

Roll No _____

Time: 3 hrs.**Max Marks: 100**

Note: Attempt **five** questions in all, including **Q.No. 9 (Section-E)** which is **compulsory** and selecting **one** question each from sections A, B, C and D. All questions carry equal marks.

Section – A

- Q1. (a) What is a bipartite graph and a complete bipartite graph? Draw $K_{3,3}$.
 (b) Show that
- No simple graph can have all the vertices with distinct degrees.
 - If a graph has Euler's path, then it has either no vertex of odd degree or two vertices of odd degree. (10, 10)
- Q2. (a) Discuss the Dijkstra's algorithm to find the shortest path from one specified vertex u to another specified vertex v , with the help of an example.
 (b) State and prove the five-color problem. (10, 10)

Section – B

- Q3. (a) In a town of 10000 families, it was found that 40% families buy newspaper A, 20% buy newspaper B and 10% buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the newspapers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.
 (b) Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n . (10, 10)
- Q4. (a) Partition the set $A = \{1, 2, 3, \dots, 10\}$ using the minsets generated by $B_1 = \{1, 7, 8\}$, $B_2 = \{1, 6, 9, 10\}$, $B_3 = \{1, 9, 10\}$. Also represent the minsets thus generated, through a Venn diagram.
 (b) If A and B are two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.
 (c) If $A = \{1, 2, 3, 4\}$. Consider the following relation in A
 $R = \{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$. Draw its directed graph. (9, 6, 5)

Section – C

- Q5. (a) Verify $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.
 (b) Construct the truth table of $\sim(p \wedge q) \rightarrow \sim p \vee \sim q$. Is it a contradiction?
 (c) Let p be "He is rich" and q be "He is honest". Write the following statements in symbolic forms using p and q :
- To be poor is to be honest
 - It is necessary to be poor in order to be honest
 - He is poor only if he is dishonest (7, 7, 6)
- Q6. (a) Prove by the method of induction that every even power of every odd number greater than 1 when divided by 8, leaves 1 as a remainder.
 (b) Write the following statements in symbolic form and give their negations:

- (i) If you work hard, you will get the first division.
(ii) If it rains, he will not go to Kathua.
(iii) If Mahatma Gandhi was a saint then Sardar Patel was an iron man.

(14, 6)

Section – D

Q7. (a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

- (b) Discuss matrix addition, scalar multiplication and multiplication of matrices by taking some suitable example. (10, 10)

Q8. (a) Solve the following system of equations using Matrix Inversion method:

$$2x - 2y + 5z = 13$$

$$2x + 3y + 4z = 20$$

$$3x - y + 3z = 10$$

- (b) Discuss the steps involved in Gauss-Elimination and Gauss-Jordan method of solving simultaneous equations. (8, 12)

Section – E (Compulsory)

Q9. Attempt and explain the following briefly:

- (i) Differentiate between simple graph and a multi graph.
(ii) What is chromatic number?
(iii) What do you mean by Cartesian product?
(iv) State the DeMorgan's law.
(v) Differentiate between countable and uncountable sets.
(vi) Give different types of quantifiers.
(vii) Explain 'equivalence' and 'implication'.
(viii) What do you mean by transpose of a matrix?
(ix) Explain the rank of a matrix.
(x) What is a proposition?

(10 X 2 = 20)