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06MAT21

Second Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***1.Answer any FIVE full questions, choosing at least two from each part.***
2.Answer all objective type questions only in OMR sheet page 5 of the answer booklet.
3.Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Select the correct answer in each of the following :

i) Curvature of a straight line is

A) ∞ B) zero C) Both A and B D) None of these.

ii) Radius of the curvature of the curve $\gamma = a\sin\theta$ at the pole is

A) $\frac{\pi}{2}$ B) $-\frac{a_n}{2}$ C) $\frac{a_n}{2}$ D) zero.

iii) If f(x) is continuous in the closed interval [a, b] differential in (a, b) then $\exists$ at least one value c of x in (a, b) such that $f'(c) =$

A) $\frac{f(b)-f(a)}{b-a}$ B) $\frac{f(b)+f(a)}{b+a}$ C) $\frac{f(b)-f(a)}{b+a}$ D) None of these

iv) Maclaurin's series expansion of $\log(1+x)$ is

A) $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots\dots\dots$ B) $x-\frac{x^2}{2!}+\frac{x^3}{3!}-\frac{x^4}{4!}+\dots\dots\dots$

C) $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\dots\dots\dots$ D) $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots\dots\dots$ **(04 Marks)**

b. Show that for the ellipse in the pedal form $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{a^2b^2}$, the radius of the curvature at the point (p, r) is a^2b^2/p^3. **(04 Marks)**

c. Verify the Roller theorem for the function $f(x) = (x-a)^m(x-b)^n$, $x \in (a, b)$. **(06 Marks)**

d. Expand $\tan(\frac{\pi}{4}+x)$ using the Maclaurin's expansion upto the 4[th] degree term. **(06 Marks)**

2 a. Select the correct answer in each of the following :

i) The basic fundamental indeterminate forms are

A) $\frac{0}{0}$ B) $\frac{\infty}{\infty}$ C) 0 D) both A and B

ii) The value of $\underset{x\to\pi/2}{\text{lt}} \frac{\log\sin x}{\left(\frac{\pi}{2}-x\right)^2}$ is

A) zero B) ½ C) – ½ D) - 2

iii) The necessary and sufficient condition for maximum and minimum is

A) $f_x(xy) = 0$ B) $f_y(xy) = 0$ C) $f_x(xy) = 0 = f_y(xy)$ D) None of these.

iv) In a plane triangle ABC, the maximum value of $\cos a \cos b \cos c$ is,

A) 3/8 B) 1/8 C) 5/8 D) 25/8. **(04 Marks)**

b. Evaluate $\lim_{x \to a}\left[2-\left(\frac{x}{a}\right)\right]^{\tan\left(\frac{\pi x}{2a}\right)}$ (04 Marks)

c. Expand $\tan^{-1}(y/x)$ about the point (1, 1) up to 2nd degree term. (06 Marks)

d. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. (06 Marks)

3 a. Select the correct answer in each of the following :

i) Value of $\int_0^1 \int_x^{\sqrt{x}} xy\, dx$ is

A) zero B) $-\frac{1}{24}$ C) $\frac{1}{24}$ D) 24

ii) R is the region of xy plane bounded by the curves $y = y_1(x)$, $y = y_2(x)$ and line $x = a$, and $x = b$. Then $\iint_R f(xy)dxdy$ is

A) $\int_{y=y_1(x)}^{y_2(x)} \int_{x=a}^{b} f(xy)dy\, dx$ B) $\int_{x=a}^{b} \int_{y=y_1(x)}^{y_2(x)} f(xy)dxdy$

C) $\int_{x=a}^{b} \int_{y=y_1(x)}^{y_2(x)} f(x,y)dydx$ D) All are correct.

iii) $\iint_R dxdy$ represents

A) Area of the region in polar form B) Area of the region in Cartesian form

C) Both A and B D) None of these.

iv) The value of $\Gamma(n+1)$ is

A) $n\Gamma(n)$ B) n! C) (n – 1)! D) Both A and B. (04 Marks)

b. If A is the area of the rectangular region bounded by the lines x = 0, x = 1 and y = 0 , y = 2 Evaluate $\int_A (x^2 + y^2)\, dA$. (04 Marks)

c. With usual notations, prove that $\sqrt{x}\ \Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(2m+1)$. (06 Marks)

d. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$, by changing the order of integration. (06 Marks)

4 a. Select the correct answer in each of the following :

i) If $\vec{F}$ is the force acted upon by the particle moves from one end of a curve to the other end. Then the total work done by $\vec{F}$ is

A) $\int_c \vec{F} \times d\vec{r}$ B) $\int_c \vec{F} \cdot d\vec{r}$ C) $\int_c d\vec{r}$ D) None of these.

ii) The line integral of $\vec{F} = x^2 i + xyj$ from O(0, 0) to P(1, 1) along the straight line is

A) 1/3 B) – 1/3 C) 2/3 D) 4/3

iii) If $\partial N/\partial x$, $\partial M/\partial y$ are continuous functions, C is a simple closed curve enclosing the region R in the xy - plane. The Green's theorem states that

A) $\oint_c Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$ B) $\oint_c Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$

C) $\oint_c Mdx + Ndy = \iint_R \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)dxdy$ D) $\oint_c Mdx - Ndy = \iint_R \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right)dxdy$

iv) The cylindrical co-ordinate system is
A) Not orthogonal B) Orthogonal C) Coplanar D) Non-coplanar. **(04 Marks)**

b. Find the total work done by the force represented by $\vec{F} = 3xyi - yj - 2zxk$, in moving a particle round the circle $x^2 + y^2 = 4$. **(04 Marks)**

c. Verify the Green's theorem for $\oint_c (xy + y^2)dx + x^2dy$, where c is the alosed curve of the region bounded by $y = x$ and $y = x^2$. **(06 Marks)**

d. Express the vector $\vec{A} = zi - 2xj + yk$, in cylindrical coordinates. **(06 Marks)**

PART – B

5 a. Select the correct answer in each of the following :
i) Solution of the differential equation $(D^2 - a^2)y$ is
A) $a_1e^{-ax} + c_2e^{ax}$ B) $(ax + b)e^{ax}$ C) $(c_1 + c_2x + c_3x^2)e^{ax}$ D) $(c_1x + c_2x^2)e^{ax}$
ii) Particular integral of the differential equation $(D^2 + 5D + 6)y = e^x$ is
A) e^x B) $e^x/12$ C) $e^x/30$ D) $e^x/6$.
iii) Complementary function of $y'' - 2y' + y = x\,e^x \sin x$ is
A) $c_1e^x + c_2e^{-x}$ B) $(c_1x + c_2)e^x$ C) $(c_1 + c_2x)e^{-x}$ D) None of these.
iv) Particular integral of $(D^2 - 4)\,y = \sin 3x$ is
A) 1/4 B) – 1/13 C) 1/5 D) None of these. **(04 Marks)**

b. Solve $(D^3 + D^2 + 4D + 4)y = 0$ **(04 Marks)**

c. Solve $y'' + 16y = x \sin 3x$. **(06 Marks)**

d. Solve $(D^2 - D - 2)y = 1 - 2x - 9e^{-x}$ by the method of undetermined coefficients. **(06 Marks)**

6 a. Select the correct answer in each of the following :
i) The wronskin of cos x and sin x is
A) 0 B) 1 C) 2 D) 4
ii) To transform $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+n)]$ into a L.D.E. with constant coefficients put $(1+x) =$
A) e^t B) log x C) e^x D) t.
iii) The solution of the differential equation $y'' + 6y = 0$ satisfies the condition $y(0) = 1$ and $y(\pi/2) = 2$ is
A) cosx + 2sinx B) 2cosx + sinx C) cosx – sinx D) None of these.
iv) $c_1\cos ax + c_2\sin ax - \frac{x}{2a}\cos ax$ is the general solution of
A) $(D^2 + a^2)y = \sin ax$ B) $(D^2 - a^2)y = \sin ax$
C) $(D^2 + a^2)y = \cos ax$ D) $(D + a)y = \sin x$ **(04 Marks)**

b. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. **(04 Marks)**

c. Solve $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$, by variation of parameter method. **(06 Marks)**

d. Solve $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$. Give that $x(0) = 0$, $\frac{dx}{dt}(0) = 15$. **(06 Marks)**

7 a. Select the correct answer in each of the following :

i) Laplace transform of f(t), $t \geq 0$ is defined by

A) $\int_0^\infty e^{-st} f(t)dt$ B) $\int_0^\infty e^{st} f(t)dt$ C) $\int_0^\infty e^{-t} f(t)dt$ D) $\int_1^\infty e^{-st} f(t)dt$

ii) Laplace transform of cos at is

A) $\frac{a}{s^2+a^2}$ B) $\frac{s}{s^2+a^2}$ C) $\frac{1}{s^2+a^2}$ D) $\frac{s}{s^2-a^2}$

iii) $L^{-1}\left\{\frac{\bar{f}(s)}{s}\right\}$ is

A) $\int_0^t f(t)dt$ B) $\int_0^t \frac{f(t)}{t}dt$ C) $t^n f(t)$ D) None of these.

iv) Laplace transform of $f'(t)$ is

A) $s f(s) - f(0)$ B) $s f'(s) - f(0)$ C) $f(s) - f(0)$ D) None of these. **(04 Marks)**

b. Find $L\{e^{at} + 2t^n - 3\sin 3t + 4\cosh 2t\}$ **(04 Marks)**

c. If f(t) is a periodic function of period 'w', then show that

$$L\{f(t)\} = \frac{1}{1-e^{-sw}} \int_0^w e^{-st} f(t)dt$$ **(06 Marks)**

d. Express the function $f(t) = \begin{cases} \sin t & o < t \leq \pi/2 \\ \cos t & t > \pi/2 \end{cases}$, in terms of unit step function and find its Laplace transform. **(06 Marks)**

8 a. Select the correct answer in each of the following :

i) Inverse Laplace transform of $\frac{s-a}{(s-a)^2+b^2}$ is

A) $e^t\cos bt$ B) $e^{at}\cos bt$ C) $e^{-at}\cos bt$ D) $e^{at}\sin bt$

ii) Inverse Laplace transform of $\left[\frac{s^2-3s+4}{s^3}\right]$ is

A) $1 - 3t + 2t^2$ B) $10 - 3t + 2t^2$ C) $4 - 3t + 4t^2$ D) None of these.

iii) $L\{u(t-a)\}$, where $u(t-a)$ is a unit step function is

A) $\frac{e^{-as}}{a}$ B) $\frac{e^{as}}{s}$ C) e^{as} D) $s\,e^{-as}$

iv) $L\{\delta(t-a)\}$, where $\delta(t-a)$ is n unit impulse function

A) e^{as} B) e^{-as} C) e^s D) $\frac{e^{-as}}{s}$ **(04 Marks)**

b. Find the inverse Laplace transform of $\frac{3s+2}{s^2-s-2}$. **(04 Marks)**

c. Using the convolution theorem obtain $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **(06 Marks)**

d. Solve the differential equation $y''(t) + 4y'(t) + 4y(t) = e^{-t}$ with $y(0) = 0\ y'(0)$, using the Laplace transform method. **(06 Marks)**

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