

2012

NUMERICAL METHODS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

$$10 \times 1 = 10$$

- i) If $\frac{5}{3}$ is approximated to 1.6667, then absolute error is
 - a) 0.000033
- b) 0.000043
- c) 0.000034
- d) none of these.
- ii) If $f(x) = \frac{1}{x^2}$ then the divided difference f(a, b) is
 - a) $\frac{(a+b)}{(ab)^2}$
- b) $\frac{(a-b)}{(ab)^2}$
- c) $\frac{1}{a^2} \frac{1}{h^2}$
- $d) \qquad \frac{1}{a^2 b^2}.$



- iii) The value of $\frac{\Delta^2}{E}(x^3)$ is
 - a) *x*

b) 6*x*

c) 3*x*

- d) x^2 .
- iv) If $\frac{dy}{dx} = x + y$ and y(1) = 0, then y(1. 1) according to

Euler's method is, h = 0.1 (say),

a) 0·1

b) 0·3

c) 0.5

- d) 0.9.
- v) If $y_0 = 2$, $y_1 = 4$, $y_2 = 8$, $y_4 = 32$ then y_3 is equal to
 - a) 5

b) 15

c) 6

- d) 16·5.
- vi) The order of h in the error expression of Trapezoidal rule

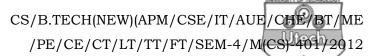
is

a) 1

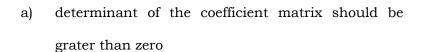
b) 2

c) 3

- d) 4.
- vii) Regula-Falsi method is
 - a) conditionally convergent
 - b) linearly convergent
 - c) divergent
 - d) none of these.



viii) Pivoting is very much essential because



- b) pivot element should not have very large value compared to the elements of the matrix
- c) it reduces the possibility of division by zero
- d) change of convergence is higher.
- Which of the following is true? ix)

a)
$$\Delta^n x^n = (n+1)$$

$$\Delta^n x^n = (n+1)!$$
 b) $\Delta^n x^n = n!$

c)
$$\Delta^n x^n = 0$$

d)
$$\Delta^n x^n = n$$
.

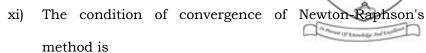
An $n \times n$ matrix A is said to be diagonally dominant if x)

a)
$$\left|a_{ii}\right| \ge \sum_{\substack{j=1 \ i \ne j}}^{n} \left|a_{ij}\right|$$
 b) $\left|a_{ii}\right| \le \sum_{\substack{j=1 \ i \ne j}}^{n} \left|a_{ij}\right|$

b)
$$\left| a_{ii} \right| \leq \sum_{\substack{j=1 \ i \neq i}}^{n} \left| a_{ij} \right|$$

c)
$$\left|a_{ii}\right| > \sum_{\substack{j=1 \ i \neq i}}^{n} \left|a_{ij}\right|$$

c)
$$\left|a_{ii}\right| > \sum_{\substack{j=1 \ i \neq j}}^{n} \left|a_{ij}\right|$$
 d) $\left|a_{ii}\right| < \sum_{\substack{j=1 \ i \neq j}}^{n} \left|a_{ij}\right|$.



a)
$$|f(x).f'(x)| < \{f''(x)\}^2$$

b)
$$|f(x).f''(x)| < \{f'(x)\}^2$$

c)
$$|f(x).f'(x)| > \{f''(x)\}^2$$

d)
$$|f(x).f''(x)| > \{f'(x)\}^2$$
.

xii) For $\frac{dy}{dx} = xy$ and y(0) = 2, the value of k_2 according to

Runge-Kutta method of 2nd order is (h = 0.2)

a) 0.1 b) 0.01

c) 0.4 d) 0.04.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following.

 $3 \times 5 = 15$

- Given $u_0 + u_6 = 3$, $u_1 + u_5 = 5$, $u_2 + u_4 = 7$. Find u_3 , where v_x 2. is a function of x.
- Using the following table find $\frac{dy}{dx}$ at x = 0 & 1.5.

x:

0

1

3

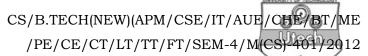
34

y: 1

2

11

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4. Solve the following system of equations using Gaussian elimination method:

$$x + y + z = 9$$

 $2x - 3y + 4z = 13$
 $3x + 4y + 5z = 40$

- 5. Find the value of $(19)^{\frac{1}{3}}$ correct to four decimal points by Newton-Raphson method.
- 6. Find the cubic polynomial by Lagrange's interpolation formula which takes the following value :

$$x : 0 4 5 8$$

 $f(x): 1 2 1 10$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

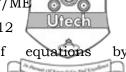
- 7. a) Find a root of the equation $x^4 x 10 = 0$ that lies between 1 & 2 using Newton-Raphson method correct to 3 places of decimal.
 - b) Solve the system of equations

$$x + y + 54z = 110$$

 $27x + 6y - z = 85$
 $6x + 15y + 2z = 72$

by Gauss-Seidel method.

7 + 8



8. a) Solve the following system of

LU-factorization method:

$$3x + 2y + 7z = 4$$

$$2x + 3y + z = 5$$

$$3x + 4y + z = 7$$

b) Using Runge-Kutta method of order 4, final y (0.2) given that $\frac{dy}{dx} = 3e^x + 2y$, y (0) = 0,taking h = 0.1.

7 + 8

- 9. a) Find the root of the equation $3x \cos x 1 = 0$ by Regula-falsi method, correct to three decimal places.
 - b) Evaluate $\int_{0}^{\frac{\pi}{2}} \sqrt{\cos x} \, dx$ by using (i) Trapezoidal and (ii) Simpson's $\frac{1}{3}$ rd rule, where $h = 15^{\circ}$. 7 + 8
- 10. a) Compute $y = (1 \cdot 4)$ by Milne's predictor & corrector's method from $\frac{dy}{dx} = \frac{1}{2}(x+y)$ where $y(1) = 3 \cdot 595$, $y(1 \cdot 1) = 3 \cdot 833$, $y(1 \cdot 2) = 4 \cdot 088$, $y(1 \cdot 3) = 4 \cdot 362$.
 - b) Derive Newton's divided difference formula.

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- c) Given that $\frac{dy}{dx} = \log_{10}(x+y)$ with the initial condition that y = 1 when x = 0. Find y for x = 0.2 and x = 0.5 using Euler's modified formula. 5 + 5 + 5
- 11. a) If y = f(x) is a polynomial degree 5 with $y_0 = f(0) = 0$, $y_1 = f(1) = 3$, $y_2 = f(2) = 14$, $y_3 = f(3) = 45$, $y_4 = f(4) = 84$, $y_5 = f(5) = 170$, $y_6 = f(6) = 258$. It is found that there is one error in the value of y_3 . Find the correct value of y_3 .
 - b) Why implicit method is preferred over explicit method though it requires more computations?
 - c) Show that the rate of convergence in Newton-Raphson method is quadratic. 8 + 3 + 4

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