y^3} (B) y^3 (C) x^3 (D) $-y^3$

iii) The necessary condition for the differential equation to be exact

(A) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$ (B) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ (C) $\frac{\partial M}{\partial y}+\frac{\partial N}{\partial x}=0$ (D) $\frac{\partial M}{\partial y}=-\frac{\partial N}{\partial x}$

iv) The orthogonal trajectory of $y^2 = 4a(x + a)$ is

(A) $y^2 = 4a(x+a)$ (B) $x^2 = 4a(y+a)$ (C) $y = mx + c$ (D) None of these. **(04 Marks)**

b. Solve $e^y\left(\frac{dy}{dx}+1\right)=e^x$ **(04 Marks)**

c. Solve $\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}$ **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$. **(06 Marks)**

6 a. i) If $\lim_{n\to\infty}\frac{U_{n+1}}{U_n}=l$, then the series is convergent if

(A) $l<1$ (B) $l>1$ (C) $l=1$ (D) $l=0$

ii) $\sum\frac{1}{n(n+2)}$ series is

(A) Convergent (B) Divergent (C) Oscillatory (D) Absolutely convergent.

iii) Every absolutely convergent series is necessarily

(A) Divergent (B) Convergent (C) Conditionally convergent (D) None of these

iv) The convergence of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}$.......... is tested by

(A) Ratio test (B) Raabe's test (C) Leibnitz test (D) Cauchy Riot test. **(04 Marks)**

b. Examine the series $\frac{1}{1.3.5}+\frac{2}{3.5.7}+\frac{3}{5.7.9}$.......... for convergence. **(04 Marks)**

c. Test the series for convergence $1+\frac{2}{3}x+\frac{2.3}{3.5}x^2+\frac{2.3.4}{3.5.7}x^3$.......... , $x > 0$. **(06 Marks)**

d. Find the nature of the series $\frac{x}{1.2}-\frac{x^2}{2.3}+\frac{x^3}{3.4}-\frac{x^4}{4.5}+$.......... , $x > 0$. **(06 Marks)**

7 a. i) if $2x + 3y + 4z + 5 = 0$ is the equation of a plane, then 2, 3, 4 represent

(A) Direction ratios of the normal to the plane

(B) Direction cosines of the normal to the plane

(C) Direction ratios of a line parallel to the plane

(D) None of these

ii) A line makes angles α, β, γ with the co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(A) 1 (B) 2 (C) 8/3 (D) 4/3

iii) The length of the perpendicular from the origin onto the plane $3x + 4y + 12z = 52$ is

(A) 4 (B) 3 (C) 0 (D) - 1

iv) The two lines are said to be parallel if

(A) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (B) $a_1/a_2 = b_1/b_2 = c_1/c_2$

(C) $a_1/b_1 + a_2/b_2 + c_1/c_2 = 0$ (D) None of these. **(04 Marks)**

b. Show that the angle between any two diagonals of a cube is $\cos^{-1}(1/3)$. **(04 Marks)**

c. Show that the lines $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ and $x + 2y + 3z - 8 = 0 = 2x + 3y + 4z - 11$ intersect. Find their point of intersection and the equation of the plane containing them. **(06 Marks)**

d. Find the image of the point $(2, -1, 3)$ in the plane $2x + 4y + z - 24 = 0$. **(06 Marks)**

8 a. i) The velocity of the moving particle along the curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ is

(A) $-e^{-t}i - 6\sin 3tj + 6\cos 3tk$ (B) $e^{-t}i - 18\cos 3tj - 18\sin 3tk$

(C) $e^{-t}i + 2\cos 3tj + 2\sin 3tk$ (D) $e^{-t} - 6\sin 3t$

ii) The resultant of a gradient is
(A) Vector (B) Scalar (C) Irrotational (D) Field

iii) If the vector $\vec{F} = (x+3y)i + (y-2z)j + (x+az)k$ is Solenoidal then a is equal to
(A) 2 (B) - 2 (C) 0 (D) 1

iv) If $F = x^2 + y^2 + z^2$, then curl grad F is
(A) 1 (B) 0 (C) - 1 (D) 2 **(04 Marks)**

b. Find the angle between the surfaces $\phi = x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,–1, 2). **(04 Marks)**

c. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both Solenoidal & irrotational. **(06 Marks)**

d. Prove that curl curl $\vec{F}$ = grad div $\vec{F} - \nabla^2 \vec{F}$ **(06 Marks)**

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