FERMI DIRAC DISTRIBUTION

Fermi Dirac function provides the probability of occupancy of energy levels by fermions.

At OK the highest energy level occupied by an electron is called fermilevel.

Fermi dirac distribution function

$$f(E) = \frac{1}{1 + e^{(E - E_F)/RT}}$$

$f(E)$ - Fermi distribution function

E - Energy of level (whose occupancy is considered

E_F - Fermi Energy

K - Boltzman constant

T - Absolute temperature

FERMI DIRAC DISTRIBUTION VARIATION WITH TEMPERATURE

CASE I:-

* At absolute zero electrons occupy energy levels in pairs starting from bottom to upper level E_f.
* Fermilevel is defined as the upper most filled energy level in a conductor at OK.

* Fermi energy is defined as the maximum energy that a free electron can have at OK.
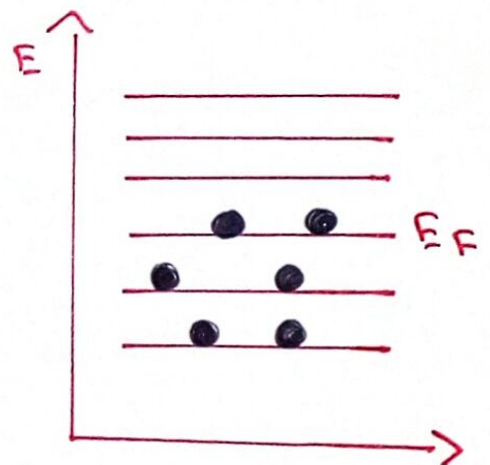
CASE I

Let Energy level E lie below E_F

$$\boxed{\text{at } T = 0K \qquad E < E_F}$$

$$f = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$E - E_F$ term becomes negative

$$f(E) = \frac{1}{1 + e^{\left(\frac{negative}{0}\right)}} \implies \frac{1}{1 + e^{-\alpha}}$$

$$e^{-\alpha} = 0 \qquad f(E) = \frac{1}{1+0} \implies \boxed{f(E) = 1}$$

This implies that all energy bands below E_F is filled

CASE II

Let energy level E lie above E_F at $T = 0K$

$$T = 0K \qquad E > E_F$$
$$E - E_F - \text{positive value}$$

$$f(E) = \frac{1}{1 + e^{\left(\frac{positive}{0}\right)}}$$

$$f(E) = \frac{1}{1 + e^{\alpha}} \implies f(E) = \frac{1}{1+\alpha}$$

$$f(E) = 0$$

This implies that levels above E_F are vacant at 0K
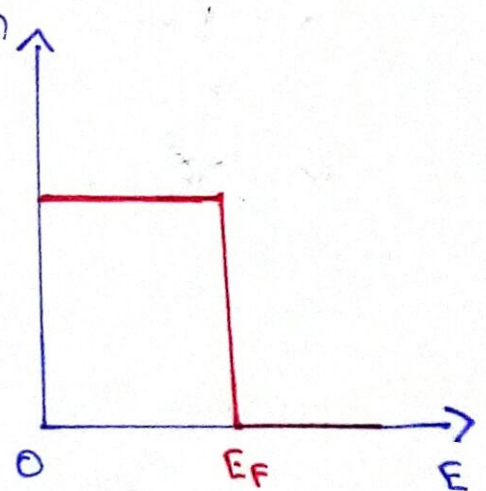
CASE III

when $T > 0K$ (on heating the conductor)

Electrons below E_F Jump into levels with energy above E_F and a new distribution of electrons is obtained.
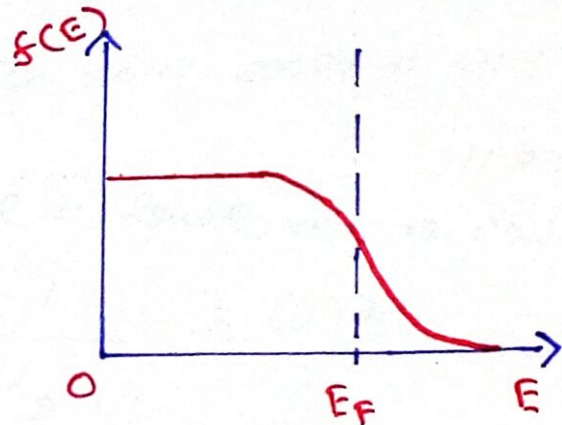
At $T > 0K$ $E = E_F$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

$$f(E) = \frac{1}{1 + e^{(0/kT)}} \implies \frac{1}{1+1} = \frac{1}{2}$$

$$\boxed{f(E) = 0.5}$$

* Probability of finding electrons below E_F decreases

* Probability of finding electrons above E_F increases

This shows that the probability of occupancy of fermi level at any temperature above zero Kelvin is 0.5 or 50%.

EXPRESSION FOR DENSITY OF ELECTRONS IN CONDUCTION BAND

Electron concentration (n) - The number of electrons in the conduction band per unit volume of the material.

The number of electrons per unit volume in the conduction band for energy range E & E+dE is given by,
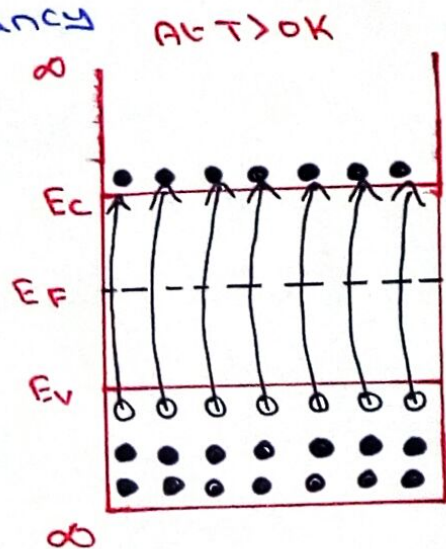
$$dn = Z(E) \, F(E) \, dE$$

Z(E) - Density of states in the energy range E & E+dE

F(E) - Electron probability occupancy

At T > 0K

Number of electrons in the entire region E_c & α is,

$$n = \int\limits_{\infty}^{\infty} dn$$

$$n = \int\limits_{E_c}^{\infty} Z(E) \, F(E) \, dE \longrightarrow \text{①}$$

Density of states in CB is given by,

$$Z(E) \, dE = \frac{4\pi}{h^3} \left(2m_e^*\right)^{3/2} \left(E - E_c\right)^{1/2} dE \longrightarrow \text{②}$$

Probability of electron occupation is given by,

$$F(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \longrightarrow \text{③}$$

Substitute equation ② & ③ in equation ①

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} \times \frac{1}{1+e^{(E-E_F)/kT}} \, dE \rightarrow \text{④}$$

Since kT is very small, $e^{(E-E_F)/kT}$ is large compared to 1, we can take $1+e^{(E-E_F)/kT}$ as $e^{(E-E_F)/kT}$

Equation ④ becomes,

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} \times \frac{1}{e^{(E-E_F)/kT}} \, dE$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_F)/kT} \, dE$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{E_F/kT} e^{-E/kT} \, dE$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{E_F/kT} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-E/kT} \, dE$$

To solve integral, Let us assume

$E-E_c = x$

$dE = dx$

$E = x + E_c$

when

$E = E_c$

$x = 0$

Lower limit

$E = \infty$

$x = \infty$

Upper limit

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{E_F/kT} \int_{0}^{\infty} x^{1/2} e^{(-x-E_c)/kT} \, dx$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{(E_F-E_c)/kT} \int_{0}^{\infty} x^{1/2} e^{-x/kT} \, dx$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \boxed{\int_0^\infty x^{1/2} e^{-x/kT} dx} \rightarrow ⑤$$

This integral can be solved using Gamma function. General solution for this integral is,

$$\int_0^\infty x^{1/2} e^{-x/kT} dx = \frac{\sqrt{\pi} (kT)^{3/2}}{2} \rightarrow ⑥$$

Substitute equation ⑥ in equation ⑤, we get,

$$n = \frac{\overset{2}{\cancel{4}}\pi}{h^3} (2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \times \frac{\pi^{1/2} (kT)^{3/2}}{\cancel{2}}$$

$$n = \frac{2}{h^3} (2m_e^* \pi kT)^{3/2} e^{(E_F - E_C)/kT}$$

$$\boxed{n = 2 \left(\frac{2\pi kT m_e^*}{h^2} \right)^{3/2} e^{(E_F - E_C)/kT}}$$

The above equation is the expression for Concentration of electrons in the conduction band for an intrinsic semiconductor.

EXPRESSION FOR DENSITY OF HOLES IN VALENCE BAND

when electrons are transferred from valence band to conduction band, a hole is created in the valence band.

dp - Number of holes in valence band between energy E & $E+dE$

$$dp = Z(E)(1-F(E))dE \longrightarrow \text{①}$$

$Z(E)$ - Density of states in the energy range E & $E+dE$

$1-F(E)$ - Hole probability occupancy

$$Z(E) = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_v-E)^{1/2}dE \longrightarrow \text{②}$$
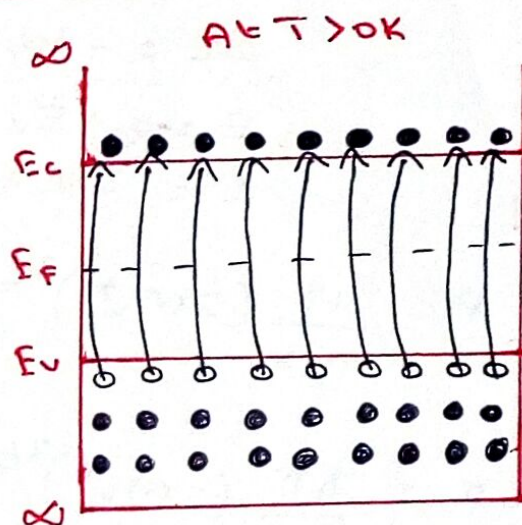
$$1-F(E) = 1 - \frac{1}{1+e^{(E-E_f)/kT}}$$

$$1-F(E) = \frac{\cancel{1}+e^{(E-E_f)/kT}-\cancel{1}}{1+e^{(E-E_f)/kT}}$$

$$1-F(E) = \frac{e^{(E-E_f)/kT}}{1+e^{(E-E_f)/kT}}$$

At T > 0K

Since E is very small than E_f in valence band
$$1+e^{(E-E_f)/kT} \approx 1$$

$$\therefore \quad 1-F(E) = e^{(E-E_f)/kT} \longrightarrow \text{③}$$

On substituting eqn ② & ③ in eqn ① we get,

$$dp = \frac{4\pi}{h^3} \left(2m_h^*\right)^{3/2} \left(E_v - E\right)^{1/2} e^{(E-E_F)/kT} \, dE$$

Density of holes in the entire energy range

$$p = \int_{-\infty}^{E_v} \frac{4\pi}{h^3} \left(2m_h^*\right)^{3/2} \left(E_v - E\right)^{1/2} e^{(E-E_F)/kT} \, dE$$

$$P = \frac{4\pi}{h^3} \left(2m_h^*\right)^{3/2} e^{-E_F/kT} \int_{-\infty}^{E_v} \left(E_v - E\right)^{1/2} e^{E/kT} \, dE$$

In order to solve the integral, substitute

$$E_v - E = x$$

$$dE = -dx$$

$$E = -x + E_v$$

$$E = -x + E_v$$

when

$E = -\infty$

$x = \infty$

lower limit

$E = E_v$

$x = 0$

upper limit

$$P = \frac{4\pi}{h^3} \left(2m_h^*\right)^{3/2} e^{-E_F/kT} \int_{\infty}^{0} x^{1/2} e^{(-x+E_v)/kT} (-dx)$$

$$P = \frac{4\pi}{h^3} \left(2m_h^*\right)^{3/2} e^{(E_v-E_F)/kT} \int_{\infty}^{0} x^{1/2} e^{-x/kT} (-dx)$$

Limits change due to $(-dx) \rightarrow dx$

$$P = \frac{4\pi}{h^3} \left(2m_h^*\right)^{3/2} e^{(E_v-E_F)/kT} \boxed{\int_{0}^{\infty} x^{1/2} e^{-x/kT} \, dx}$$

General solution for this integral using Gamma function

$$\boxed{\int_{0}^{\infty} x^{1/2} e^{-x/kT} \, dx = \frac{\sqrt{\pi} \, (kT)^{3/2}}{2}}$$

$$\therefore \quad P = \frac{4\pi^2}{h^3}(2m_h^*)^{3/2} e^{(E_v - E_F)/kT} \times \frac{\pi^{1/2}(kT)^{3/2}}{2}$$

$$P = \frac{2\pi}{h^3}(2m_h^*)^{3/2} e^{(E_v - E_F)/kT} \times \pi^{1/2}(kT)^{3/2}$$

$$\boxed{P = 2\left(\frac{2\pi kT\, m_h^*}{h^2}\right)^{3/2} e^{(E_v - E_F)/kT}}$$

INTRINSIC CARRIER CONCENTRATION

In an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valence band.

$$\therefore \quad n = P = n_i$$

n_i = intrinsic carrier concentration

$$n_i^2 = n \times P \longrightarrow ①$$

we know

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_F - E_C)/kT}$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_V - E_F)/kT}$$

Substituting n & P values in eq ① we get,

$$n_i^2 = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_F - E_C)/kT}$$

$$\times$$

$$2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_V - E_F)/kT}$$

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3 (m_e^* m_h^*)^{3/2} e^{(E_V - E_C)/kT}$$

$$E_V - E_C = -E_g \text{ (Energy Gap)}$$

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3 (m_e^* m_h^*)^{3/2} e^{E_g/kT}$$

$$\boxed{n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT}}$$

FERMI LEVEL & ITS VARIATION WITH TEMPERATURE

In an intrinsic semiconductor, the density of electrons in the conduction band is equal to the density of holes in the valence band.

$$n = p \longrightarrow ①$$

we know that

$$n = 2\left(\frac{2\pi kT\, m_e^*}{h^2}\right)^{3/2} e^{(E_F - E_C)/kT}$$

$$p = 2\left(\frac{2\pi kT\, m_h^*}{h^2}\right)^{3/2} e^{(E_V - E_F)/kT}$$

Substituting $n = p$ values in eq ① we get,

$$\cancel{2}\left(\frac{2\pi k\cancel{T}\, m_e^*}{h^2}\right)^{3/2} e^{(E_F - E_C)/kT} = \cancel{2}\left(\frac{2\pi k\cancel{T}\, m_h^*}{h^2}\right)^{3/2} e^{(E_V - E_F)/kT}$$

$$e^{(E_F - E_C)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} e^{(E_V - E_F)/kT}$$

Rearranging we get,

$$\frac{e^{E_F/kT}}{e^{-E_F/kT}} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} \frac{e^{E_V/kT}}{e^{-E_C/kT}}$$

$$e^{2E_F/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} e^{(E_V + E_C)/kT}$$

Taking log on both the sides

$$\frac{2E_F}{kT} = \frac{3}{2} \log_e \frac{m_h^*}{m_e^*} + \frac{E_v + E_c}{kT}$$

If $m_h^* = m_e^*$

$$\log_e \frac{m_h^*}{m_e^*} = \log \frac{m_h^*}{m_h^*} \Rightarrow \log 1 \Rightarrow 0$$

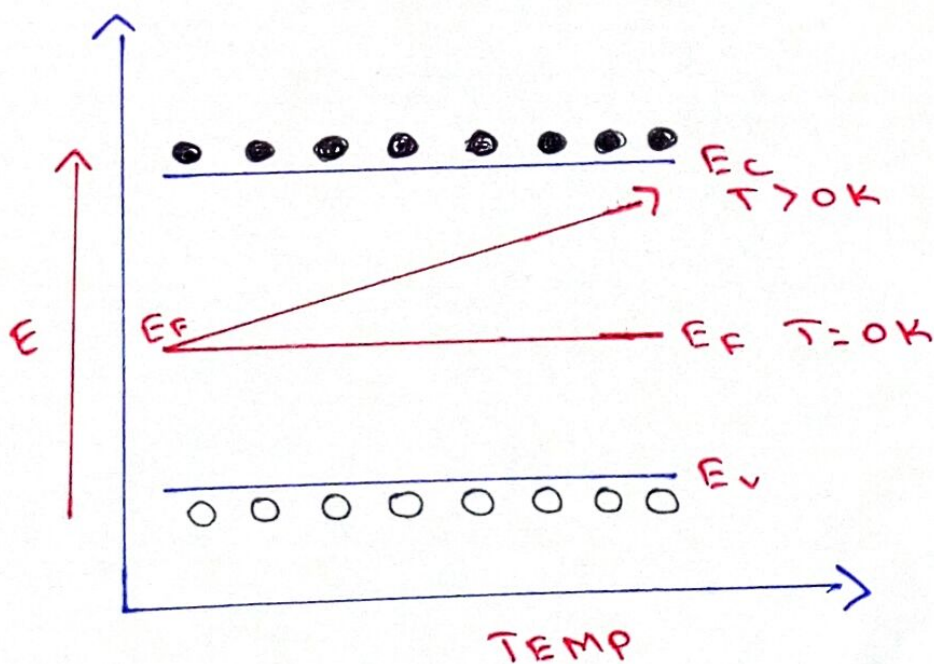$$\frac{2E_F}{kT} = 0 + \frac{E_v + E_c}{kT}$$

$$\frac{2E_F}{\cancel{kT}} = \frac{E_v + E_c}{\cancel{kT}}$$

$$\boxed{E_F = \frac{E_v + E_c}{2}}$$

Fermi level is located half way between top of valence band and bottom of conduction band.

VARIATION WITH TEMPERATURE

when the temperature is increased, the Fermi level changes as shown in the figure below
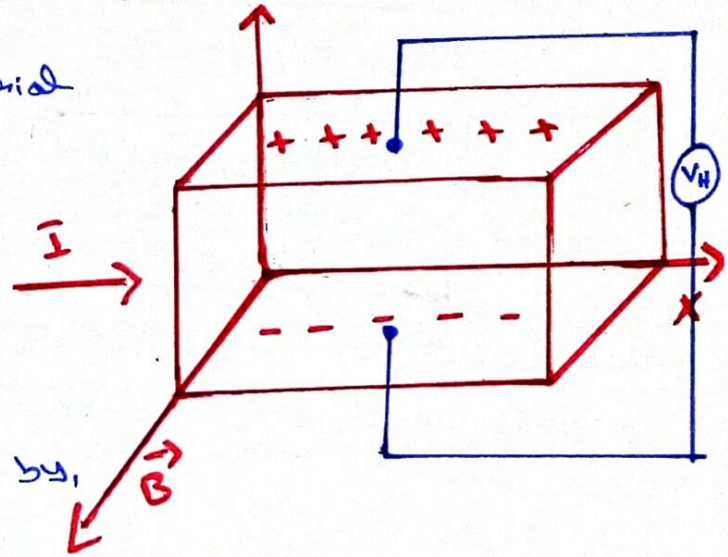
HALL EFFECT

When a current carrying conductor is placed in a magnetic field (B) perpendicular to this current, a potential difference is developed inside the conductor perpendicular to both current and magnetic field.

Consider a semiconductor material in rectangular shape

X axis - current

Y axis - voltage

Z axis - Magnetic field

The Lorentz equation is given by,

$$\vec{F} = q\vec{E} + q\vec{U} \times \vec{B}$$

Electric force   Magnetic force

Downward magnetic force experienced by electron   : BeV

V - drift velocity

e - charge of an electron

B - Applied magnetic field

Upward electric force acting on electron $\}$ = eE_H

E_H - Hall electric field

At thermal equilibrium downward force = upward force

$$BeV = eE_H \longrightarrow ①$$

current density, $J_x = -neV$

$$V = -\frac{1}{ne} J_x$$

Substituting V in equation ① we get

$$BV = E_H$$

$$E_H = -\frac{1}{ne} J_x B$$

$$E_H = -\frac{1}{ne} B J_x$$

$$\boxed{R_H = -\frac{1}{ne}}$$

R_H - Hall coefficient

$$E_H = R_H B J_x$$

n type

$$\boxed{R_H = -\frac{1}{ne}}$$

$$\boxed{R_H = \frac{E_H}{J_x B}}$$

p-type

$\rightarrow$ ②

$$\boxed{R_H = +\frac{1}{Pe}}$$

HALL COEFFICIENT IN TERMS OF HALL VOLTAGE

t - thickness of the sample
V_H - Hall voltage
E_H - Hall electric field
Relation between electric field & voltage is,

$$V_H = E_H t \qquad \rightarrow ③$$

E_H value from eu ② is

$$E_H = J_x B R_H \quad \text{on substituting in eu ③ is,}$$

$$V_H = J_x B R_H t$$

current density $J_x = \frac{I}{A \,(area)} \Rightarrow \frac{I_x}{bt}$

$$\therefore V_H = \frac{I_x}{b\cancel{t}} \times B R_H \cancel{t}$$

$$V_H = \frac{I_x B R_H}{b} \Rightarrow \boxed{R_H = \frac{V_H b}{I_x B}}$$

MOBILITY OF CHARGE CARRIERS

Electrical conductivity of a semiconductor

$$\sigma = ne\mu$$

n type   p type

$$\sigma_e = ne\mu_e \qquad \sigma_h = Pe\mu_h$$

$$\boxed{\mu_e = \frac{\sigma_e}{ne}} \qquad \boxed{\mu_h = \frac{\sigma_h}{Pe}}$$