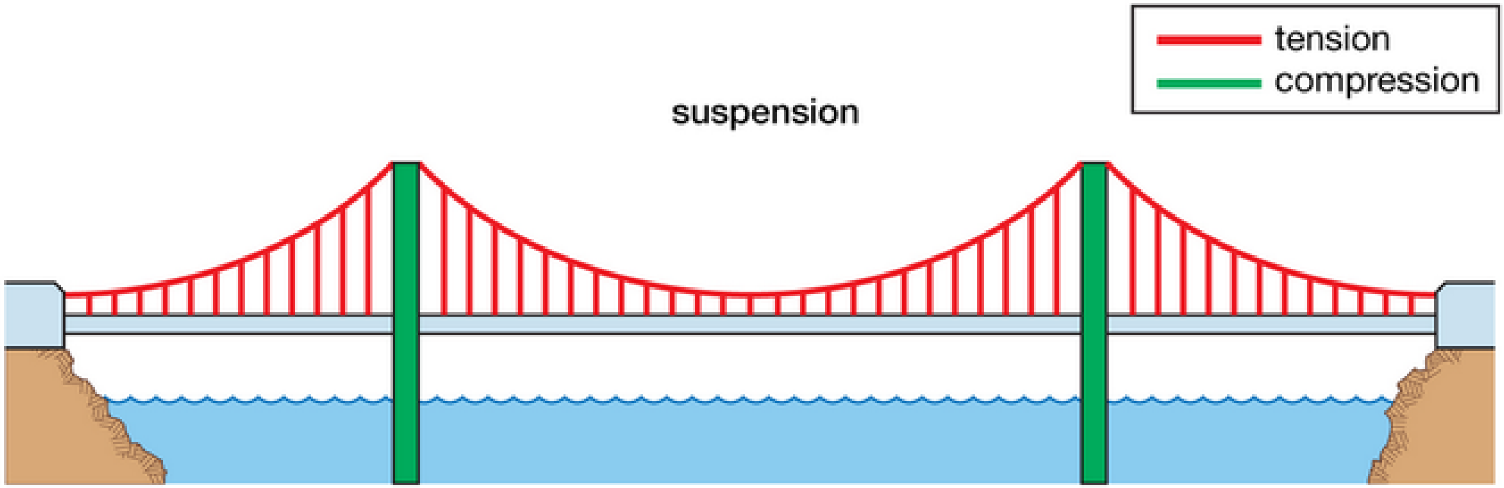
SOLID MECHANICS

CHAPTER 4: SHEAR FORCE AND BENDING MOMENT DIAGRAMS

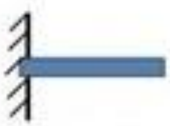


- https://www.youtube.com/watch?v=PRYtw9EQhug
- (Elastic behavior, Beam, column good video)

Beams: beams are the structural member subjected to the transverse load. Generally it is placed in the horizontal direction.

Columns : Columns are also the structural members subjected to the compressive load and are generally placed in the vertical direction.

Types of supports & reactions in the beams

S.no	Types of Support	Representation by	Reaction Force	Resisting Load
1.	**Roller Support**		Vertical	Vertical loads
2.	**Pinned Support**		Horizontal and vertical	Vertical and horizontal loads
3.	**Fixed Support**		Horizontal, vertical and moments	All types of loads Horizontal, vertical and Moments
4.	**Simple Support**		Vertical	Vertical loads

Types of supports

- Roller support:
  - free to rotate and translate along the surface upon which the **roller** rests

ROLLER SUPPORT

LOCATION OF
ROLLER BEARING
TO SUPPORT JET
ENGINE ROTOR

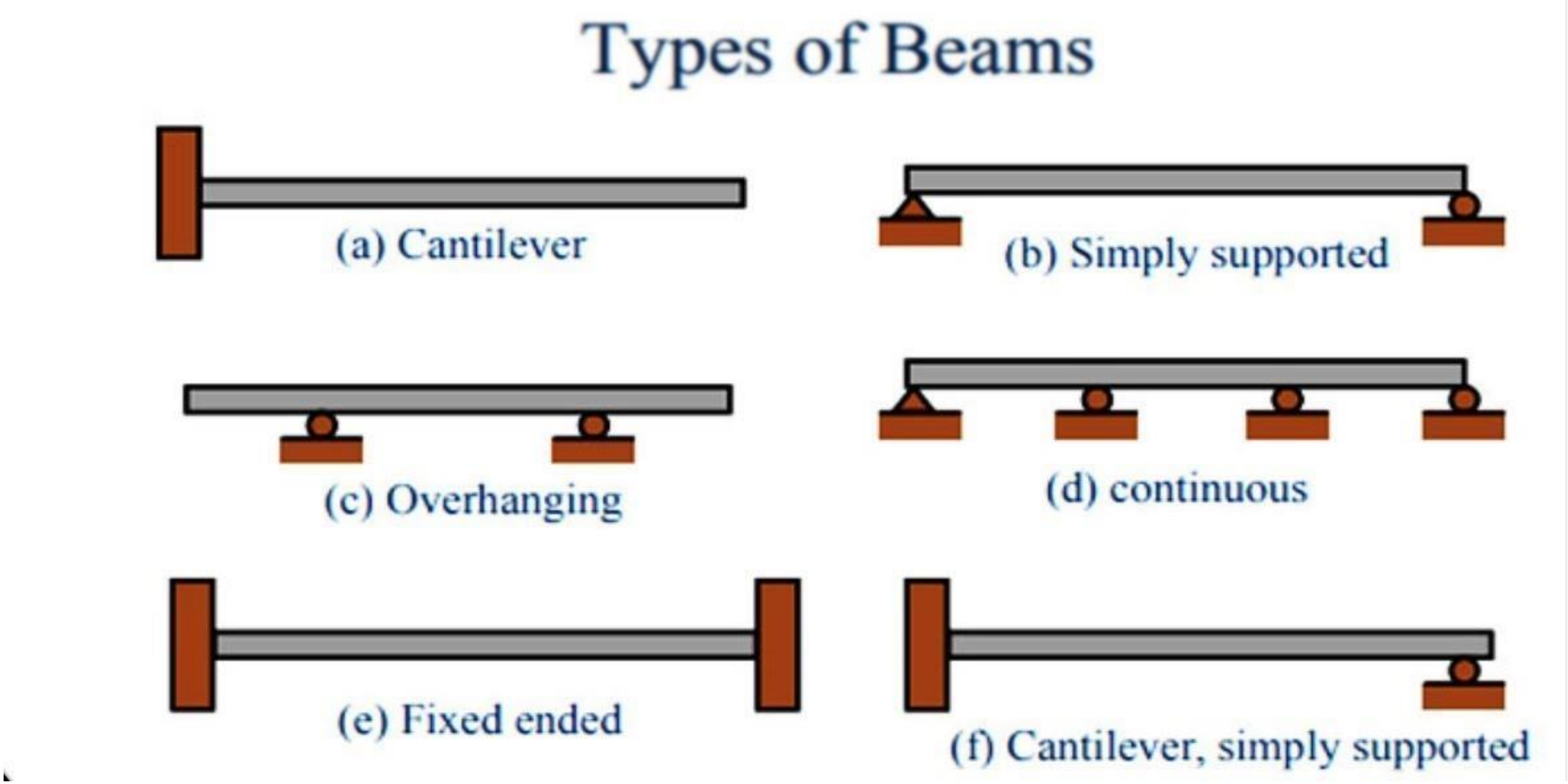
Hinge/Pinned support

- No translational displacement of beam is possible

Fixed Support

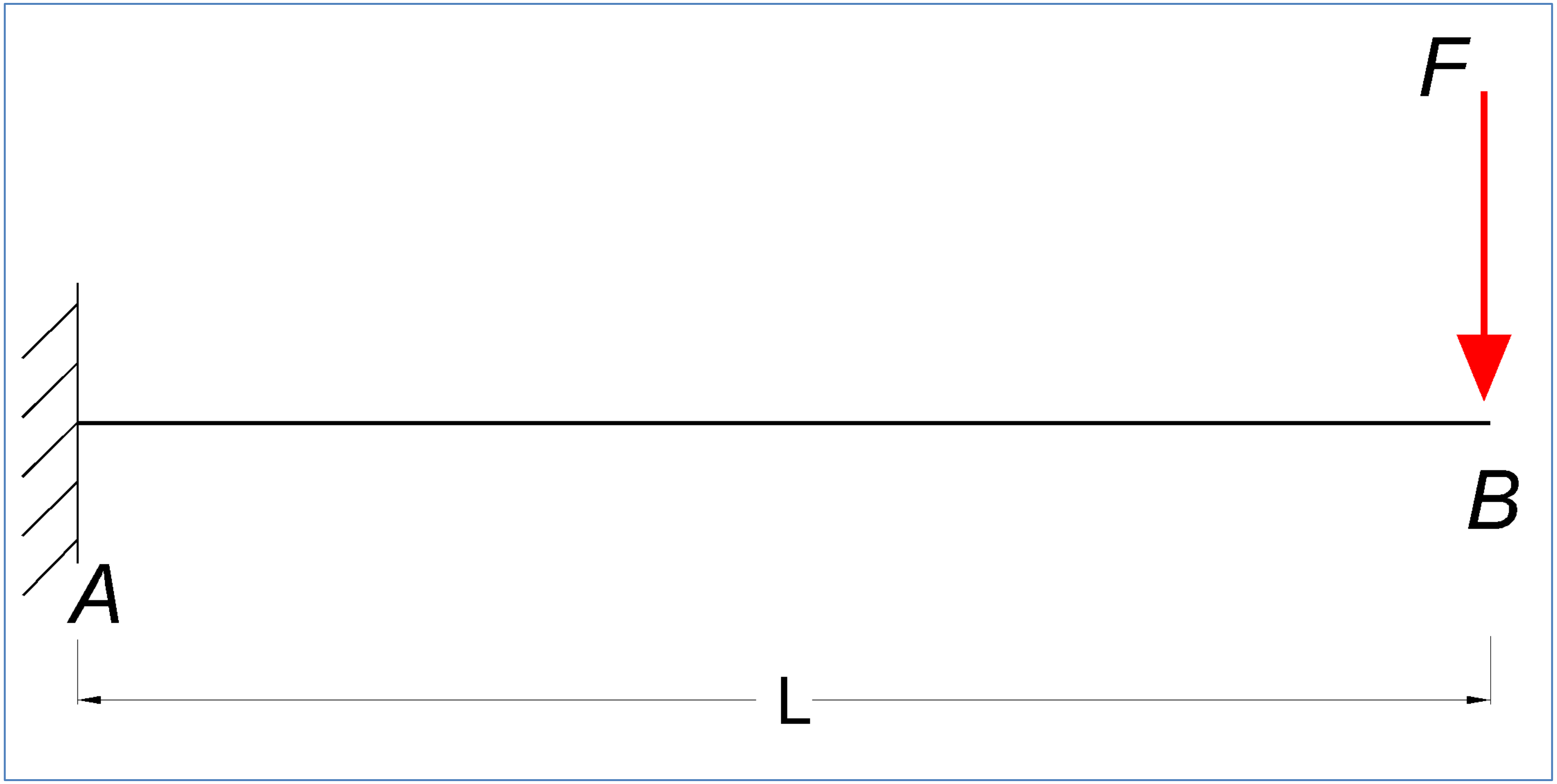
- Rigid support to the beam
- No movement is possible
  - Beams supporting the roof
  - Riveted and Welded joints

TYPES OF BEAMS



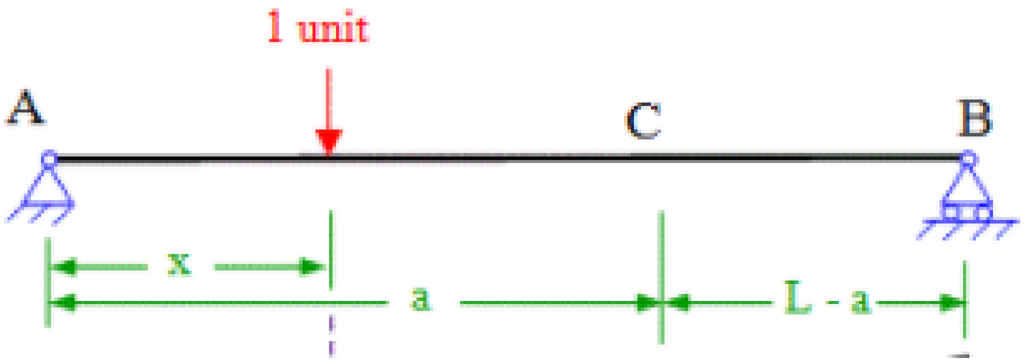
Cantilever beams: are the beams in which one end is fixed and other is free.

How to represents the cantilever beam !!



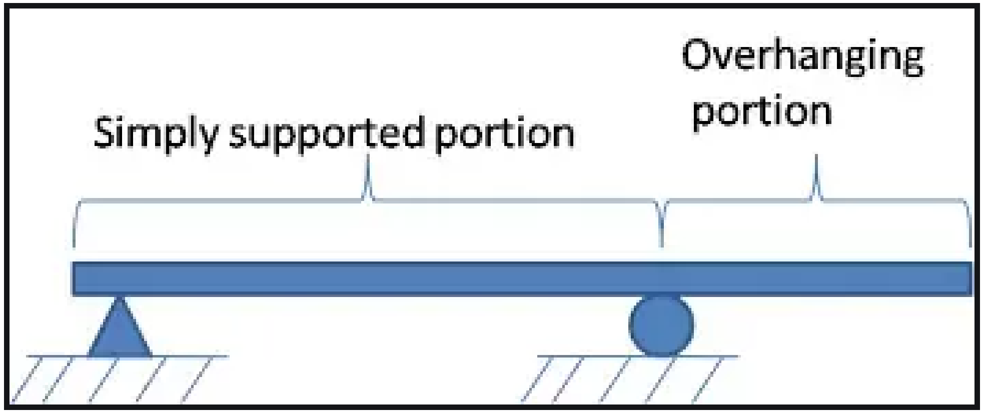
Simply Supported beam : A beam supported or resting freely on the supports at its **both the ends**.

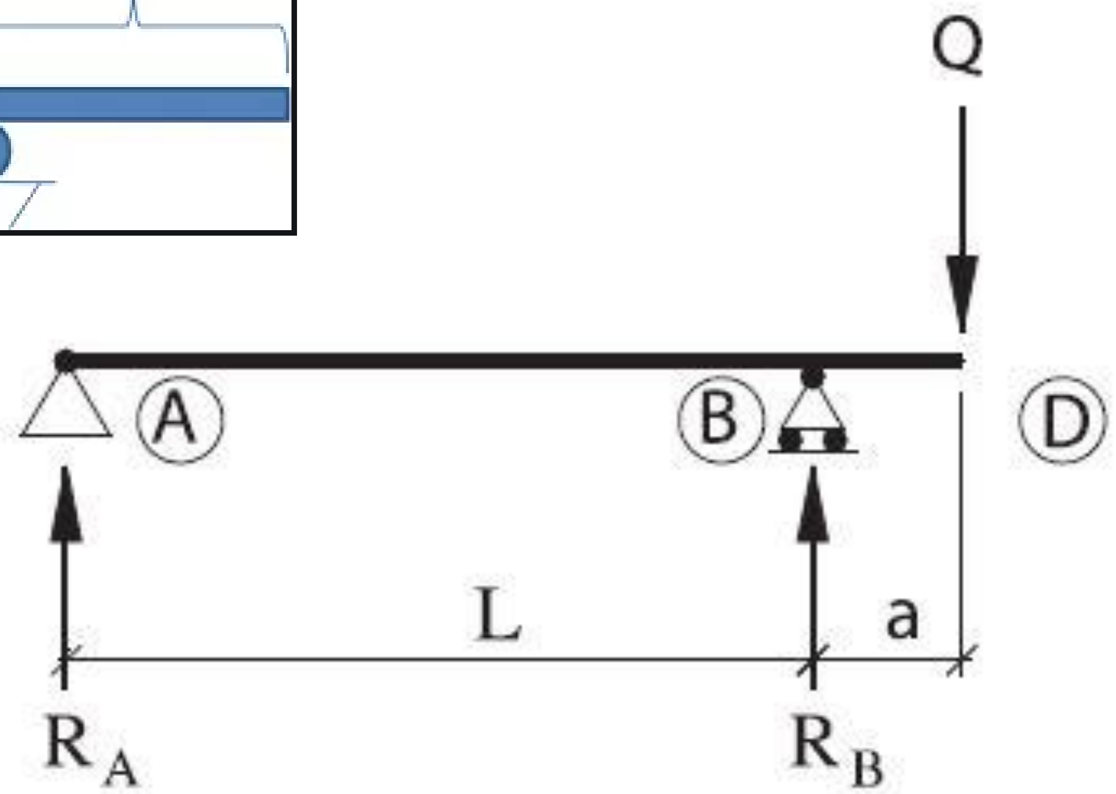
How to represents the Simply Supported beam !!



Overhanging Beam: If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam.

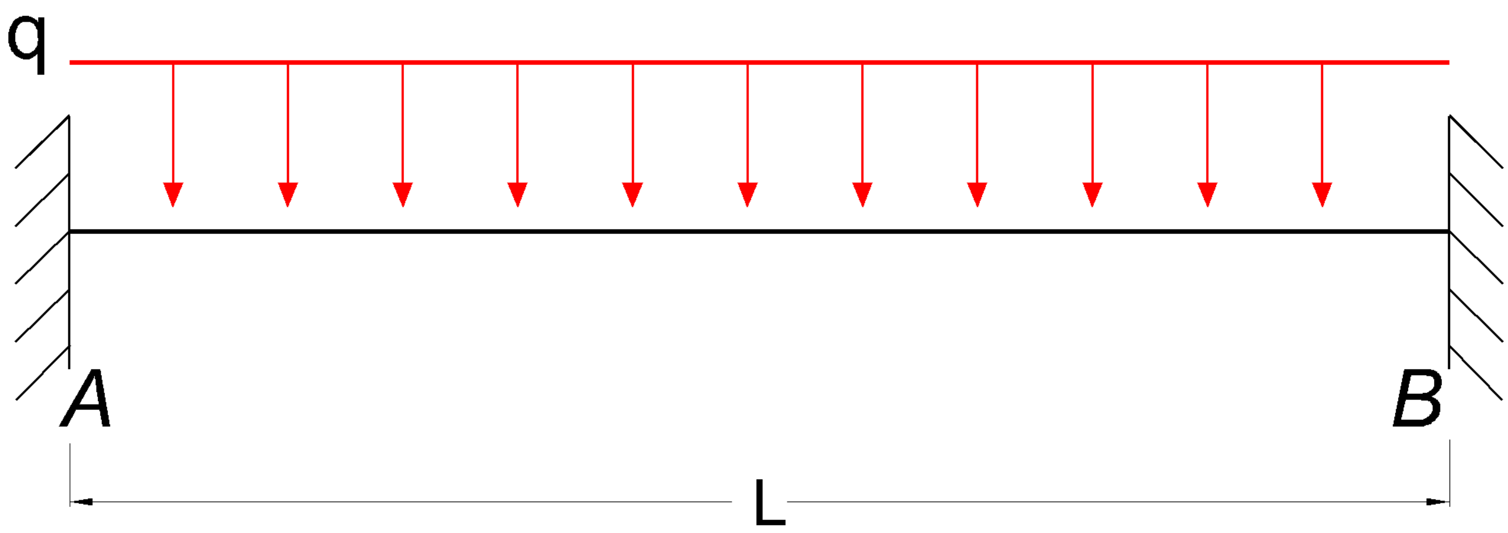
How to represents the Overhanging beam !!





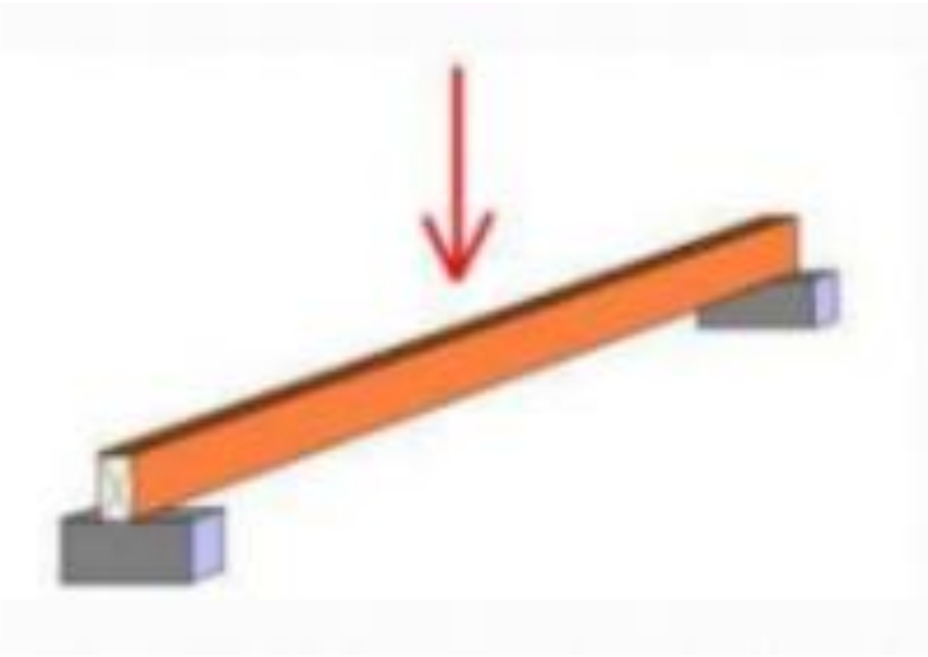
Fixed beam: A beam whose both the ends are fixed or built in walls.

How to represents the Fixed beam !!

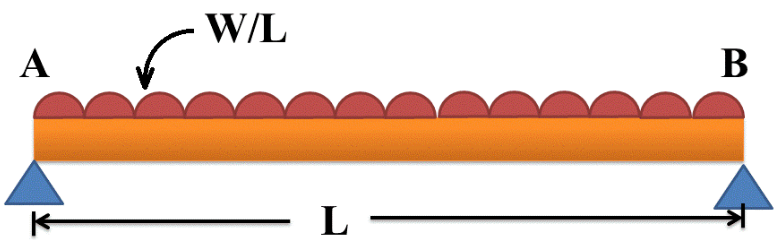


Types of loads

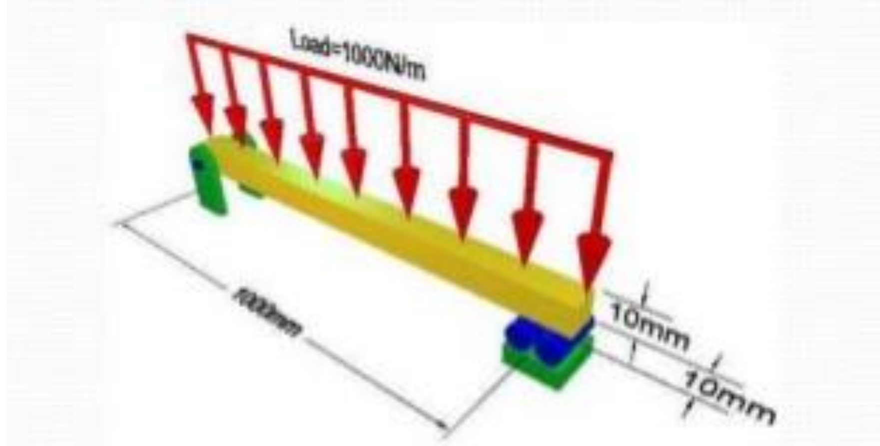
Point Load: When a load acts concentrated at a definite point then it is named as concentrated load or point load.

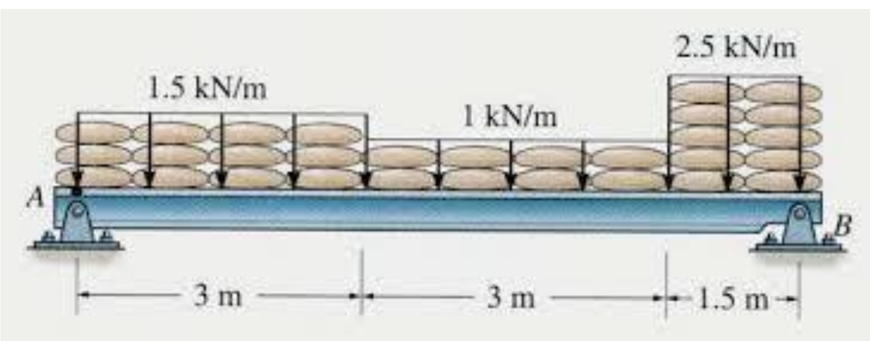


Uniformly Distributed Load (U.D.L) : A distributed load is load which is spread on some length of a beam. If the intensity is constant along the length then it is named as uniformly distributed load.

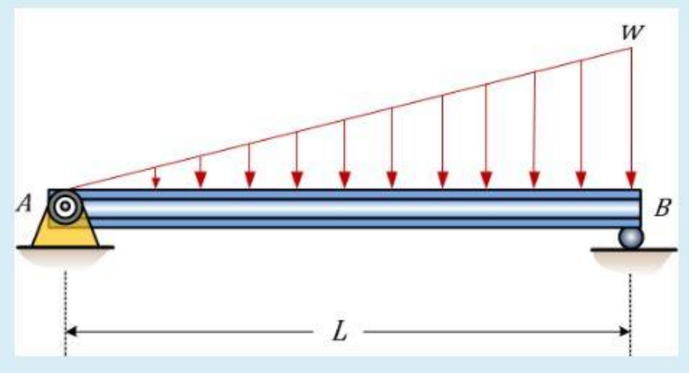


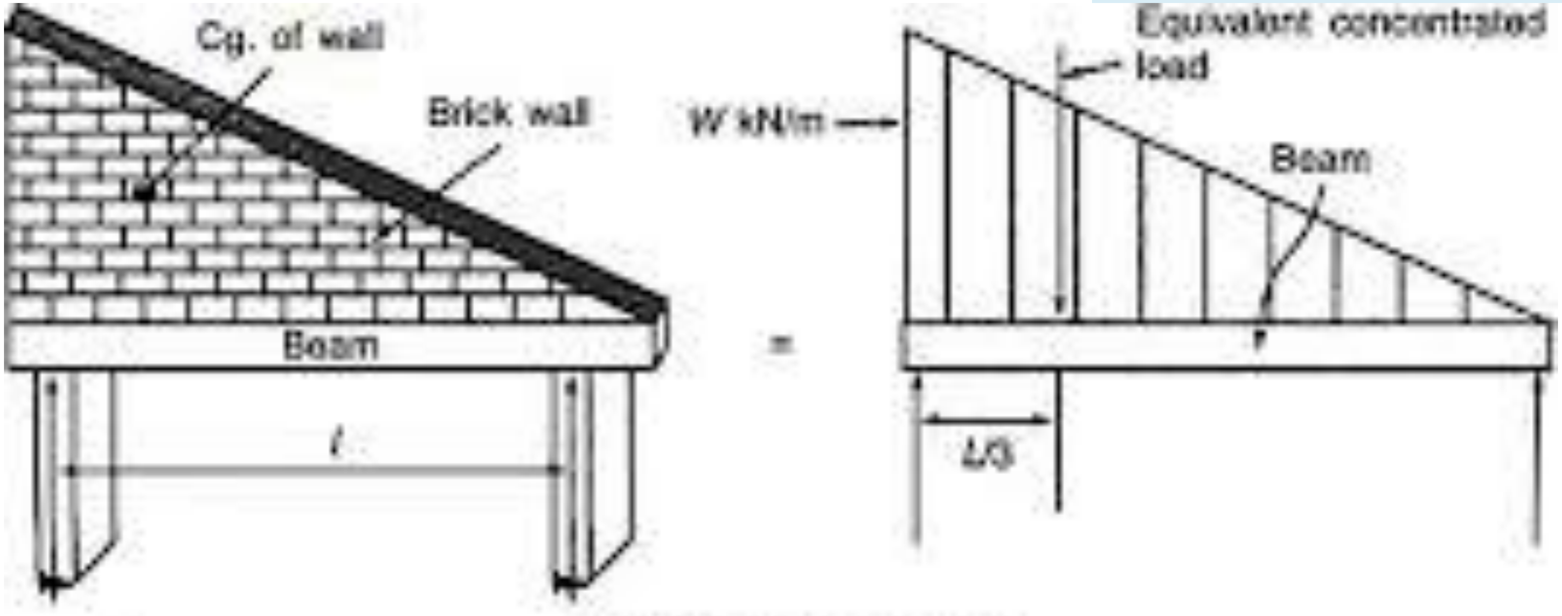
Uniformly Distributed Load(UDL)





Uniformly Varying load: Whenever the load distributed along the length of the beam varies in intensity uniformly.





(c) Uniformly varying load


OVER SIZE LOAD
TDOT


OVERSIZE LOAD
C213
WESTERN
WESTERN

TYPES OF LOADS IN THE BEAMS

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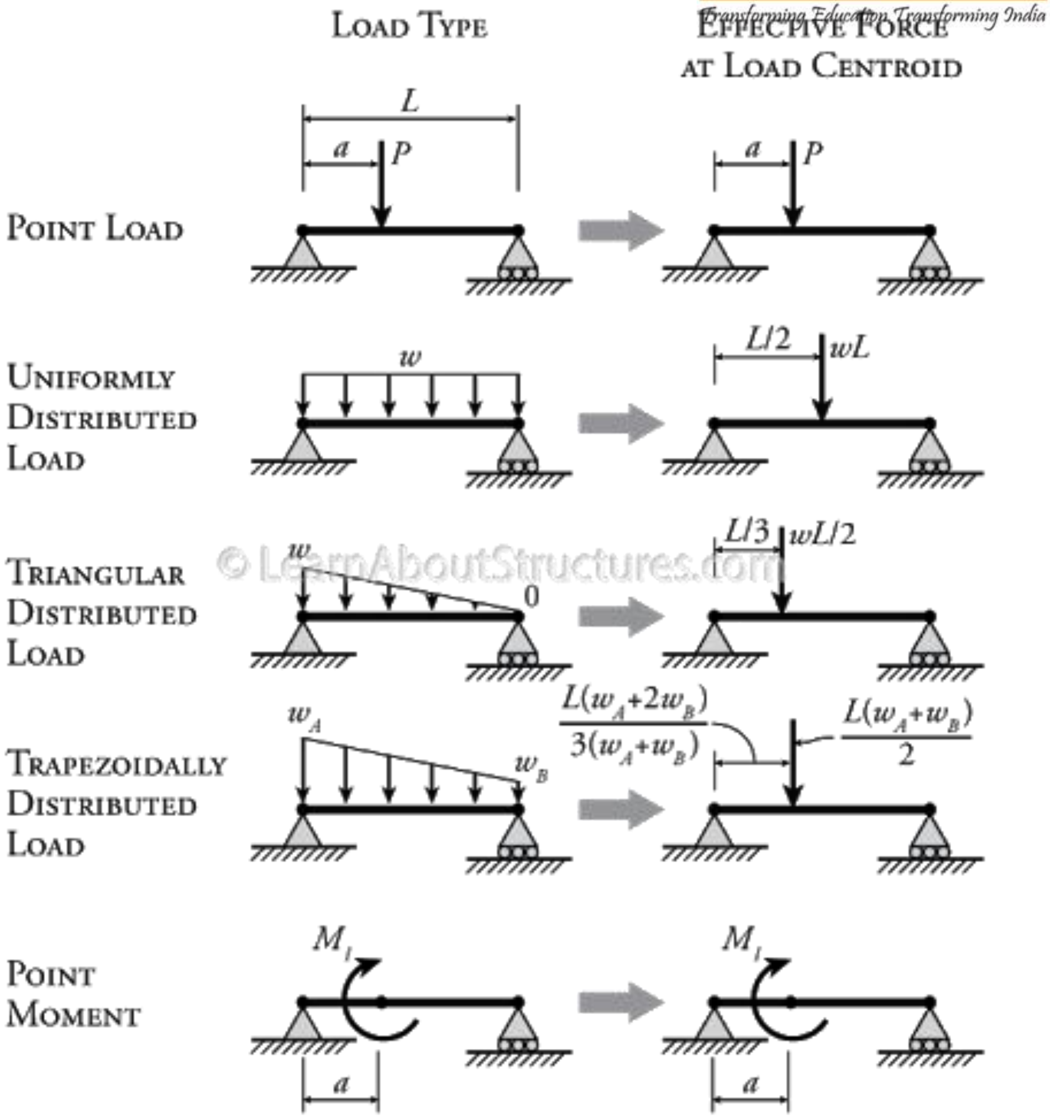
a) Point load

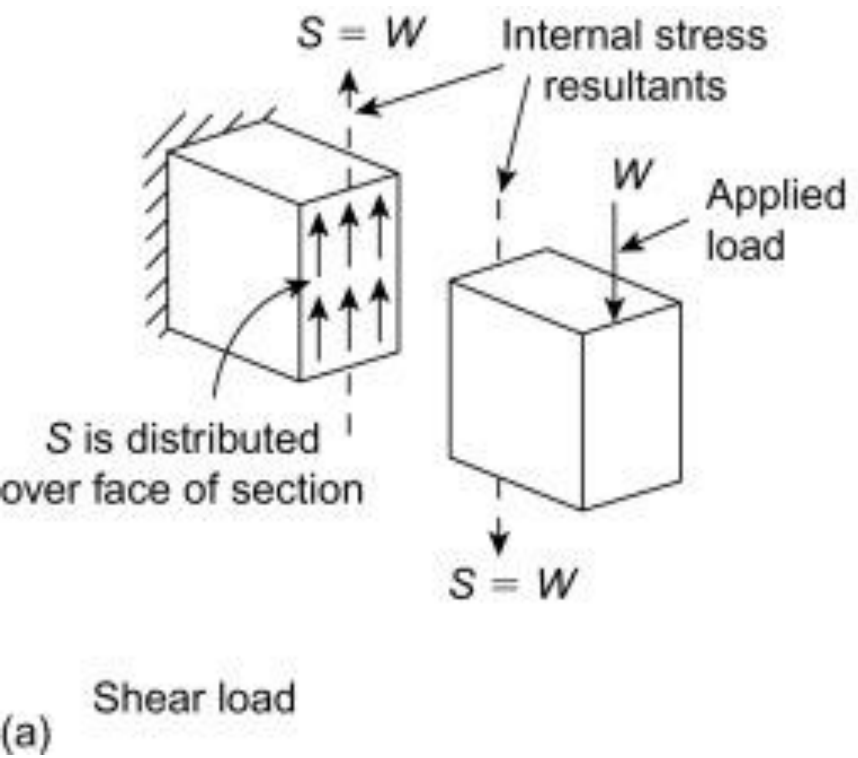
b) Uniformly Distributed Load

c) Gradually Varying Load

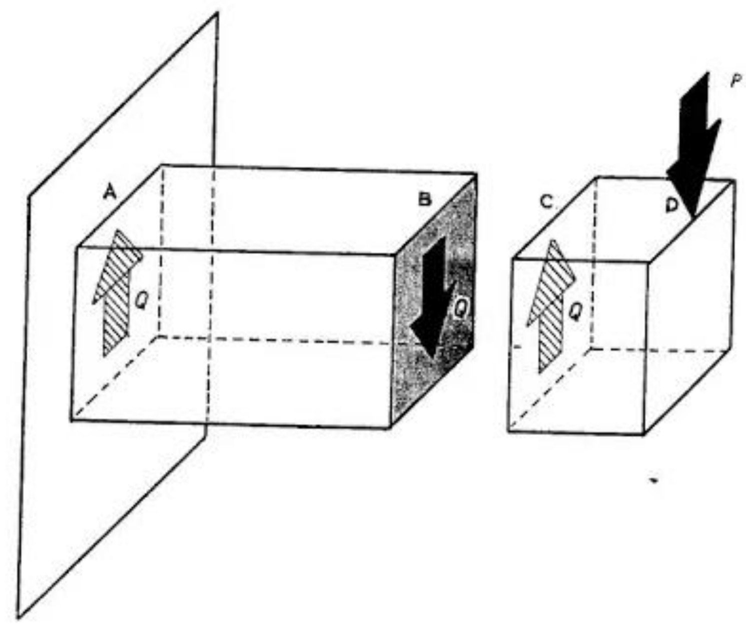
d) Point Moment (Couple)



SHEAR FORCE IN THE BEAM

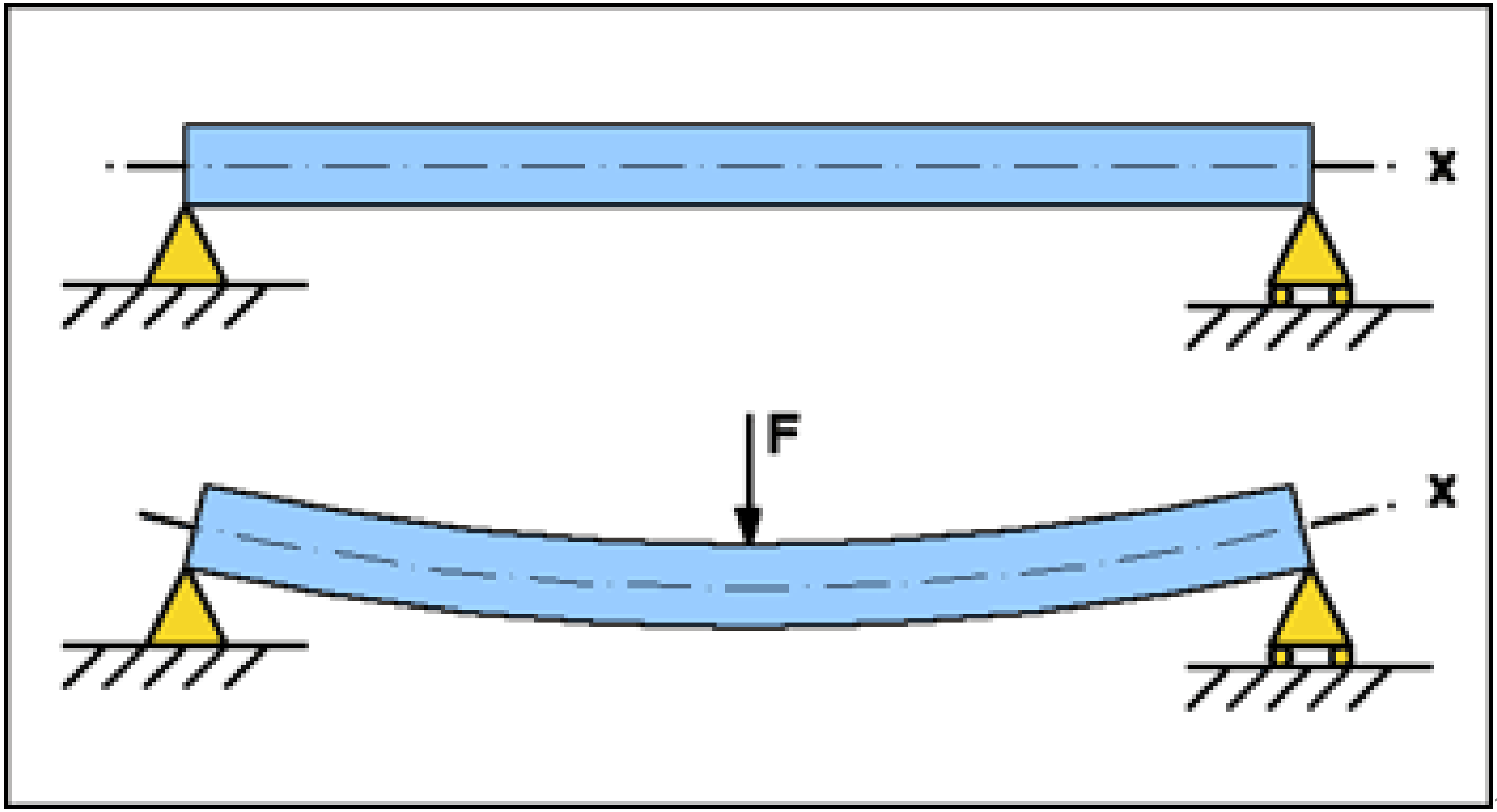


Shear force: The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force

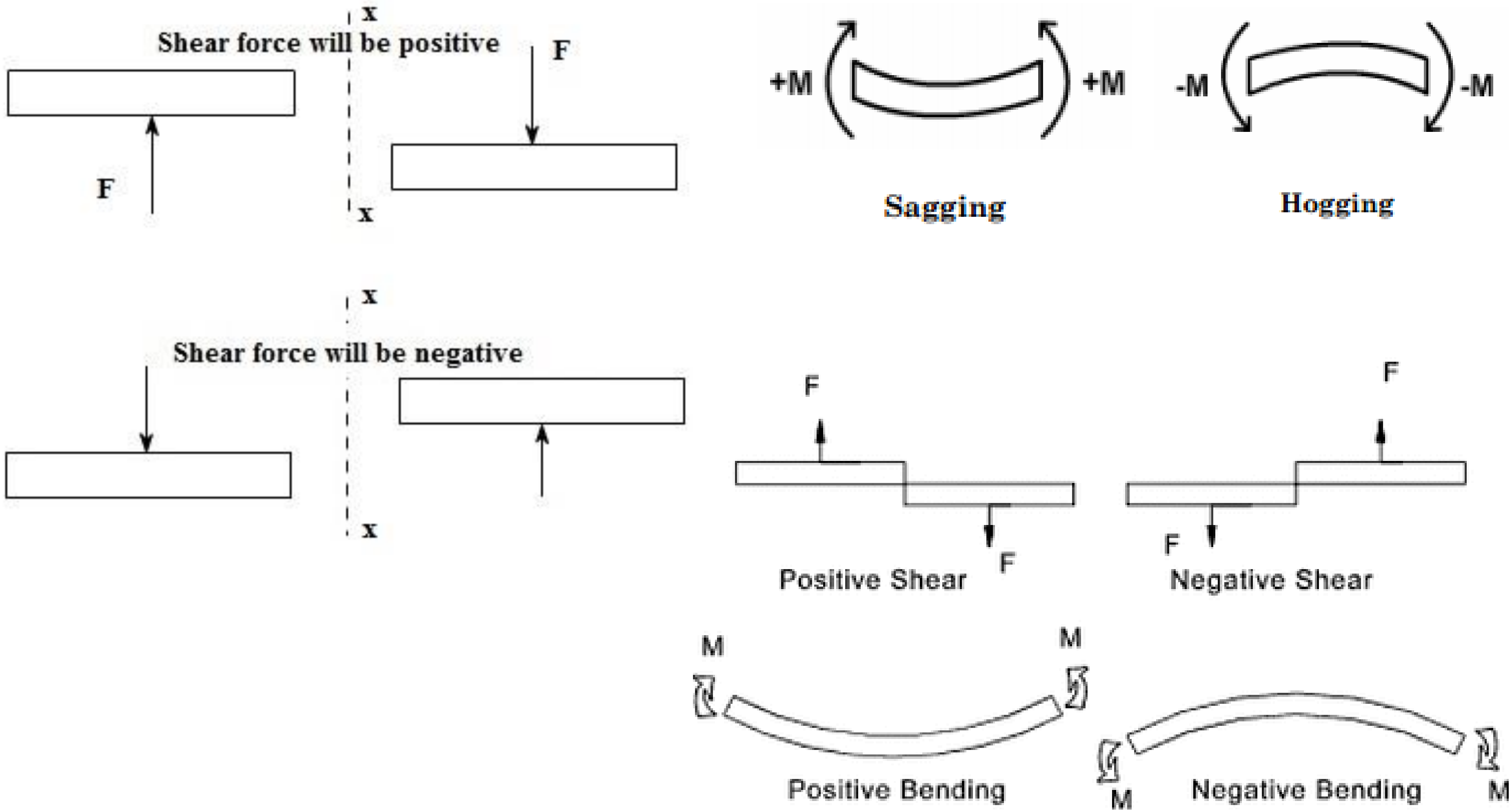


BENDING MOMENT IN THE BEAM

The algebraic sum of the moments of all the forces acting to the right or left of the section is known as bending moment.



SIGN CONVENTIONS IN THE BEAMS



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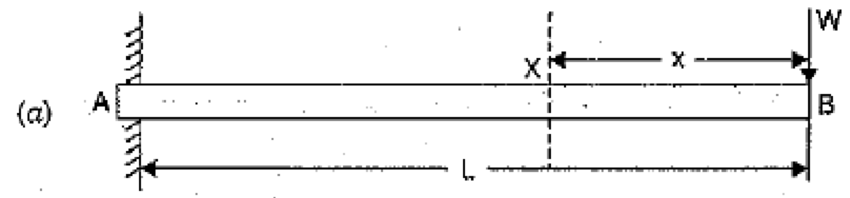
SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A CANTILEVER WITH A POINT LOAD AT THE FREE END



Let F_x = Shear force at X, and

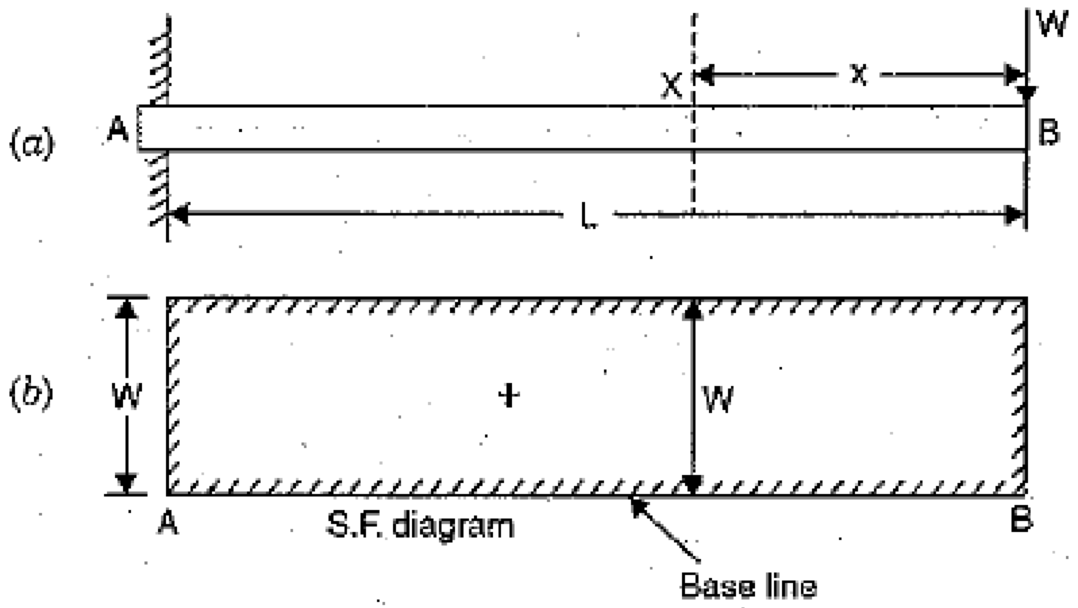
M_x = Bending moment at X.

Take a section X at a distance x from the free end. Consider the right portion of the section.

The shear force at this section is equal to the resultant force acting on the right portion at the given section. But the resultant force acting on the right portion at the section X is W and acting in the downward direction. But a force on the right portion acting downwards is considered positive. Hence shear force at X is positive.

$$\therefore \qquad F_x = +W$$

The shear force will be constant at all sections of the cantilever between A and B as there is no other load between A and B. The shear force diagram is shown in Fig.



Bending Moment Diagram

The bending moment at the section X is given by

$$M_x = -W \times x \quad \ldots(i)$$

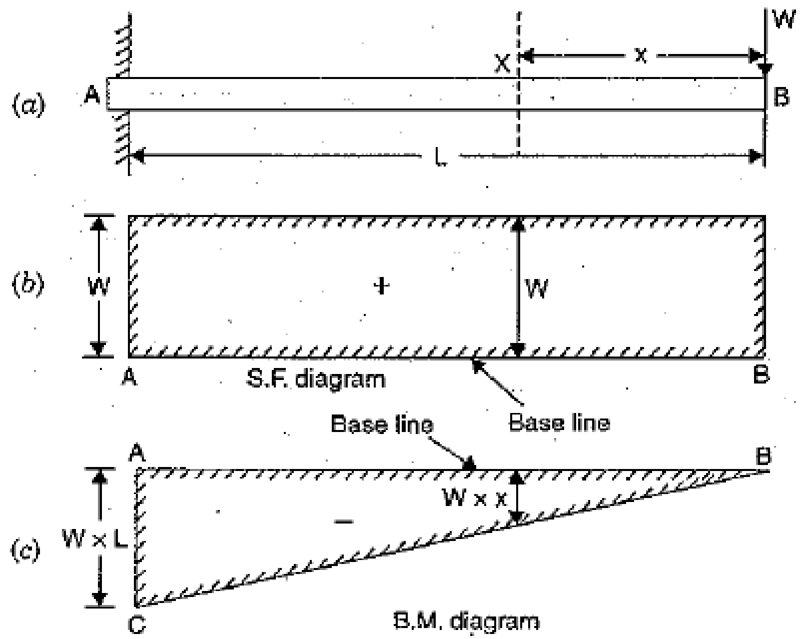
(Bending moment will be negative as for the right portion of the section, the moment of W at X is clockwise. Also the bending of cantilever will take place in such a manner that convexity will be at the top of the beam).

From equation (i), it is clear that B.M. at any section is proportional to the distance of the section from the free end.

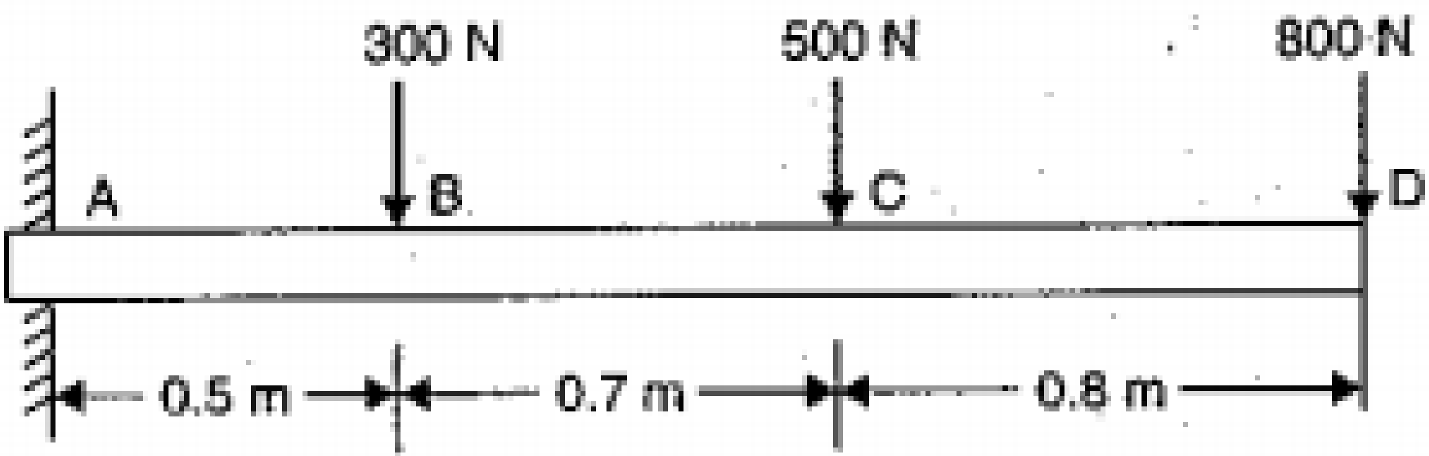
At $x = 0$ *i.e.*, at B, B.M. $= 0$

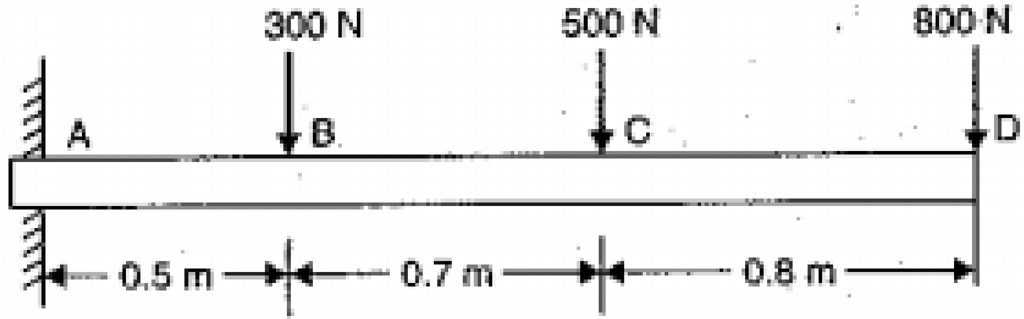
At $x = L$ *i.e.*, at A, B.M. $= W \times L$

Hence B.M. follows the straight line law. The B.M. diagram is shown in Fig. 6.14 (*c*). At point A, take $AC = W \times L$ in the downward direction. Join point B to C.



Q. A cantilever beam of 2 m carries the point loads as shown in figure. Draw the shear force and bending moment diagrams for the cantilever beam.





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Shear Force Diagram

The shear force at D is + 800 N. This shear force remains constant between D and C. At C, due to point load, the shear force becomes (800 + 500) = 1300 N. Between C and B, the shear force remains 1300 N. At B again, the shear force becomes (1300 + 300) = 1600 N. The shear force between B and A remains constant and equal to 1600 N. Hence the shear force at different points will be as given below :

S.F. at D, $F_D = +800$ N

S.F. at C, $F_C = +800 + 500 = +1300$ N

S.F. at B, $F_B = +800 + 500 + 300 = 1600$ N

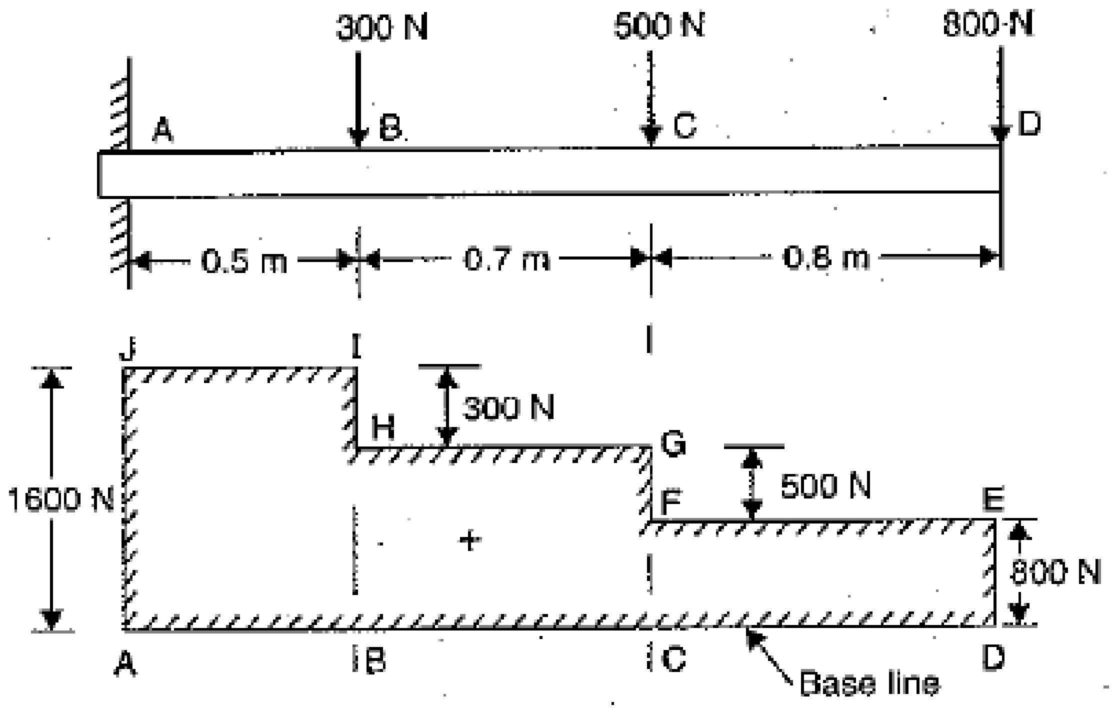
S.F. at A, $F_A = +1600$ N.

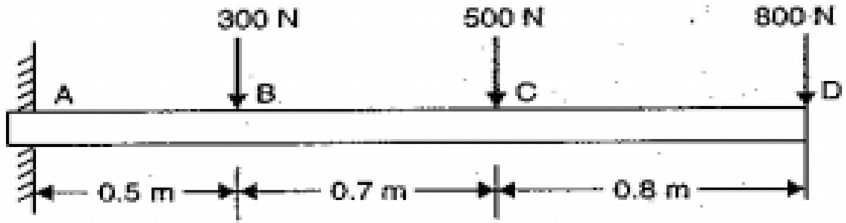
S.F. at D, $F_D = +800$ N

S.F. at C, $F_C = +800 + 500 = +1300$ N

S.F. at B, $F_B = +800 + 500 + 300 = 1600$ N

S.F. at A, $F_A = +1600$ N.





Bending Moment Diagram

The bending moment at D is zero :

(*i*) The bending moment at any section between C and D at a distance x and D is given by,

$$M_x = -800 \times x \text{ which follows a straight line law.}$$

At C, the value of $x = 0.8$ m.

$\therefore$ B.M. at C, $M_C = -800 \times 0.8 = -640$ Nm.

(*ii*) The B.M. at any section between B and C at a distance x from D is given by (At C, $x = 0.8$ and at B, $x = 0.8 + 0.7 = 1.5$ m. Hence here x varies from 0.8 to 1.5).

$$M_x = -800\,x - 500\,(x - 0.8) \quad \text{...(i)}$$

Bending moment between B and C also varies by a straight line law.

B.M. at B is obtained by substituting $x = 1.5$ m in equation (*i*),

$$\therefore \quad M_B = -800 \times 1.5 - 500\,(1.5 - 0.8)$$
$$= -1200 - 350 = -1550 \text{ Nm.}$$

(*iii*) The B.M. at any section between A and B at a distance x from D is given by (At B, $x = 1.5$ and at A, $x = 2.0$ m. Hence here x varies from 1.5 m to 2.0 m)

$$M_x = -800\,x - 500\,(x - 0.8) - 300\,(x - 1.5) \quad \text{...(ii)}$$

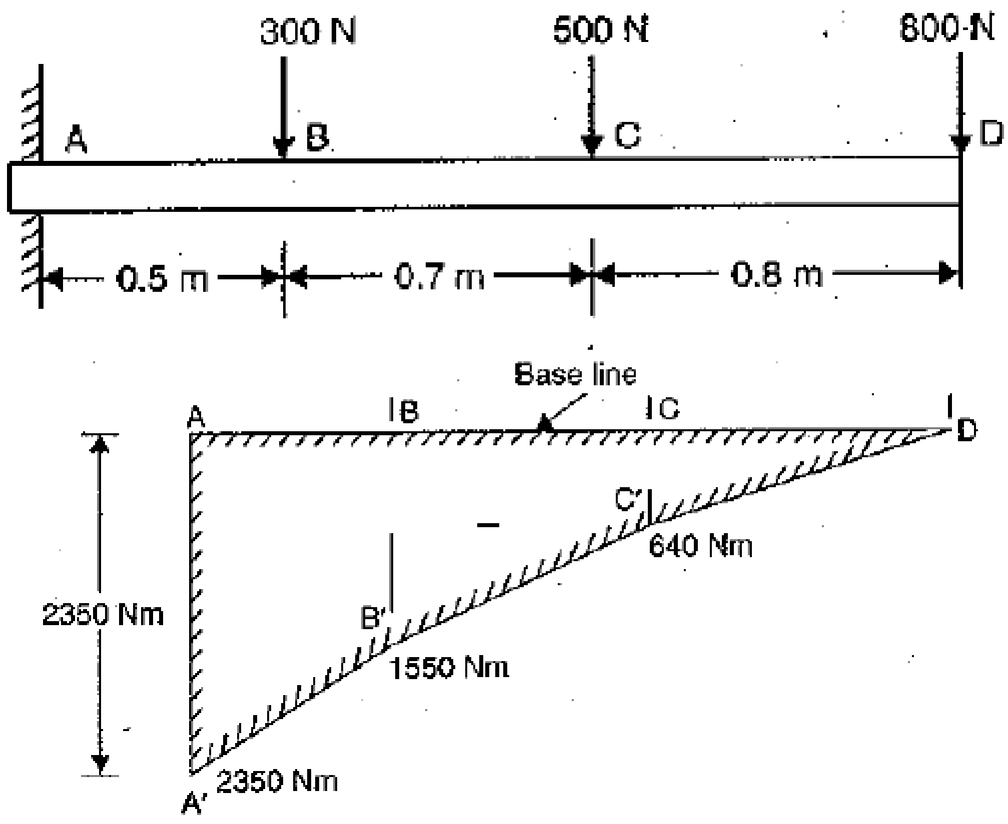
Bending moment between A and B varies by a straight line law.

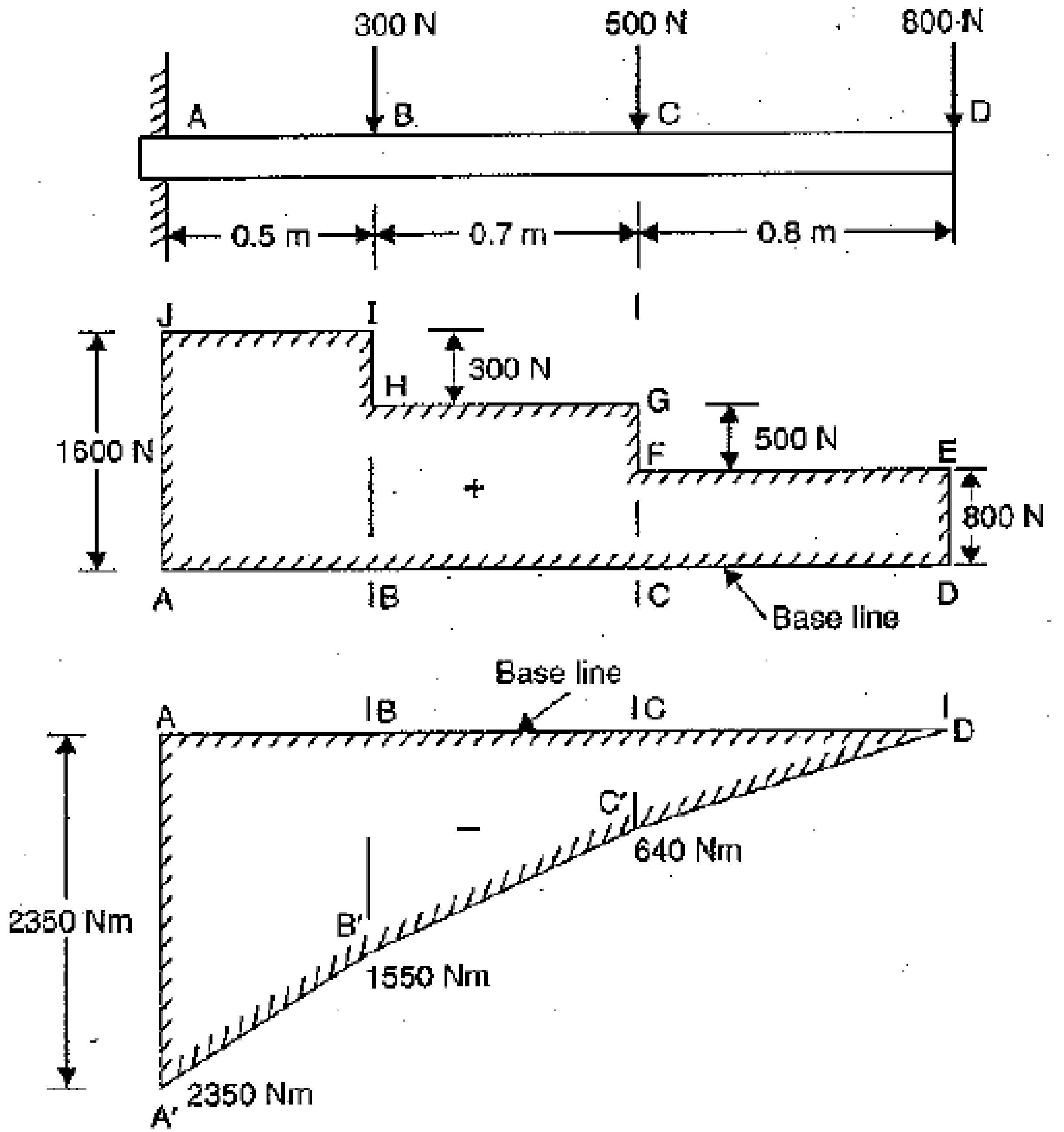
B.M. at A is obtained by substituting $x = 2.0$ m in equation (*ii*),

$$\therefore \quad M_A = -800 \times 2 - 500\,(2 - 0.8) - 300\,(2 - 1.5)$$
$$= -800 \times 2 - 500 \times 1.2 - 300 \times 0.5$$
$$= -1600 - 600 - 150 = -2350 \text{ Nm.}$$

Hence the bending moments at different points will be as given below :

$$M_D = 0$$
$$M_C = -\,640\ \text{Nm}$$
$$M_B = -\,1550\ \text{Nm}$$
$$M_A = -\,2350\ \text{Nm.}$$




300 N
500 N
800 N
A
B
C
D
0.5 m
0.7 m
0.8 m
J
I
H
300 N
G
500 N
F
E
1600 N
+
800 N
A
B
C
D
Base line
Base line
A
B
C
D
C'
640 Nm
–
2350 Nm
B'
1550 Nm
2350 Nm
A'

Q. A cantilever beam of length 10m carries point loads of 4 KN & 6 KN at 2m and 6m respectively from the fixed end along with another load of 6 KN at the free end. Draw the shear force and bending moment diagram for cantilever.

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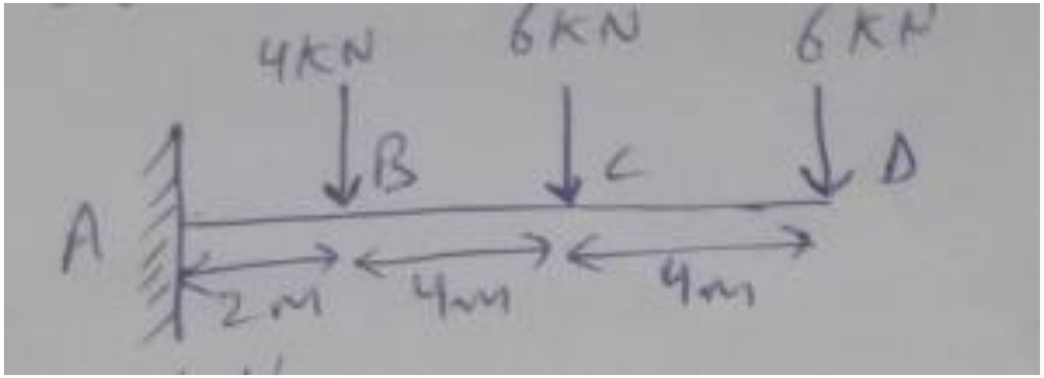


Sol: S.F. diag.

Portion CD,

Consider a sec^n at a dist. x from free end.

Force to right $F_x = 6\,kN$

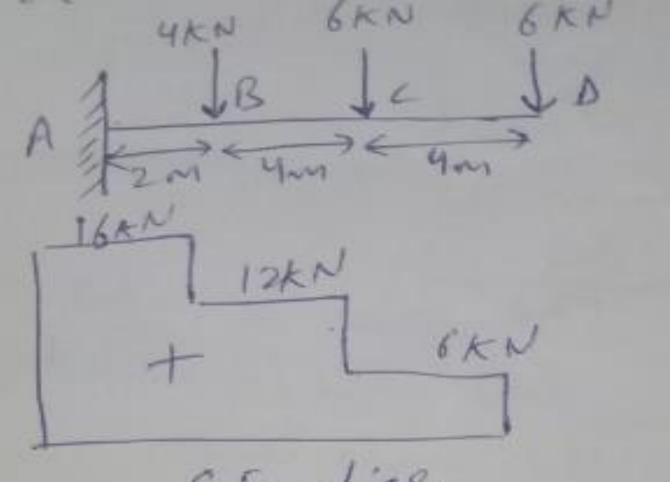
It is const. b/w C & D

||ly for Portion BC, $F_x = 6+6 = 12\,kN$

& is const.

||ly For AB, $F_x = 12+4 = 16\,kN$

(Cont.)



S.F. diag.

B.M. diag.

Portion CD:

At any section x from right,

$M = 6x$ (hogging = -ve)

(linear)

At D, $x = 0$, $M = 0$

C, $x = 4$, $M = 24$ kN-m

Portion BC:

Taking moment,

$M = 6x + 6(x-4)$

At C, $x = 4$ m

$M = 6\times4 + 6(4-4)$

$= 24$ kN-m

At B, $x = 8$ m

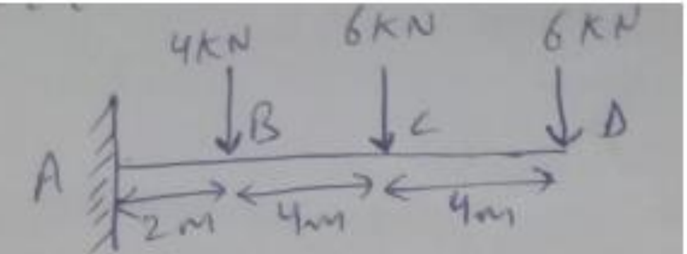
$M = 6\times8 + 6(8-4) = 48 + 24 = 72$ kN-m

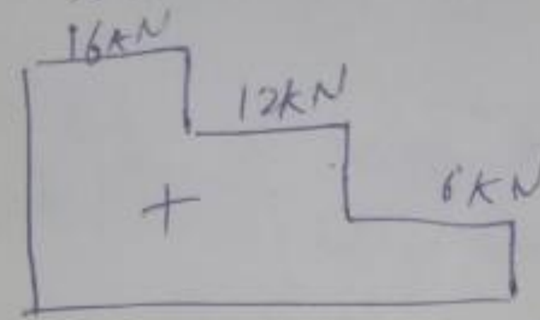
Portion AB,

$M = 6x + 6(x-4) + 4(x-8)$

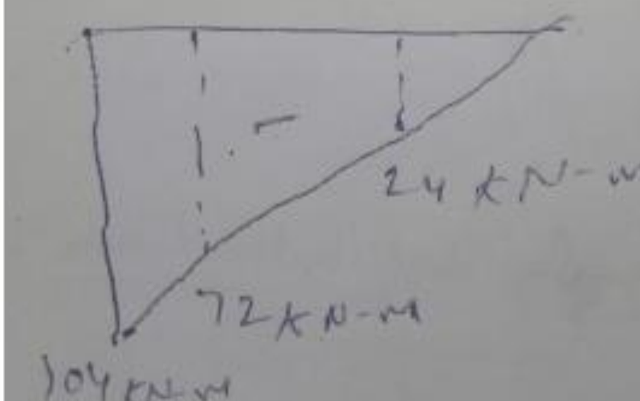
At B, $x = 8$, $M = 72$

A, $x = 10$, $M = 104$ kN-m





S.F. diag.

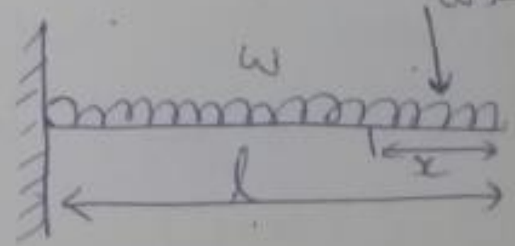


B.M. diag.

- A cantilever of span L is to withstand a downward acting load W at the free end and an upward acting load W at a distance ‘a’ from the free end. Draw SFD and BMD.

Q. A cantilever beam of length 2 m carries a point load of 1 KN at its free end and another load of 2 KN at a distance of 1m from the free end. Draw the shear force and bending moment diagram.

Uniformly distributed loads :-

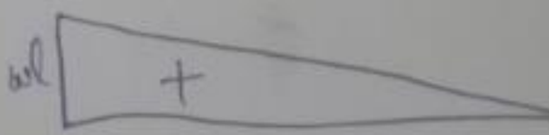


Consider a section at a dist. x from the free end.

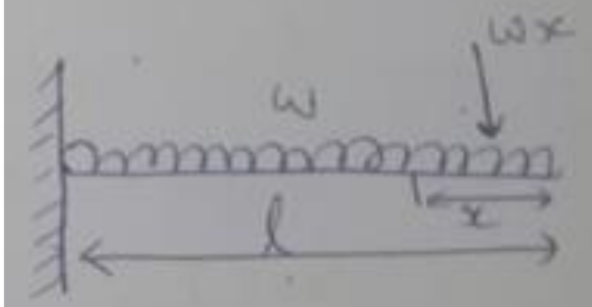
UDL is converted to pt. load by multiplying by dist. & acting at center.

S.F. dig.

Force to the right of section is wx, downwards & varies linearly along the whole length

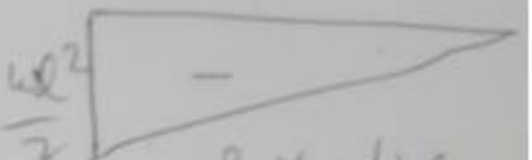


S.F. diagram



B.M. diag.

Force wx to the right of section is a concentrated load at a dist. x/2 from free end.



B.M. diag.

Taking moment about the section,

$$M = W.x = wx.\frac{x}{2} = \frac{wx^2}{2}$$

clockwise moment ∴ −ve

$$Max.\ B.M. = \frac{wl^2}{2} \text{ at fixed end}$$

- A cantilever of length 2m carries a UDL of 1kN/m run over a length of 1.5m from the free end. Draw SFD and BMD

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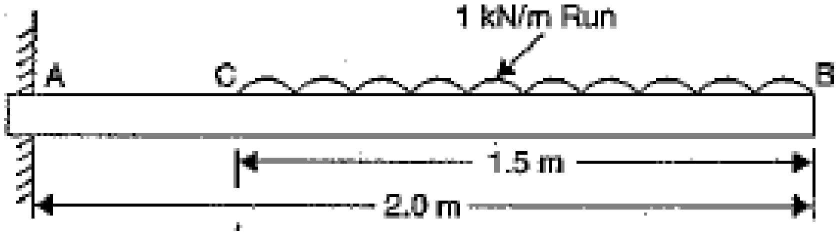


Shear Force Diagram

Consider any section between C and B a distance of x from the free end B. The shear force at the section is given by

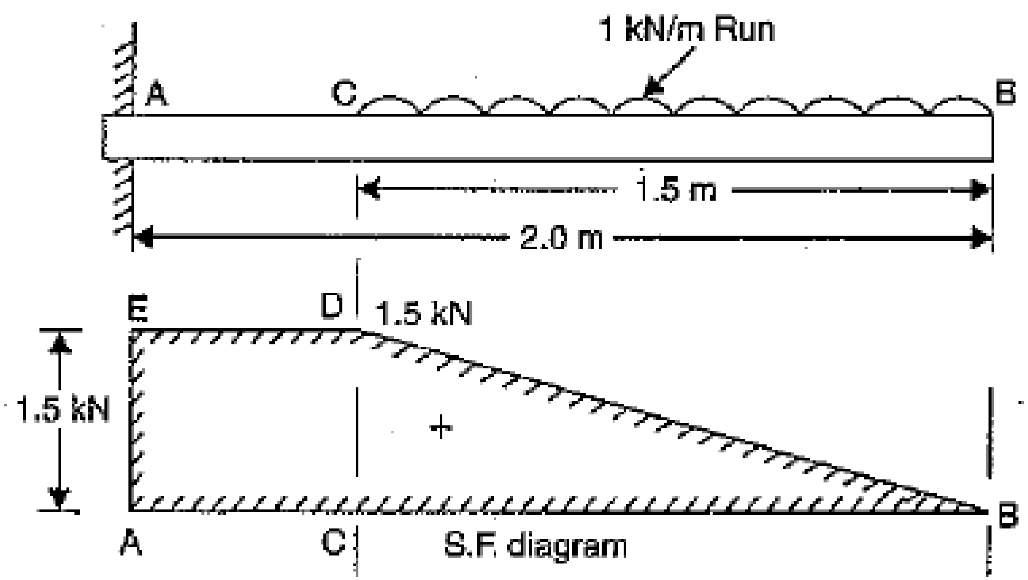
$$F_x = w.x \quad \text{(+ve sign is due to downward force on right portion of the section)}$$

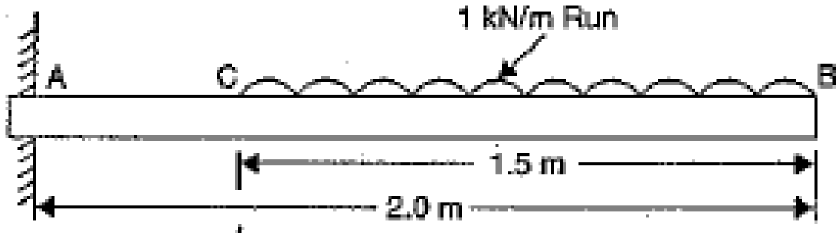
$$= 1.0 \times x \qquad (\because \quad w = 1.0 \text{ kN/m run})$$

At B, $x = 0$ hence $F_x = 0$

At C, $x = 1.5$ hence $F_x = 1.0 \times 1.5 = 1.5$ kN.

The shear force follows a straight line law between C and B. As between A and C there is no load, the shear force will remain constant. Hence shear force between A and C will be represented by a horizontal line.





Bending Moment Diagram

(*i*) The bending moment at any section between C and B at a distance x from the free end B is given by

$$M_x = -(w.x.).\frac{x}{2} = -\left(1.\frac{x^2}{2}\right) = -\frac{x^2}{2} \quad ...(i)$$

(The bending moment will be negative as for the right portion of the section the moment of load at x is clockwise).

At B, $x = 0$ hence $M_B = -\frac{0^2}{2} = 0$

At C, $x = 1.5$ hence $M_C = -\frac{1.5^2}{2} = -1.125$ Nm

From equation (*i*) it is clear that the bending moment varies according to parabolic law between C and B.

(*ii*) The bending moment at any section between A and C at a distance x from the free end B is obtained as : (here x varies from 1.5 m to 2.0 m)

Total load due to U.D.L. $= w \times 1.5 = 1.5$ kN.

This load is acting at a distance of $(x - 0.75)$ from any section between A and C.

∴ Moment of this load at any section between A and C at a distance x from free end

$$= (\text{Load due to U.D.L.}) \times (x - 0.75)$$

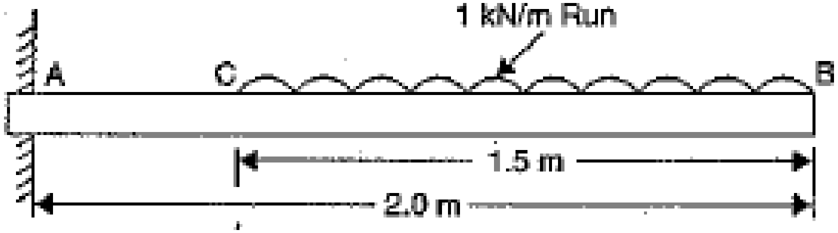
∴ $$M_x = -1.5 \times (x - 0.75) \quad \ldots(ii)$$

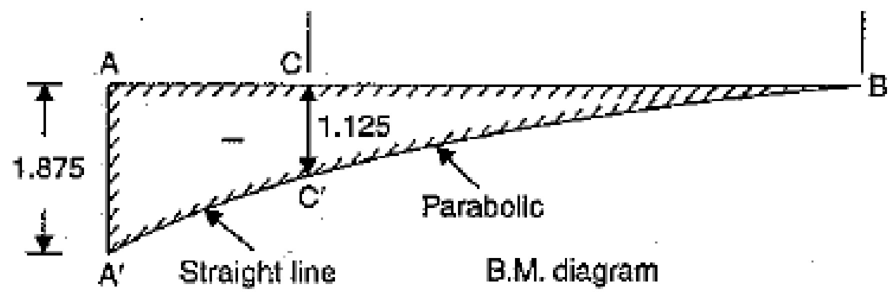
(– ve sign is due to clockwise moment for right portion)

From equation (*ii*) it is clear that the bending moment follows straight line law between A and C.

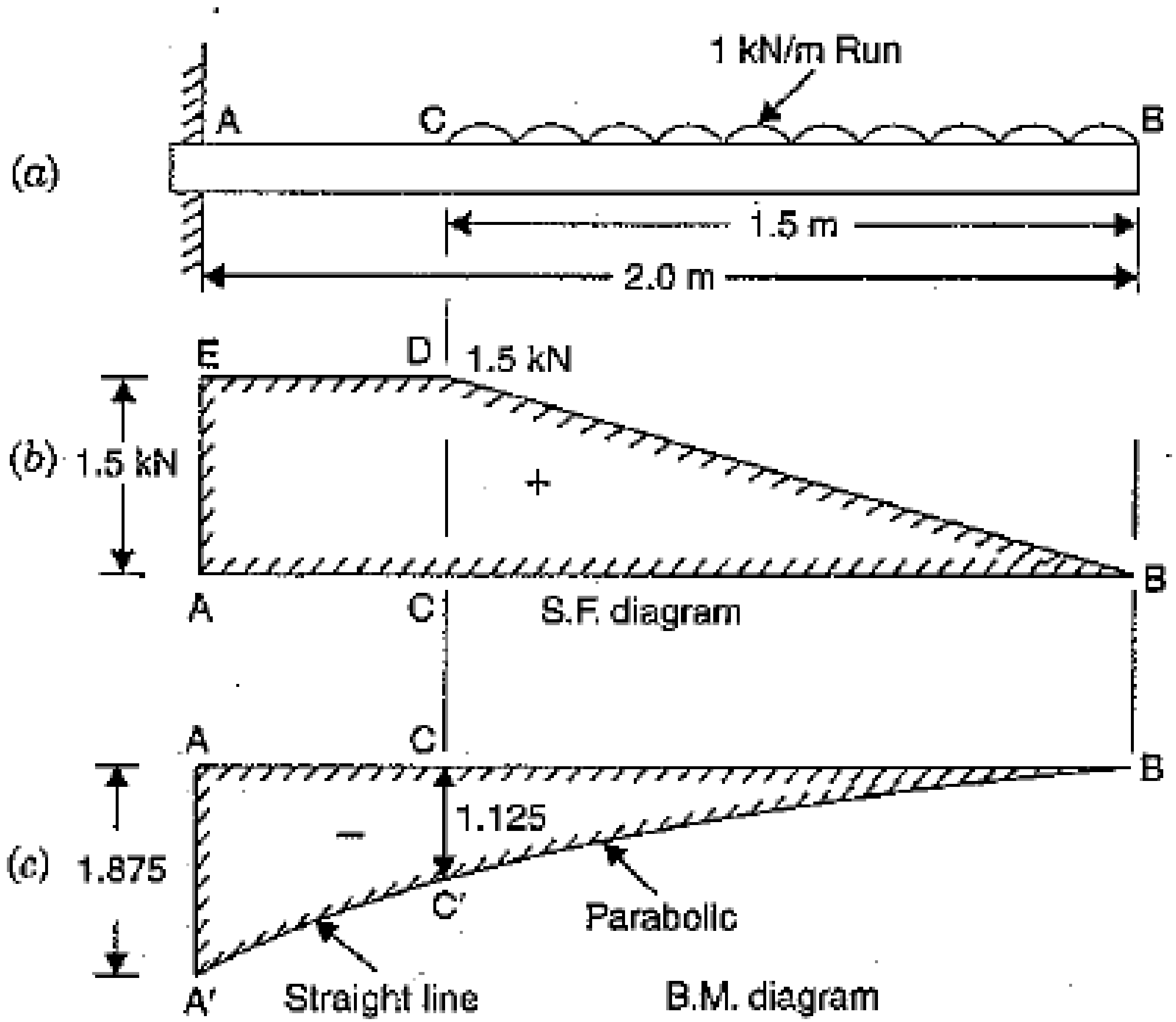
At C, $x = 1.5$ m hence $M_C = -1.5\ (1.5 - 0.75) = -1.125$ Nm

At A, $x = 2.0$ m hence $M_A = -1.5\ (2 - 0.75) = -1.875$ Nm.



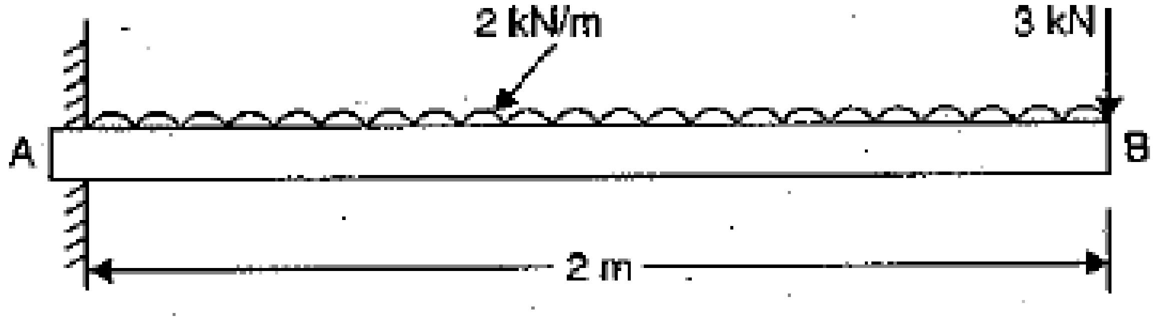


B.M. diagram


1 kN/m Run
A
C
B
(a)
1.5 m
2.0 m
E
D
1.5 kN
(b) 1.5 kN
+
A
C
B
S.F. diagram
A
C
B
1.125
(c) 1.875
−
C′
Parabolic
A′
Straight line
B.M. diagram

- A cantilever of length 2m carries a UDL of 2kN/m length over the whole length and a point load of 3kN at the free end Draw SFD and BMD.

- A cantilever of length 2m carries a UDL of 2kN/m length over the whole length and a point load of 3kN at the free end Draw SFD and BMD.



Shear Force Diagram

The shear force at $B = 3$ kN

Consider any section at a distance x from the free end B. The shear force at the section is given by,

$$F_x = 3.0 + w.x \qquad \text{(+ve sign is due to downward force on right portion of the section)}$$

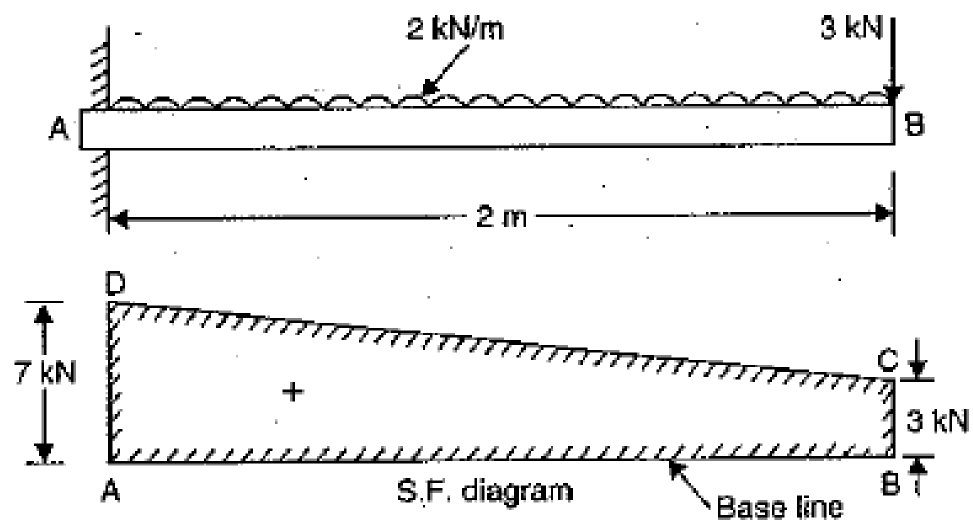
$$= 3.0 + 2 \times x \qquad (\because \quad w = 2 \text{ kN/m})$$

The above equation shows that shear force follows a straight line law.

At B, $x = 0$ hence $\quad F_B = 3.0$ kN

At A, $x = 2$ m hence $\quad F_A = 3 + 2 \times 2 = 7$ kN.

The shear force diagram is shown in Fig. 6.18 (b) in which $F_B = BC = 3$ kN and $F_A = AD = 7$ kN. The points C and D are joined by a straight line.



Bending Moment Diagram

The bending moment at any section at a distance x from the free end B is given by,

$$M_x = -\left(3x + wx \cdot \frac{x}{2}\right)$$

$$= -\left(3x + \frac{2x^2}{2}\right) \qquad (\because \quad w = 2 \text{ kN/m})$$

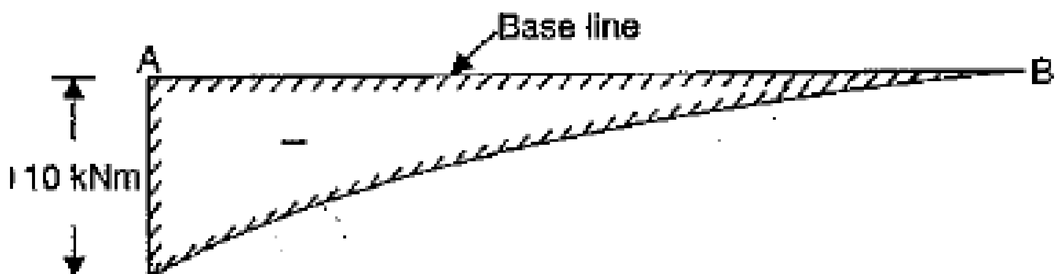
$$= -(3x + x^2) \qquad \ldots(i)$$

(The bending moment will be negative as for the right portion of the section, the moment of loads at x is clockwise).

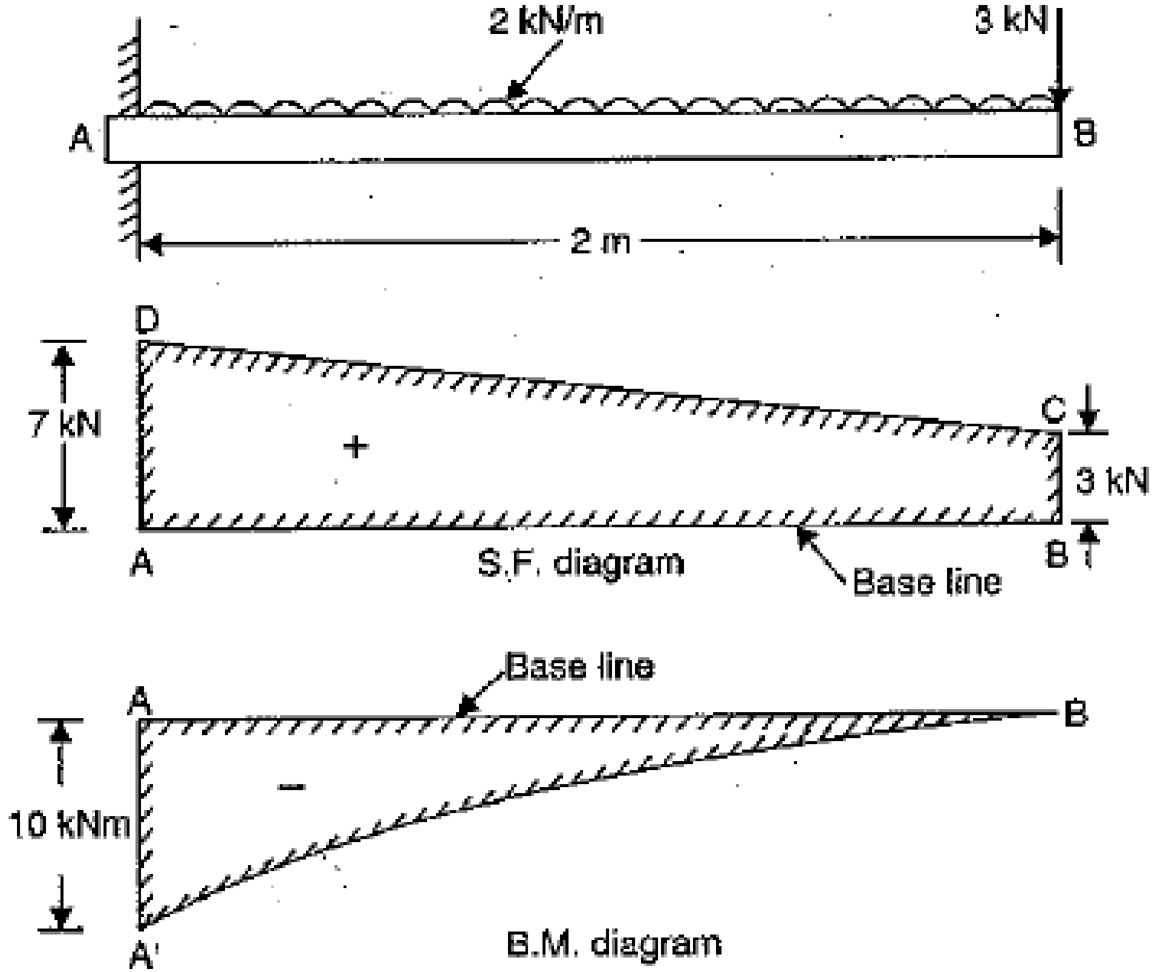
The equation (i) shows that the B.M. varies according to the parabolic law. From equation (i), we have

At B, $x = 0$ hence $\qquad M_B = -(3 \times 0 + 0^2) = 0$

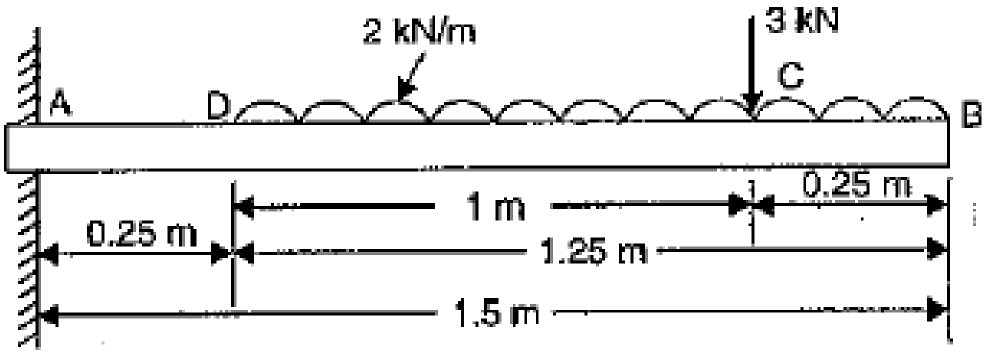
At A, $x = 2$ m hence $\qquad M_A = -(3 \times 2 + 2^2) = -10$ kN/m

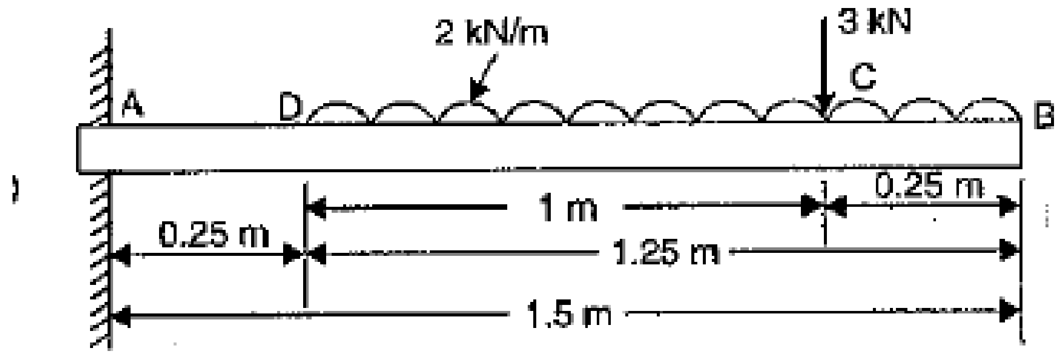


B.M. diagram

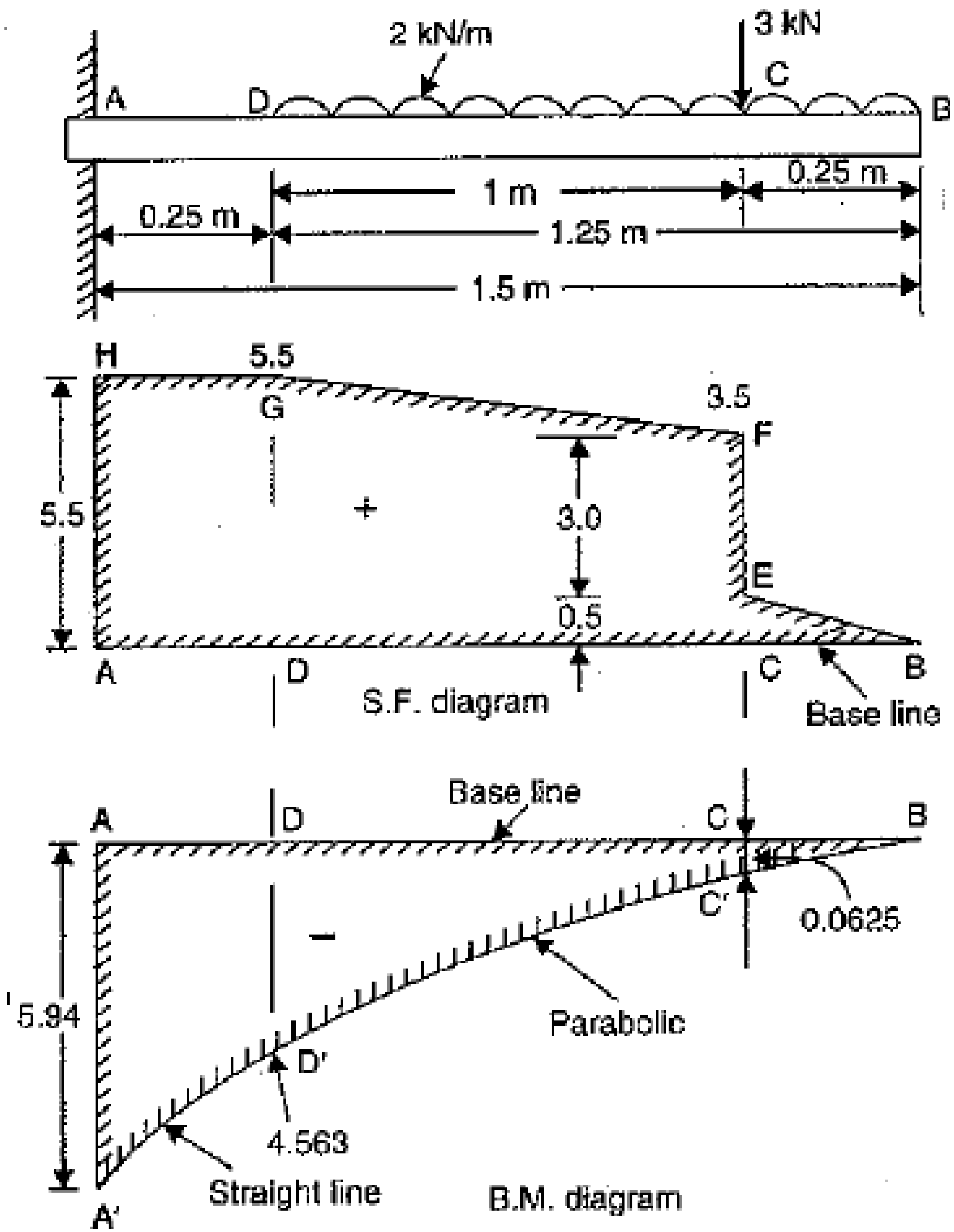

2 kN/m
3 kN
A
B
2 m
D
7 kN
+
C
3 kN
A
S.F. diagram
B
Base line
Base line
A
B
10 kNm
–
A'
B.M. diagram

A cantilever 1.5 m long is loaded with a uniformly distributed load of 2 kN/m run over a length of 1.25 m from the free end. It also carries a point load of 3 kN at a distance of 0.25 m from the free end. Draw the shear force and bending moment diagrams of the cantilever.

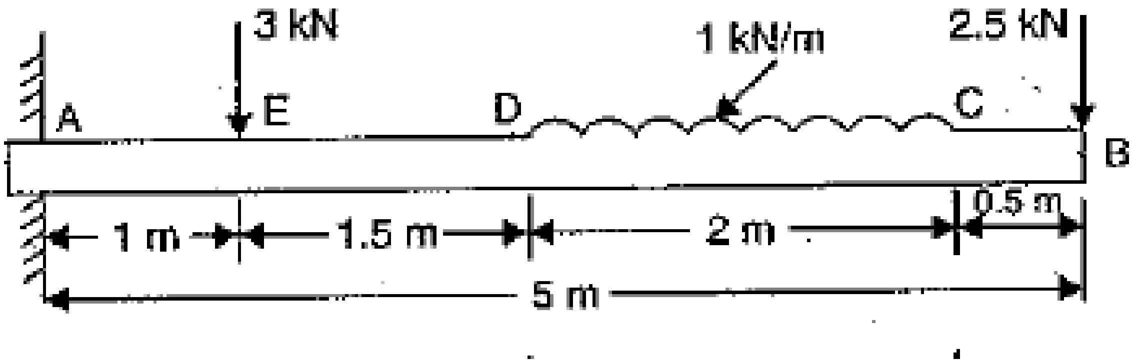


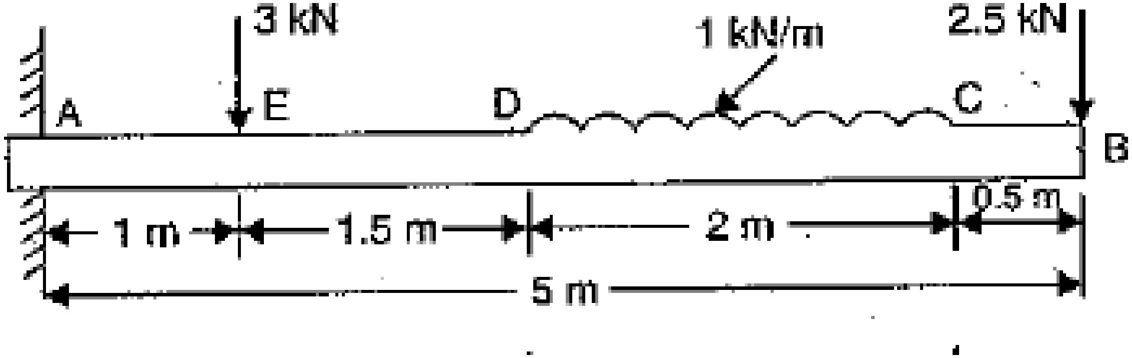


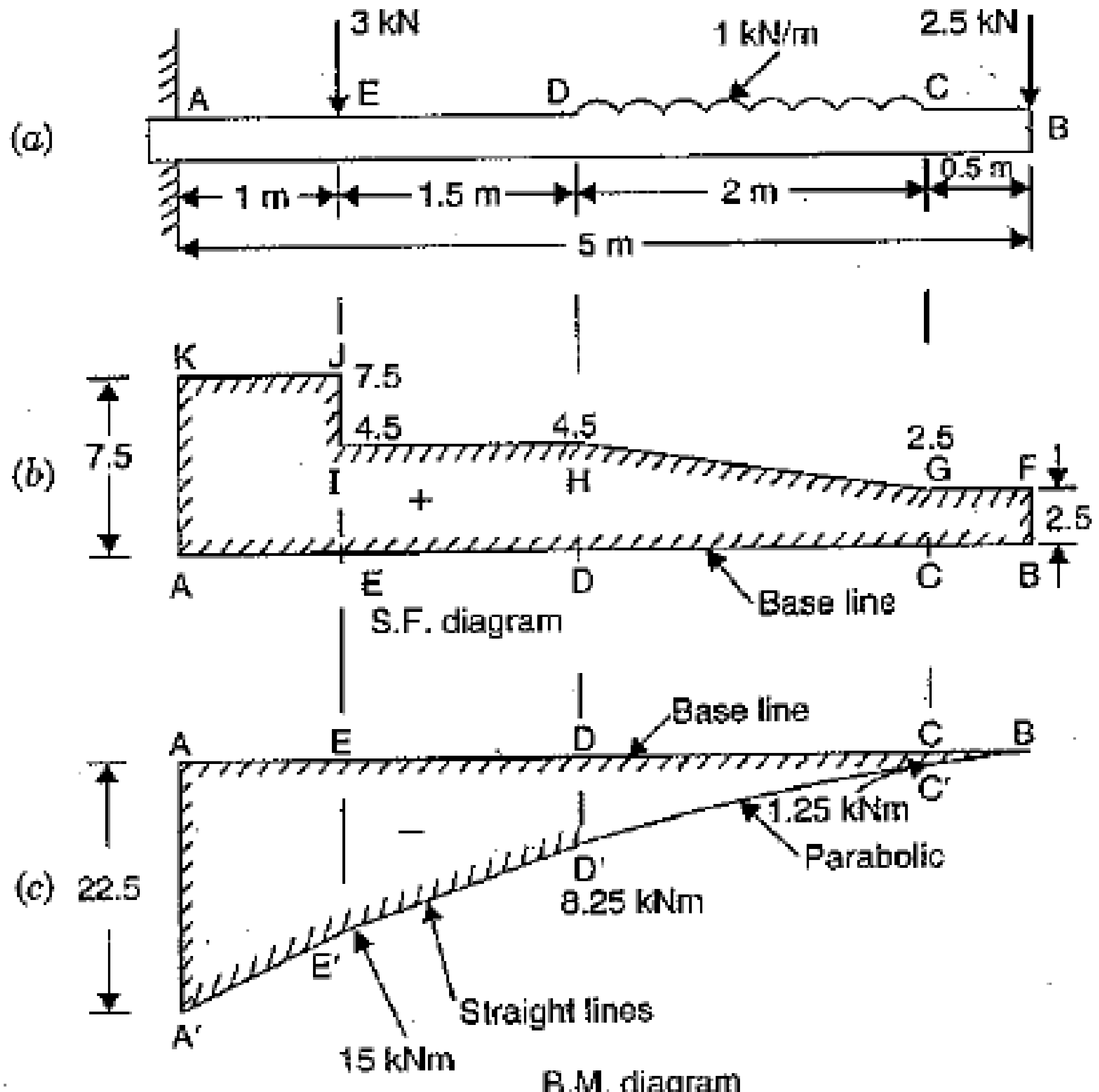
2 kN/m
3 kN
A
D
C
B
1 m
0.25 m
0.25 m
1.25 m
1.5 m


2 kN/m
3 kN
A
D
C
B
1 m
0.25 m
0.25 m
1.25 m
1.5 m
H
5.5
G
3.5
F
5.5
+
3.0
E
0.5
A
D
C
B
S.F. diagram
Base line
Base line
A
D
C
B
C'
0.0625
−
5.94
Parabolic
D'
4.563
Straight line
A'
B.M. diagram

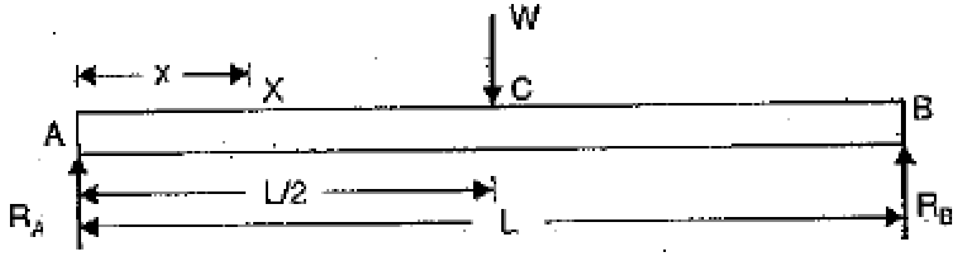
- A cantilever of length 5m is loaded as shown in figure. Draw SFD and BMD.




3 kN
1 kN/m
2.5 kN
A
E
D
C
B
1 m
1.5 m
2 m
0.5 m
5 m


3 kN
1 kN/m
2.5 kN
A
E
D
C
B
(a)
1 m
1.5 m
2 m
0.5 m
5 m
K
J
7.5
4.5
4.5
2.5
G
F
(b)
7.5
I
H
+
2.5
A
E
D
C
B
Base line
S.F. diagram
Base line
A
E
D
C
B
C'
1.25 kNm
Parabolic
(c)
22.5
D'
8.25 kNm
E'
A'
Straight lines
15 kNm
B.M. diagram

. SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM WITH A POINT LOAD AT MID-POINT



Symmetrical/Unsymmetrical loading

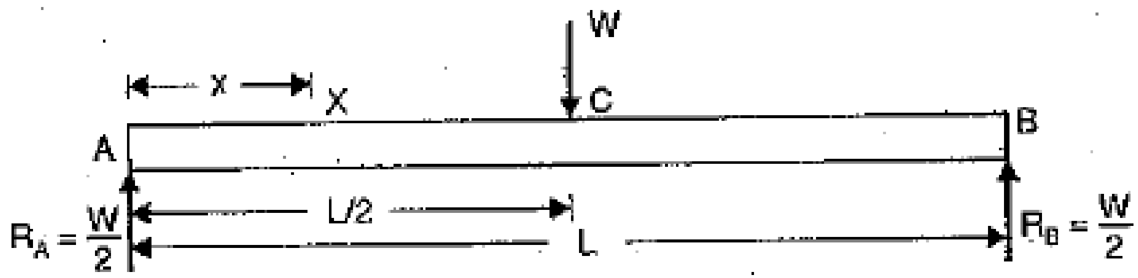
Fig. shows a beam AB of length L simply supported at the ends A and B and carrying a point load W at its middle point C.

The reactions at the support will be equal to $\frac{W}{2}$ as the load is acting at the middle point of the beam. Hence $R_A = R_B = \frac{W}{2}$.

Take a section X at a distance x from the end A between A and C.

Let F_x = Shear force at X,

and M_x = Bending moment at X.



Here we have considered the *left portion* of the section. The shear force at X will be equal to the resultant force acting on the left portion of the section. But the resultant force on the left portion is $\frac{W}{2}$ acting upwards. But according to the sign convention, the resultant force on the *left portion* acting upwards is considered positive. Hence shear force at X is positive and its magnitude is $\frac{W}{2}$.

$$\therefore \qquad F_x = +\frac{W}{2}$$

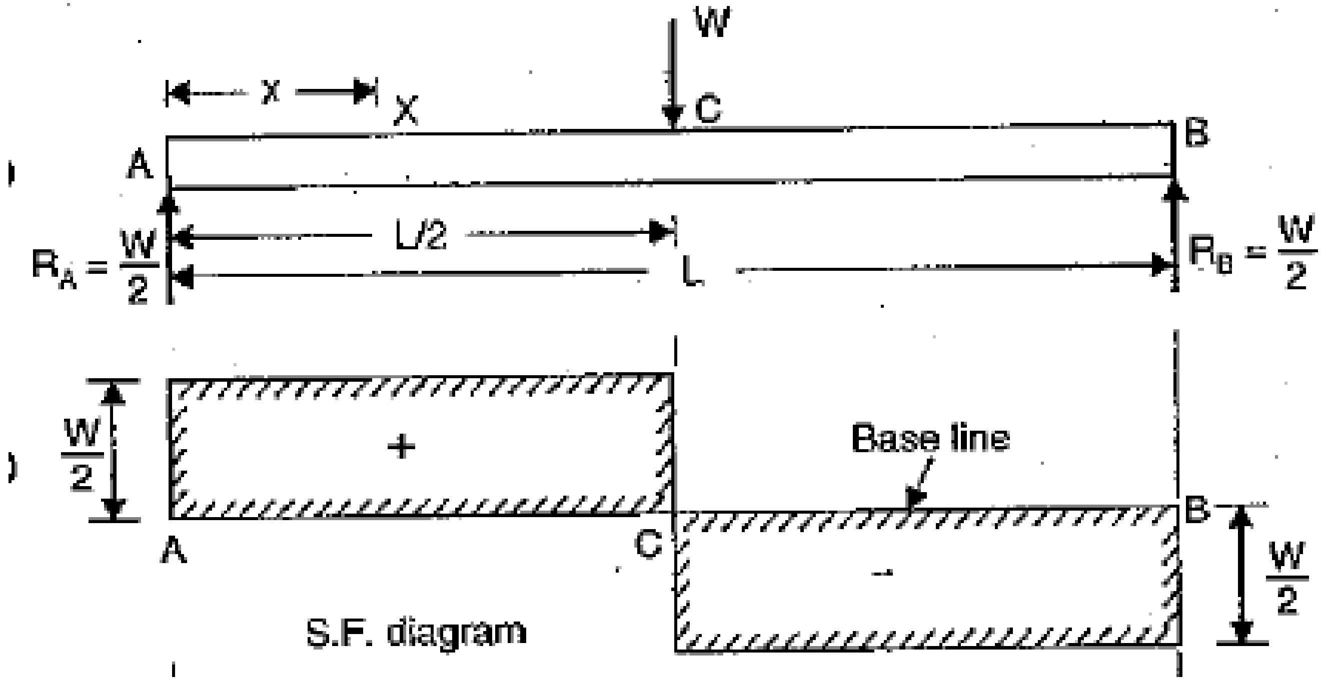
Hence the shear force between A and C is constant and equal to $+\frac{W}{2}$.

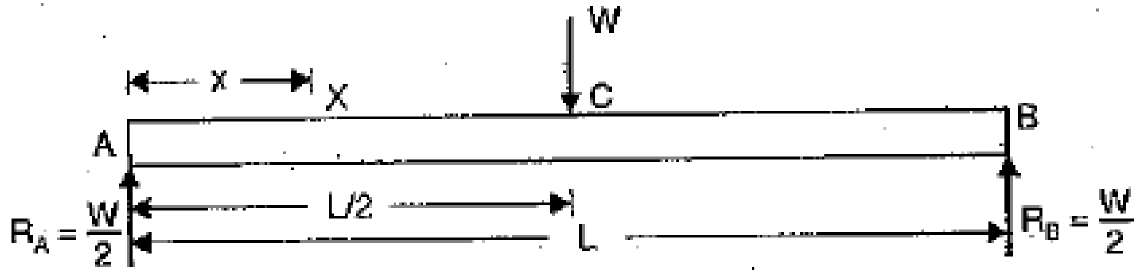
Now consider any section between C and B at distance x from end A. The resultant force on the left portion will be

$$\left(\frac{W}{2} - W\right) = -\frac{W}{2}.$$

This force will also remain constant between C and B. Hence shear force between C and B is equal to $-\frac{W}{2}$.

At the section C the shear force changes from $+\frac{W}{2}$ to $-\frac{W}{2}$.


W
x
X
C
A
B
L/2
L
$R_A = \frac{W}{2}$
$R_B = \frac{W}{2}$
$\frac{W}{2}$
+
Base line
B
A
C
–
$\frac{W}{2}$
S.F. diagram



Bending Moment Diagram

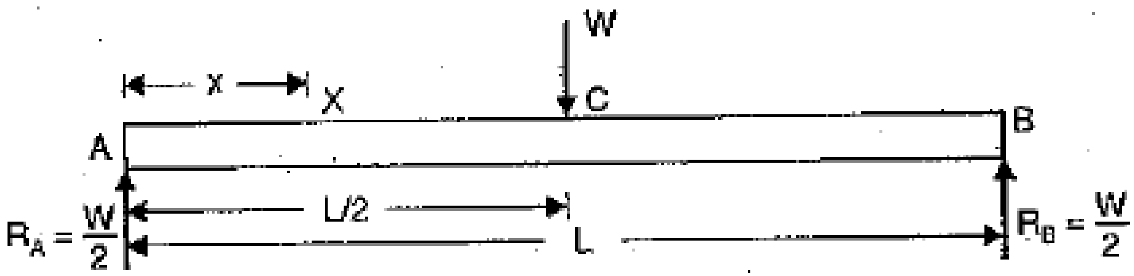
(*i*) The bending moment at any section between A and C at a distance of x from the end A, is given by

$$M_x = R_A . x \quad \text{or} \quad M_x = + \frac{W}{2} . x \qquad ...(i)$$

(B.M. will be positive as for the *left portion* of the section, the moment of all forces at X is clockwise. Moreover, the bending of beam takes place in such a manner that concavity is at the top of the beam).

At A, $x = 0$ hence $\qquad M_A = \frac{W}{2} \times 0 = 0$

At C, $x = \frac{L}{2}$ hence $\qquad M_C = \frac{W}{2} \times \frac{L}{2} = \frac{W \times L}{4}$.



From equation (*i*), it is clear that B.M. varies according to straight line law between A and C. B.M. is zero at A and it increases to $\frac{W \times L}{4}$ at C.

(*ii*) The bending moment at any section between C and B at a distance x from the end A, is given by

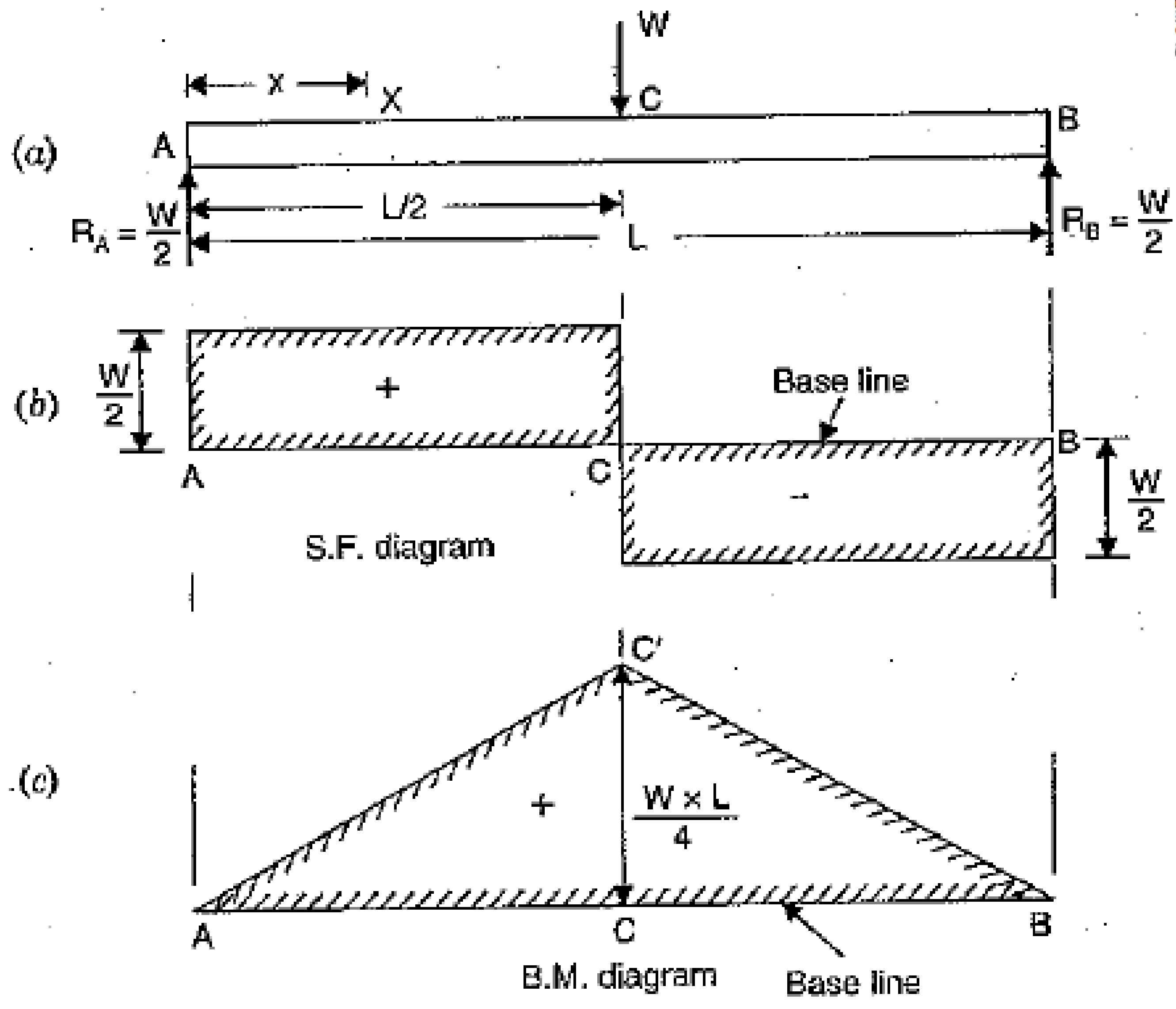
$$M_x = R_A . x - W \times \left(x - \frac{L}{2}\right) = \frac{W}{2} . x - Wx + W \times \frac{L}{2} = \frac{WL}{2} - \frac{2x}{2}$$

At C, $x = \frac{L}{2}$ hence $\quad M_C = \frac{WL}{2} - \frac{W}{2} \times \frac{L}{2} = \frac{W \times L}{4}$

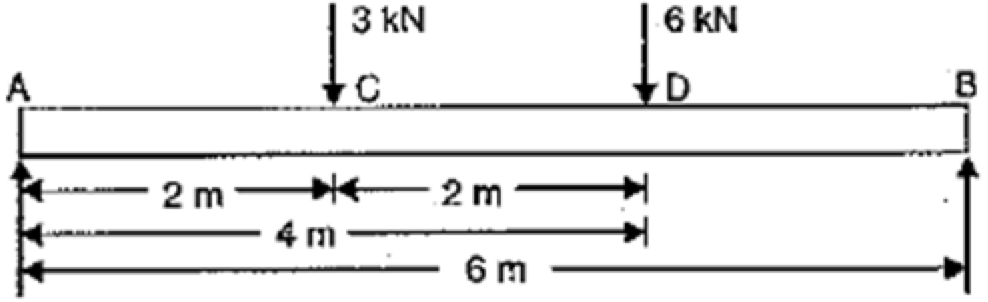
At B, $x = L$ hence $\quad M_B = \frac{WL}{2} - \frac{W}{2} \times L = 0.$

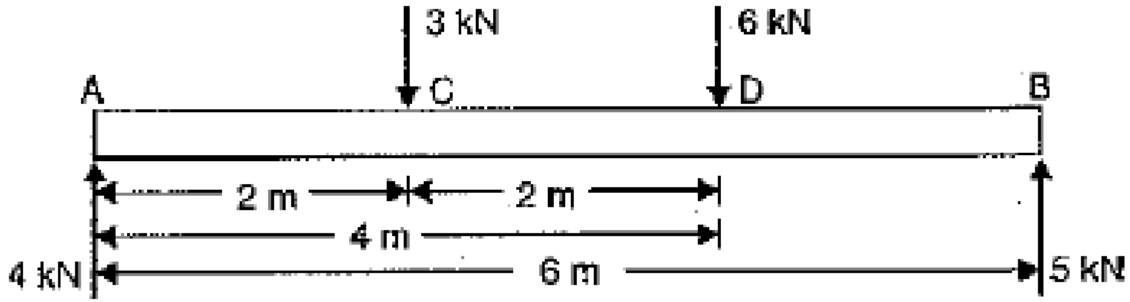
Hence bending moment at C is $\frac{WL}{4}$ and it decreases to zero at B. Now the B.M. diagram can be completed as shown in Fig. 6.24 (*c*).

Note. The bending moment is maximum at the middle point C, where the shear force changes its sign.


W
x
X
C
B
(a)
A
L/2
$R_A = \frac{W}{2}$
L
$R_B = \frac{W}{2}$
(b)
$\frac{W}{2}$
+
Base line
B
A
C
–
$\frac{W}{2}$
S.F. diagram
C′
(c)
+
$\frac{W \times L}{4}$
A
C
B
B.M. diagram
Base line

A simply supported beam of length 6 m, carries point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.





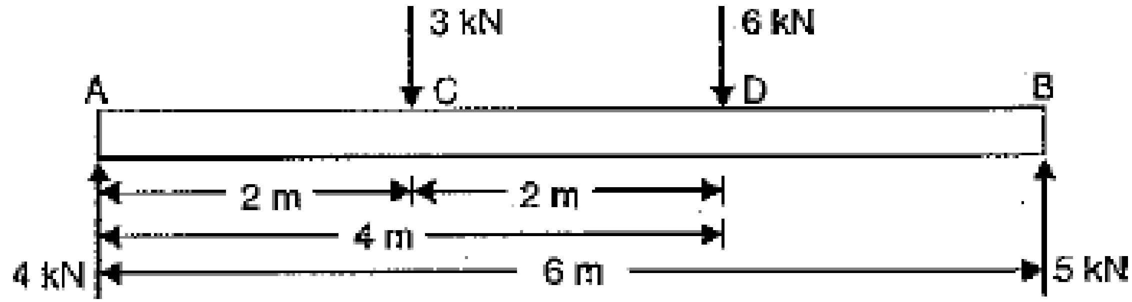
Sol. First calculate the reactions R_A and R_B.

Taking moments of the force about A, we get

$$R_B \times 6 = 3 \times 2 + 6 \times 4 = 30$$

$$\therefore \quad R_B = \frac{30}{6} = 5 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load on beam} - R_B = (3 + 6) - 5 = 4 \text{ kN}$$



Shear Force Diagram

Shear force at A, $F_A = +R_A = +4$ kN

Shear force between A and C is constant and equal to + 4 kN

Shear force at C, $F_C = +4 - 3.0 = +1$ kN

Shear force between C and D is constant and equal to + 1 kN.

Shear force at D, $F_D = +1 - 6 = -5$ kN

The shear force between D and B is constant and equal to – 5 kN.

Shear force at B, $F_B = -5$ kN

The shear force diagram is drawn as shown in Fig. 6.26 (*b*).

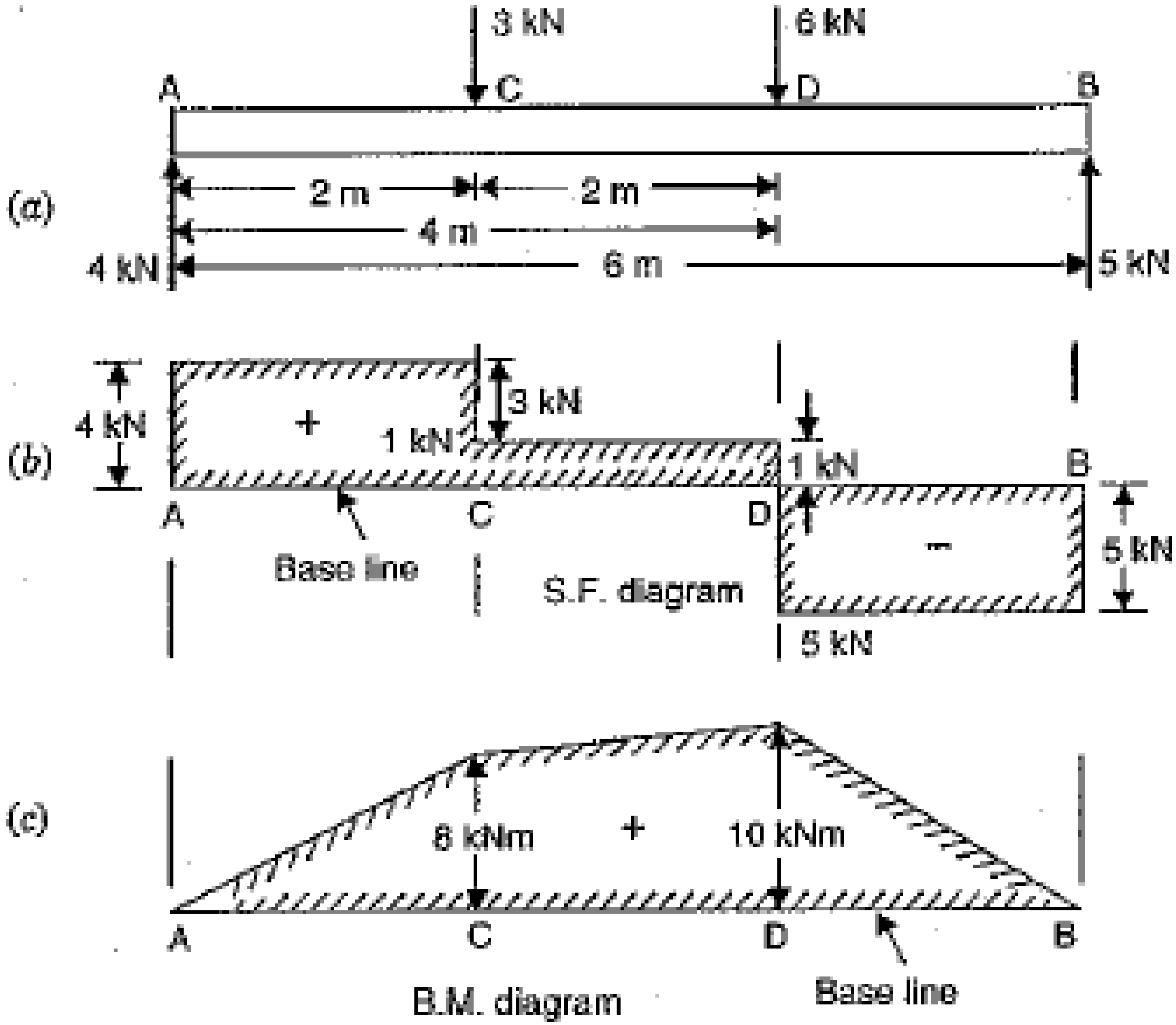
Bending Moment Diagram

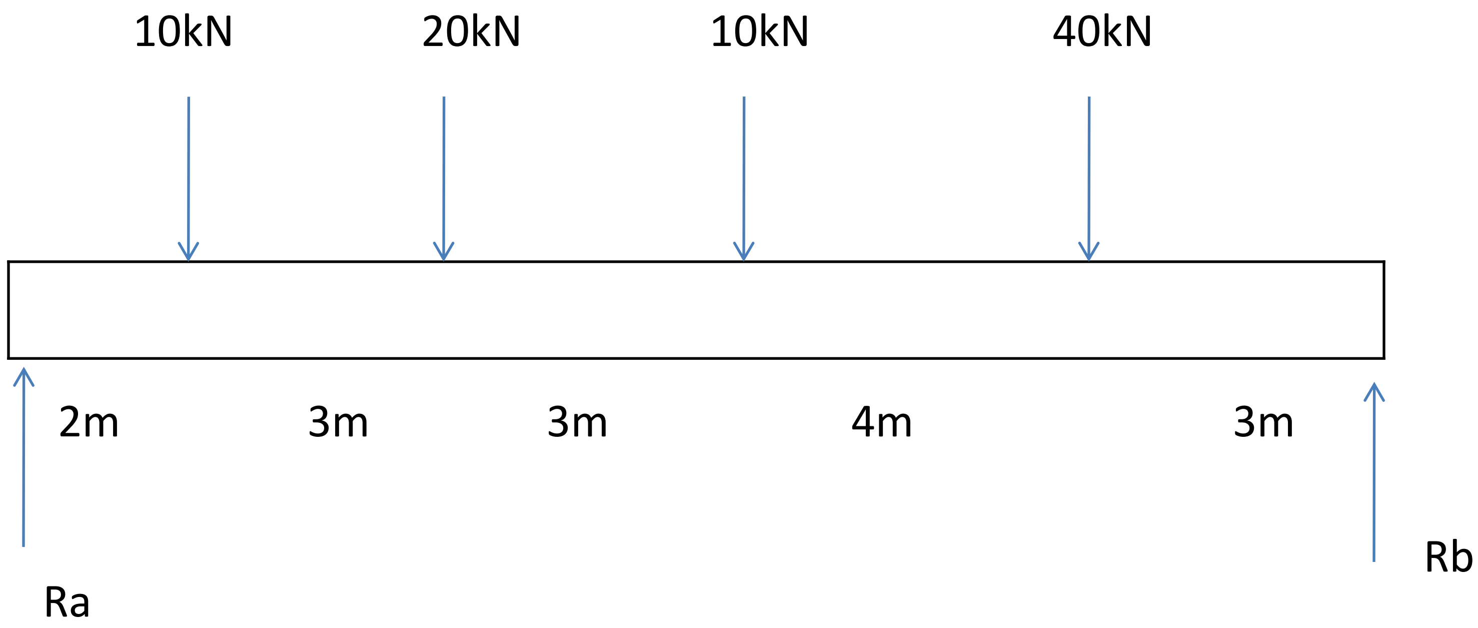
B.M. at A, $M_A = 0$

B.M. at C, $M_C = R_A \times 2 = 4 \times 2 = +8$ kNm

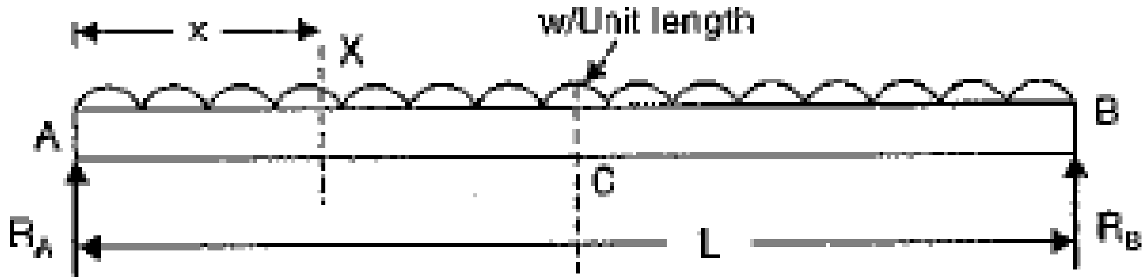
B.M. at D, $M_D = R_A \times 4 - 3 \times 2 = 4 \times 4 - 3 \times 2 = +10$ kNm

B.M. at B, $M_B = 0$


3 kN
6 kN
A
C
D
B
(a)
2 m
2 m
4 m
4 kN
6 m
5 kN
(b)
4 kN
+
1 kN
3 kN
1 kN
B
A
C
D
Base line
S.F. diagram
5 kN
5 kN
(c)
6 kNm
+
10 kNm
A
C
D
B
B.M. diagram
Base line


10kN
20kN
10kN
40kN
2m
3m
3m
4m
3m
Ra
Rb

SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM CARRYING A UNIFORMLY DISTRIBUTED LOAD



Let R_A = Reaction at A, and

R_B = Reaction at B

$$\therefore \quad R_A = R_B = \frac{w.L}{2}$$

Consider any section X at a distance x from the *left end A*. The shear force at the section (*i.e.*, F_x) is given by,

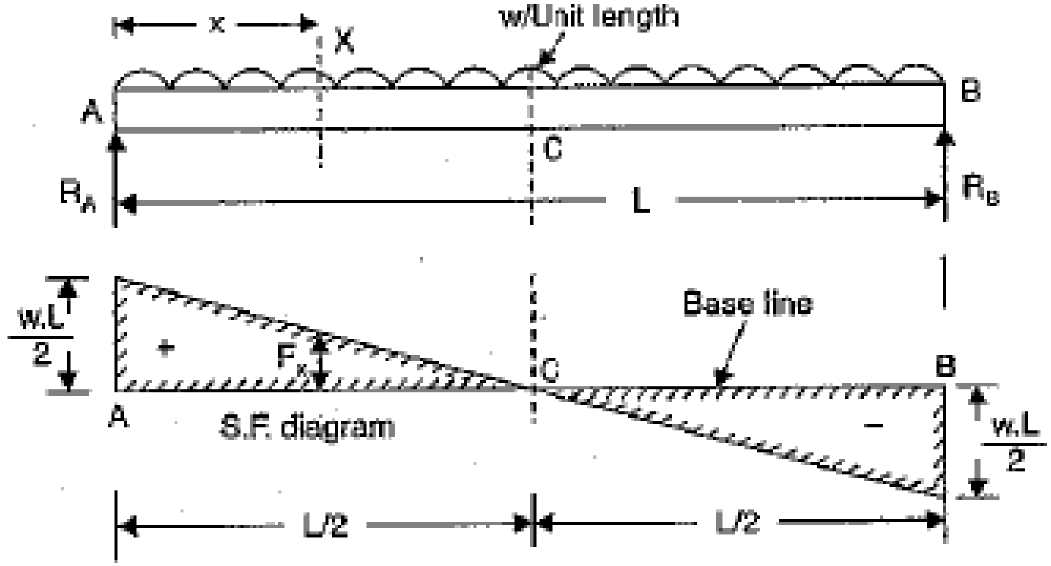
$$F_x = +R_A - w \cdot x = +\frac{w.L}{2} - w \cdot x \quad \ldots(i)$$

From equation (i), it is clear that the shear force varies according to straight line law. The values of shear force at different points are :

At A, $x = 0$ hence $\quad F_A = +\frac{w.L}{2} - \frac{w.0}{2} = +\frac{w.L}{2}$

At B, $x = L$ hence $\quad F_B = +\frac{w.L}{2} - w.L = -\frac{w.L}{2}$

At C, $x = \frac{L}{2}$ hence $\quad F_C = +\frac{w.L}{2} - w.\frac{L}{2} = 0$



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The bending moment at the section X at a distance x from left end A is given by,

$$M_x = +R_A \cdot x - w \cdot x \cdot \frac{x}{2}$$

$$= \frac{w.L}{2} \cdot x - \frac{w.x^2}{2} \qquad \left(\because \; R_A = \frac{w.L}{2}\right) \quad ...(ii)$$

From equation (*ii*), it is clear that B.M. varies according to parabolic law.

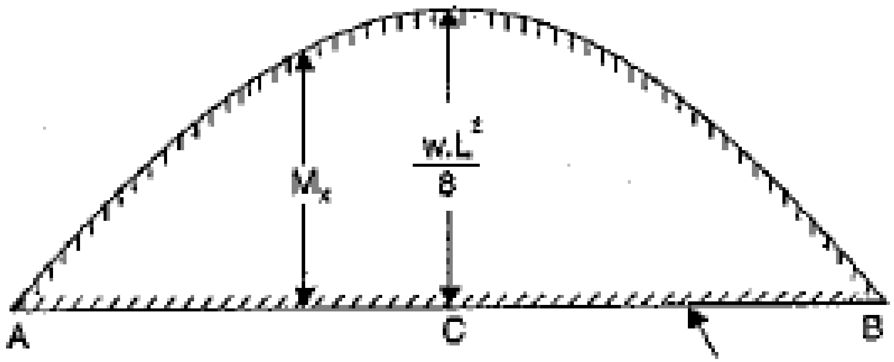
The values of B.M. at different points are :

At A, $x = 0$ hence $\quad M_A = \frac{w.L}{2} \cdot 0 - \frac{w.0}{2} = 0$

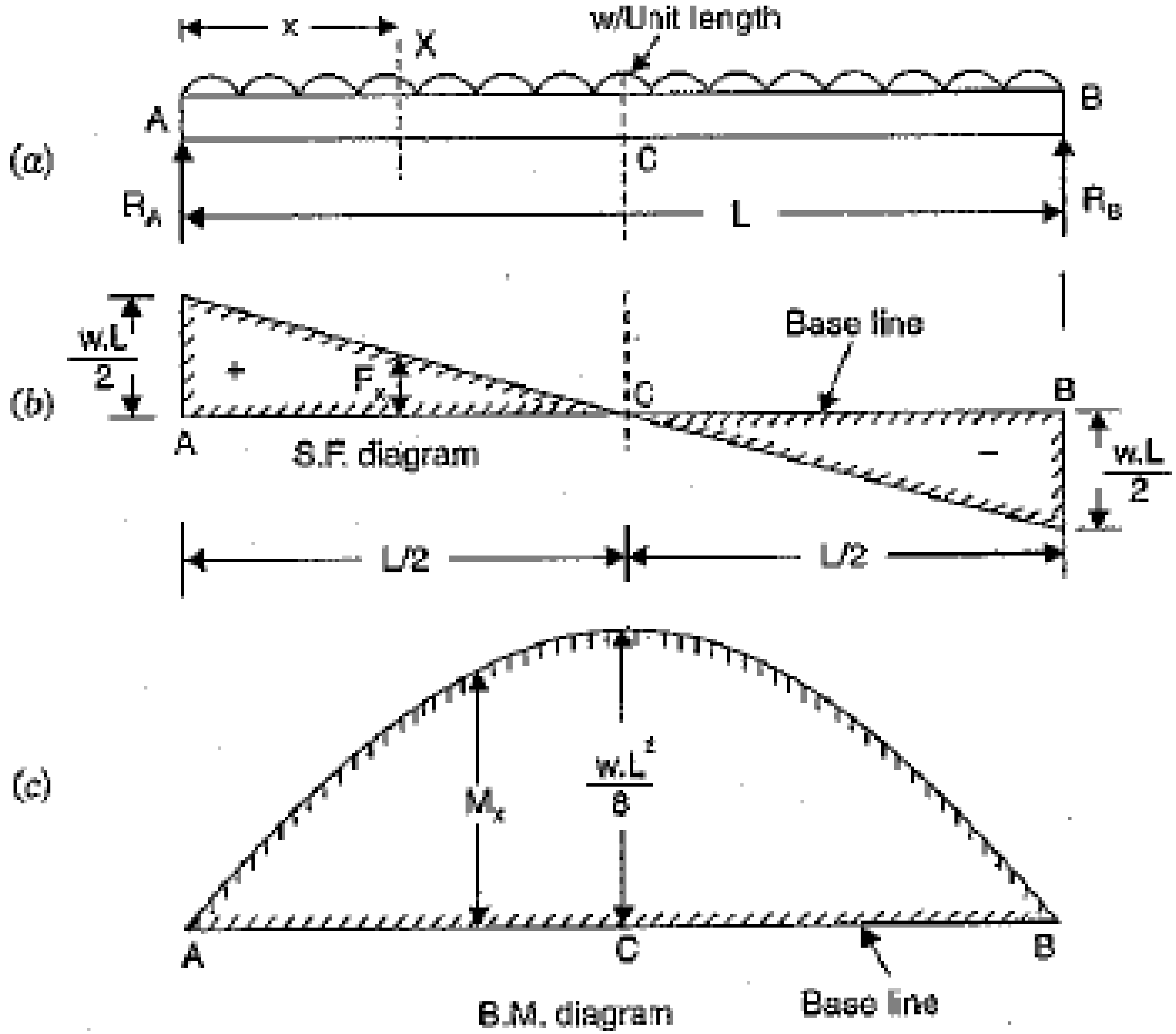
At B, $x = L$ hence $\quad M_B = \frac{w.L}{2} \cdot L - \frac{w}{2} \cdot L^2 = 0$

At C, $x = \frac{L}{2}$ hence $\quad M_C = \frac{w.L}{2} \cdot \frac{L}{2} - \frac{w}{2} \cdot \left(\frac{L}{2}\right)^2 = \frac{w.L^2}{4} - \frac{w.L^2}{8} = +\frac{w.L^2}{8}$.

Thus the B.M. increases according to parabolic law from zero at A to $+\frac{w.L^2}{8}$ at the middle point of the beam and from this value the B.M. decreases to zero at B according to the parabolic law.

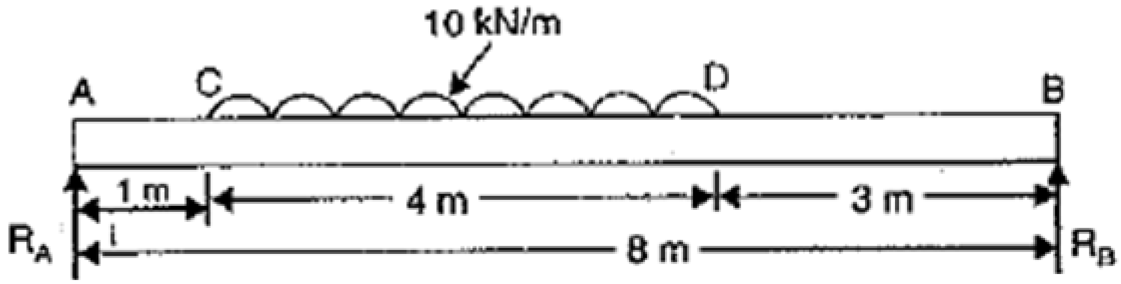


B.M. diagram



x
X
w/Unit length
A
B
(a)
C
R_A
L
R_B
$\frac{w.L}{2}$
+
F_x
Base line
(b)
C
B
A
S.F. diagram
–
$\frac{w.L}{2}$
L/2
L/2
$\frac{w.L^2}{8}$
M_x
(c)
A
C
B
B.M. diagram
Base line

Draw the shear force and B.M. diagrams for a simply supported beam of length 8 m and carrying a uniformly distributed load of 10 kN/m for a distance of 4 m as shown



Sol. First calculate the reactions R_A and R_B.

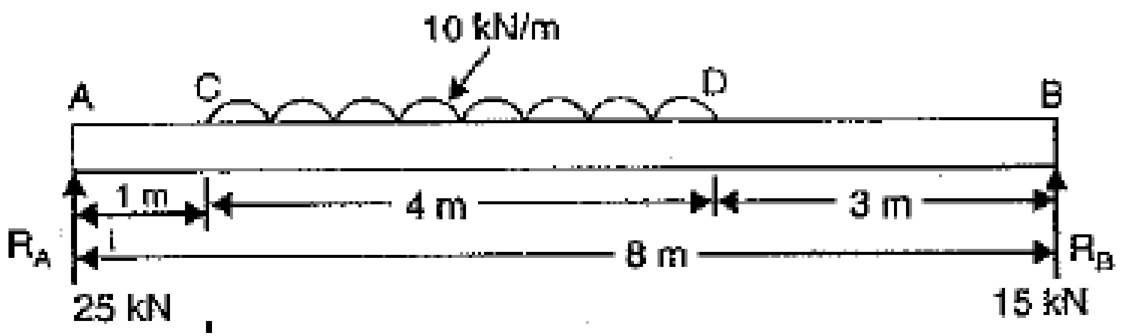
Taking moments of the forces about A, we get

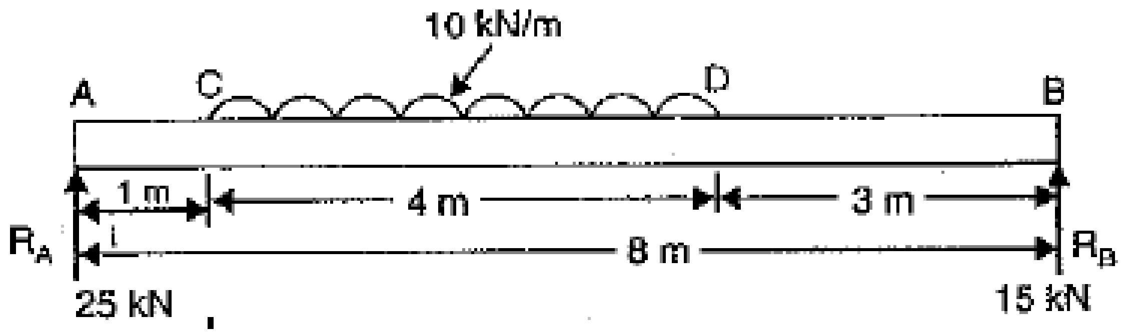
$$R_B \times 8 = 10 \times 4 \times \left(1 + \frac{4}{2}\right) = 120$$

$$R_B = \frac{120}{8} = 15 \text{ kN}$$

$$R_A = \text{Total load on beam} - R_B$$

$$= 10 \times 4 - 15 = 25 \text{ kN}$$





Shear Force Diagram

The shear force at A is + 25 kN. The shear force remains constant between A and C and equal to + 25 kN. The shear force at B is – 15 kN. The shear force remains constant between B and D and equal to – 15 kN. The shear force at any section between C and D at a distance x from A is given by,

$$F_x = +25 - 10(x-1) \qquad \ldots(i)$$

At C, $x = 1$ hence $\quad F_C = +25 - 10(1-1) = +25$ kN

At D, $x = 5$ hence $\quad F_D = +25 - 10(5-1) = -15$ kN

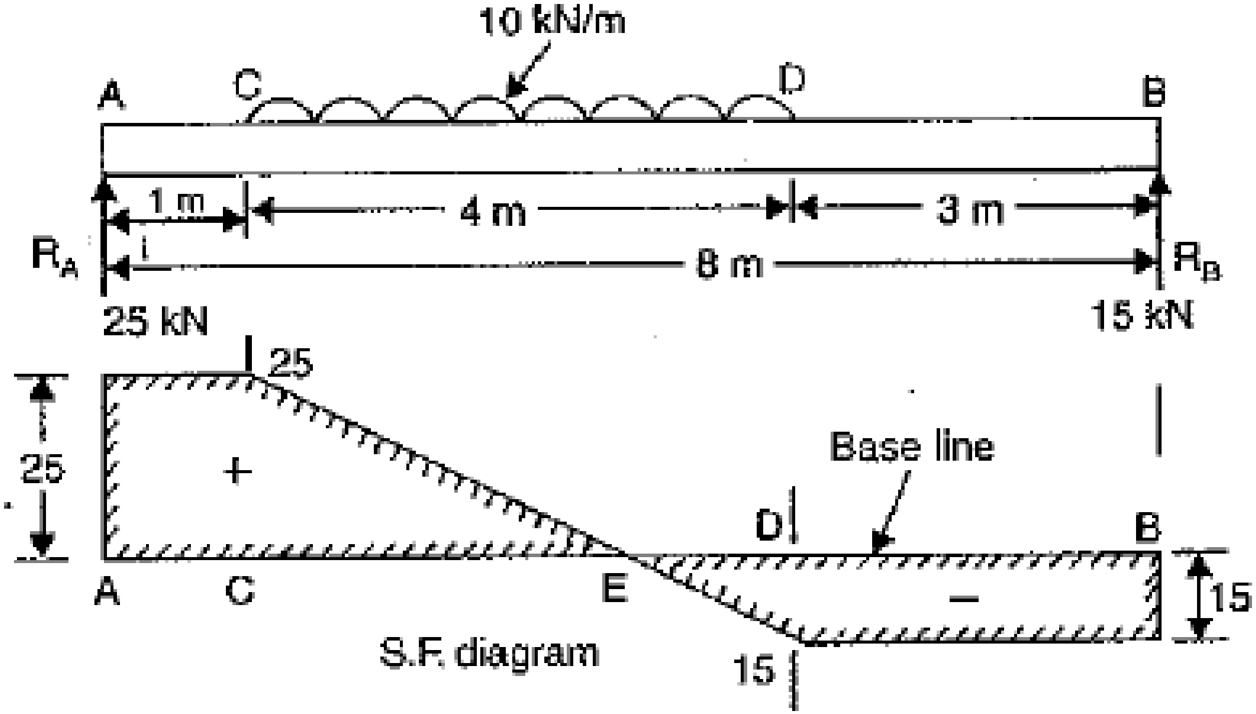
The shear force at C is + 25 kN and at D is – 15 kN. Also shear force between C and D varies by a straight line law. This means that somewhere between C and D, the shear force is zero. Let the S.F. be zero at x metre from A. Then substituting the value of S.F. (*i.e.*, F_x) equal to zero in equation (*i*), we get

$$0 = 25 - 10(x-1)$$

or $\quad 0 = 25 - 10x + 10 \quad$ or $\quad 10x = 35$

$$\therefore \quad x = \frac{35}{10} = 3.5 \text{ m}$$

Hence the shear force is zero at a distance 3.5 m from A.


10 kN/m
A
C
D
B
1 m
4 m
3 m
8 m
R_A
R_B
25 kN
15 kN
25
25
+
Base line
D
B
A
C
E
15
–
15
S.F. diagram

B.M. Diagram

B.M. at A is zero

B.M. at B is also zero

B.M. at $C = R_A \times 1 = 25 \times 1 = 25$ kNm

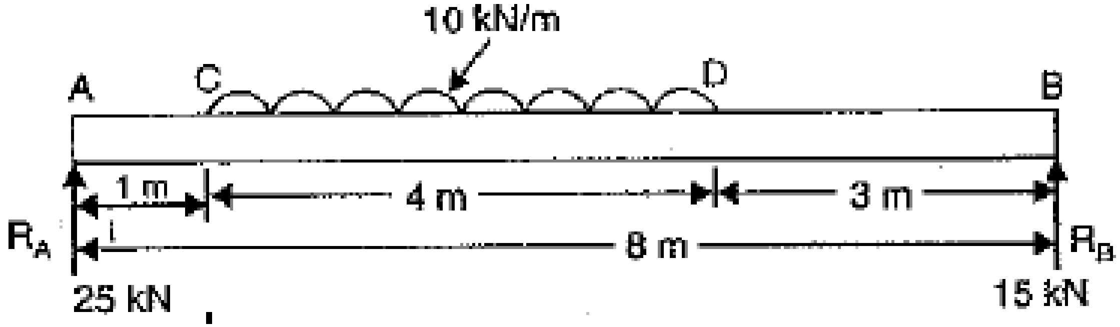
The B.M. at any section between C and D at a distance x from A is given by,

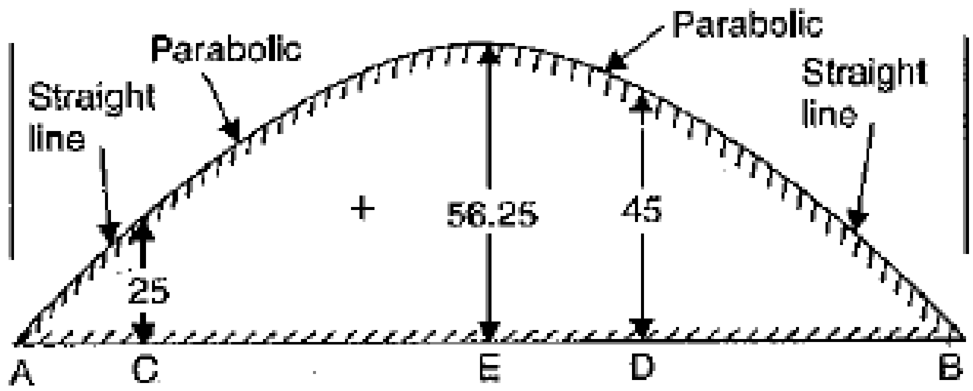
$$M_x = R_A \cdot x - 10(x-1) \cdot \frac{(x-1)}{2} = 25 \times x - 5(x-1)^2 \qquad \ldots(ii)$$

At C, $x = 1$ hence $M_C = 25 \times 1 - 5(1-1)^2 = 25$ kNm

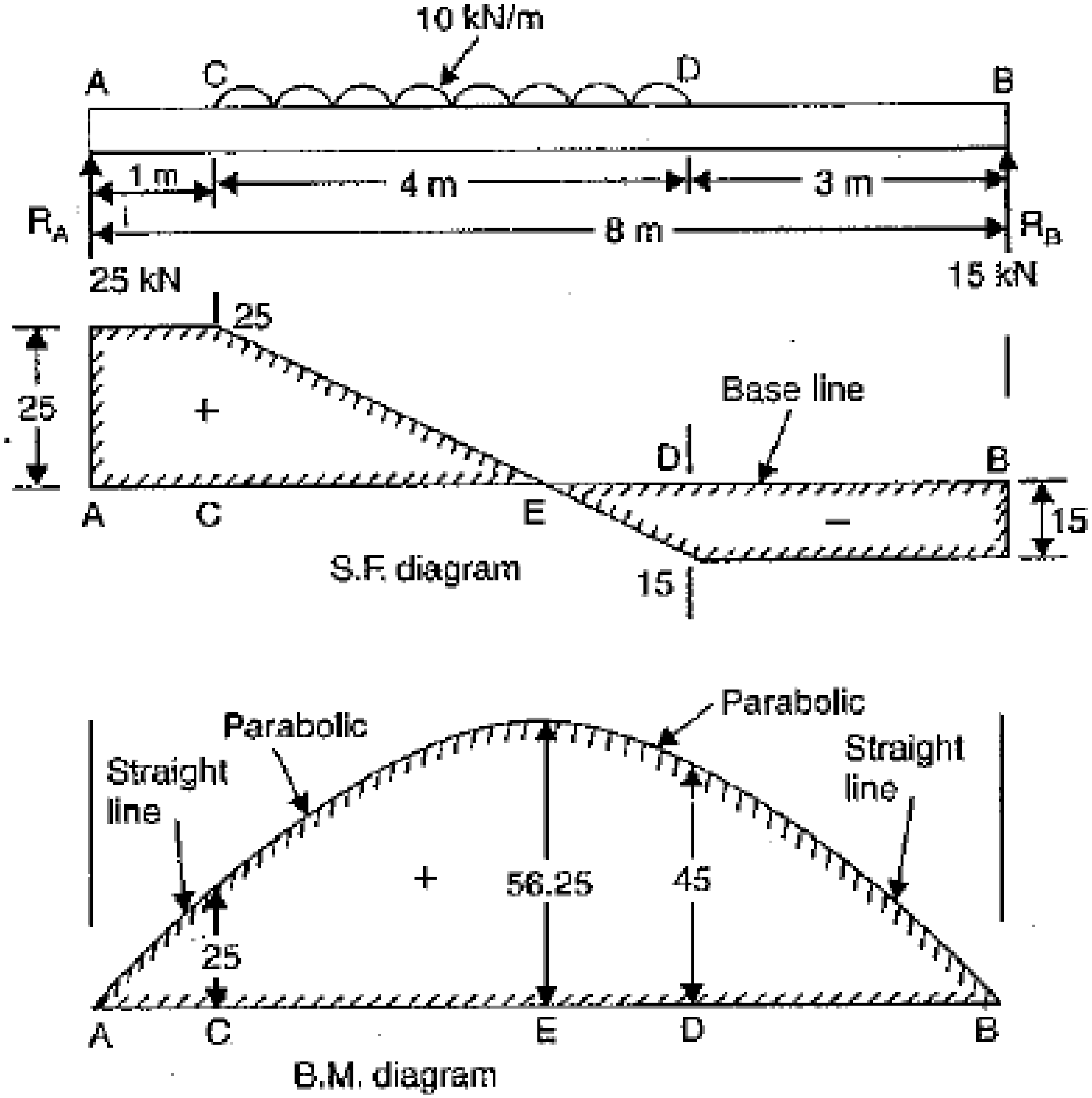
At D, $x = 5$ hence $M_D = 25 \times 5 - 5(5-1)^2 = 125 - 80 = 45$ kNm

At E, $x = 3.5$ hence $M_E = 25 \times 3.5 - 5(3.5-1)^2 = 87.5 - 31.25 = 56.25$ kNm

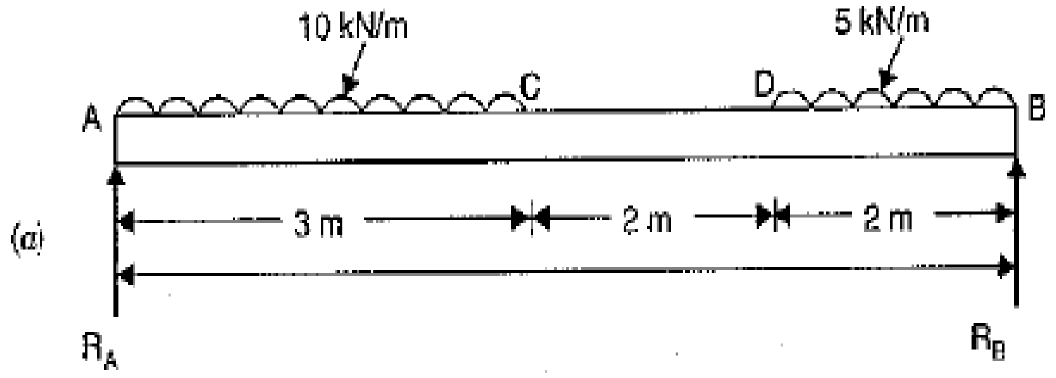




B.M. diagram


10 kN/m
A
C
D
B
1 m
4 m
3 m
8 m
RA
RB
25 kN
15 kN
25
25
+
Base line
D
B
A
C
E
–
15
15
S.F. diagram
Parabolic
Parabolic
Straight line
Straight line
+
56.25
45
25
A
C
E
D
B
B.M. diagram

Draw the S.F. and B.M. diagrams of a simply supported beam of length 7 m carrying uniformly distributed loads as shown in Fig. 6.30.



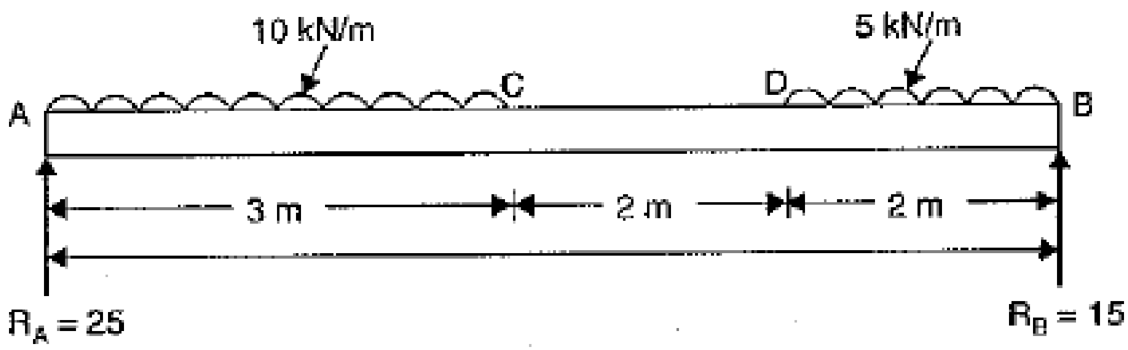
Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 7 = 10 \times 3 \times \frac{3}{2} + 5 \times 2 \times \left(3 + 2 + \frac{2}{2}\right) = 45 + 60 = 105$$

$$\therefore \quad R_B = \frac{105}{7} = 15 \text{ kN}$$

$$R_A = \text{Total load on beam} - R_B$$
$$= (10 \times 3 + 5 \times 2) - 15 = 40 - 15 = 25 \text{ kN}$$



S.F. Diagram

The shear force at A is + 25 kN

The shear force at $C = R_A - 3 \times 10 = +25 - 30 = -5$ kN

The shear force varies between A and C by a straight line law.

The shear force between C and D is constant and equal to – 5 kN

The shear force at B is – 15 kN

The shear force between D and B varies by a straight line law.

The shear force diagram is drawn as shown in Fig. 6.30 (b).

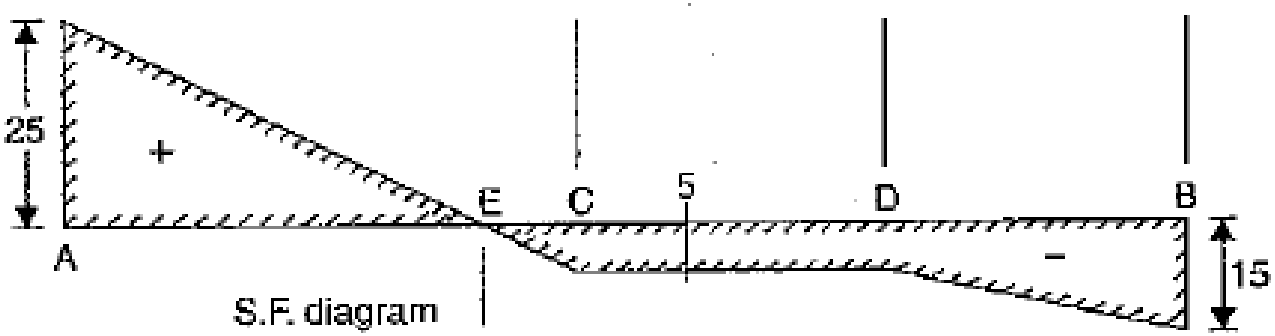
The shear force is zero at point E between A and C. Let us find the location of E from A. Let the point E be at a distance x from A.

The shear force at $E = R_A - 10 \times x = 25 - 10x$

But shear force at $E = 0$

$\therefore \quad 25 - 10x = 0 \quad$ or $\quad 10x = 25$

or $\quad x = \frac{25}{10} = 2.5$ m



S.F. diagram

B.M. Diagram

B.M. at A is zero

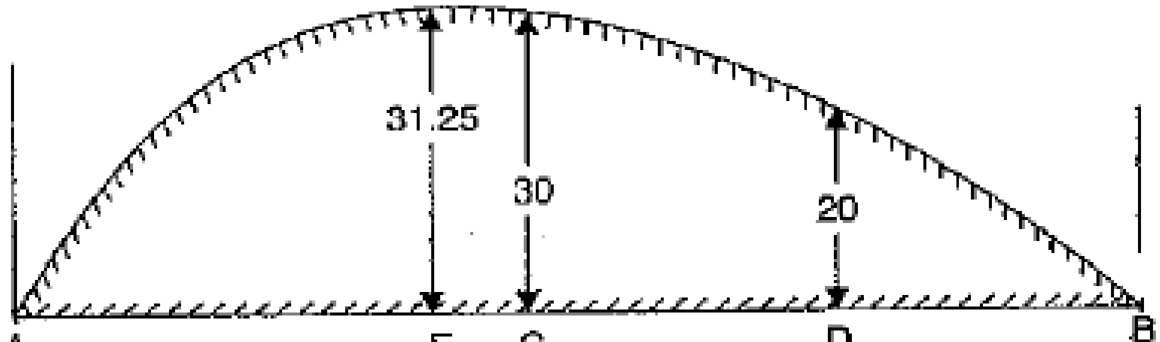
B.M. at B is zero

B.M. at C, $$M_C = R_A \times 3 - 10 \times 3 \times \frac{3}{2} = 25 \times 3 - 45 = 75 - 45 = 30 \text{ kNm}$$

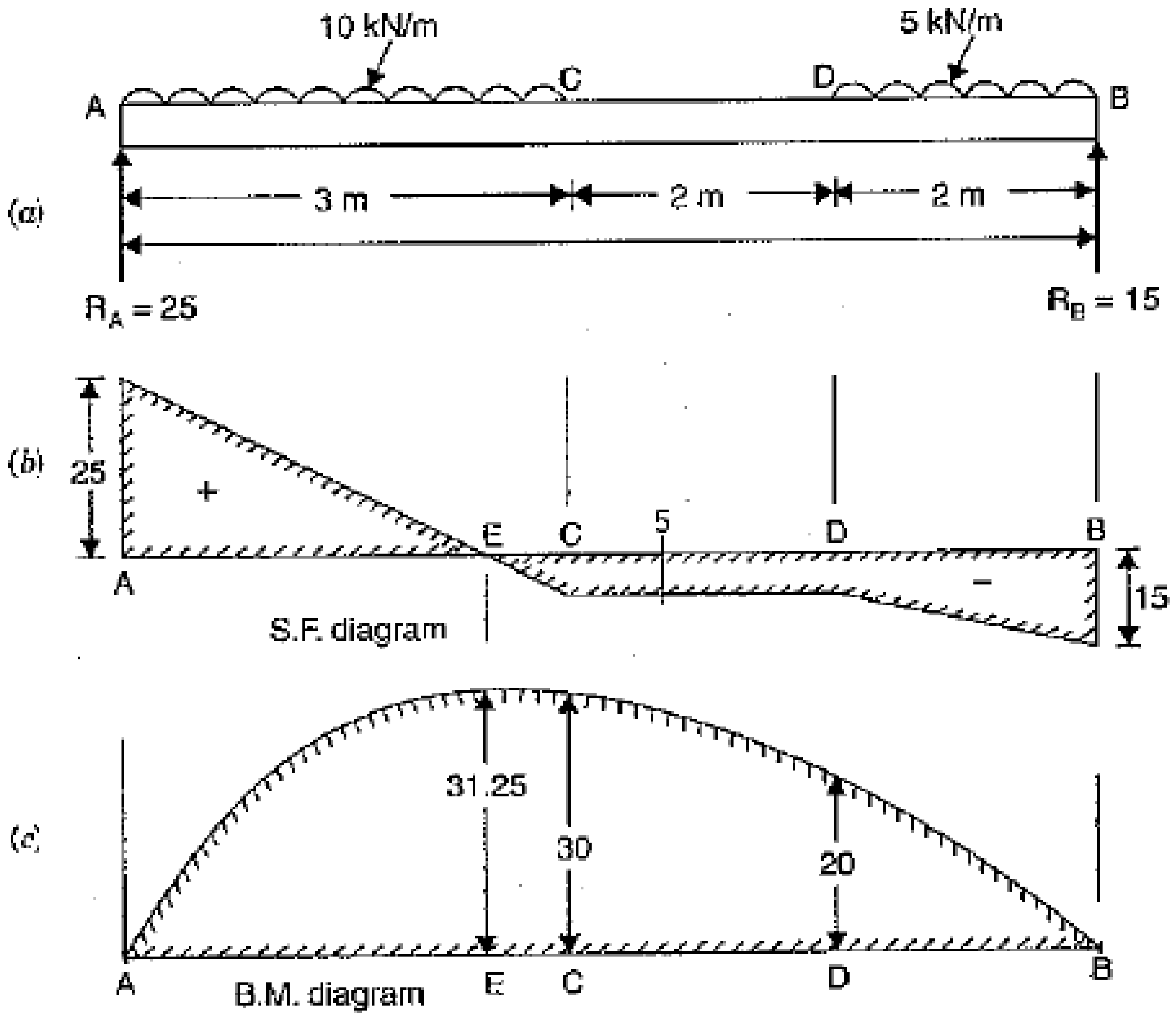
At E, $x = 2.5$ and hence

B.M. at E, $$M_E = R_A \times 2.5 - 10 \times 2.5 \times \frac{2.5}{2} = 25 \times 2.5 - 5 \times 6.25$$
$$= 62.5 - 31.25 = 31.25 \text{ kNm}$$

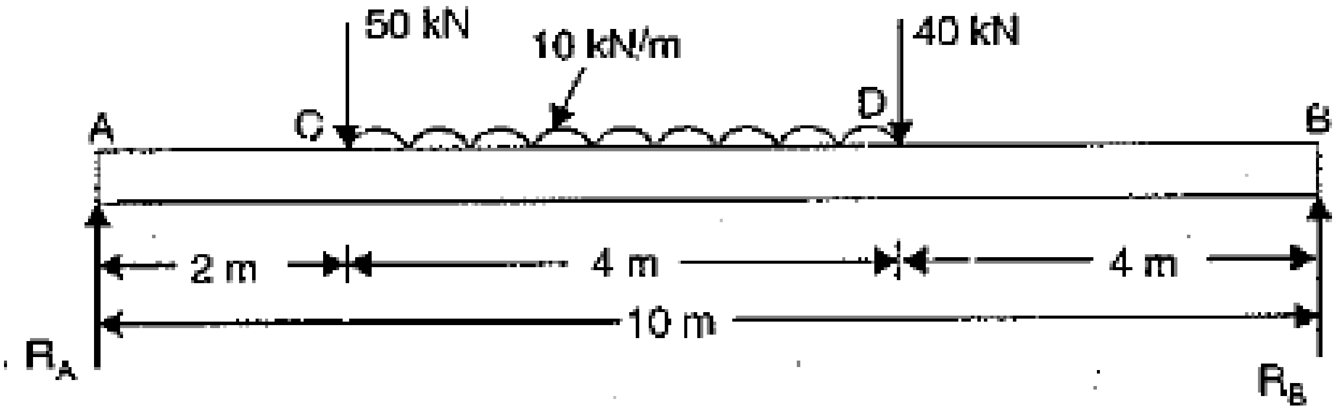
B.M. at D, $$M_D = 25(3 + 2) - 10 \times 3 \times \left(\frac{3}{2} + 2\right) = 125 - 105 = 20 \text{ kNm}$$

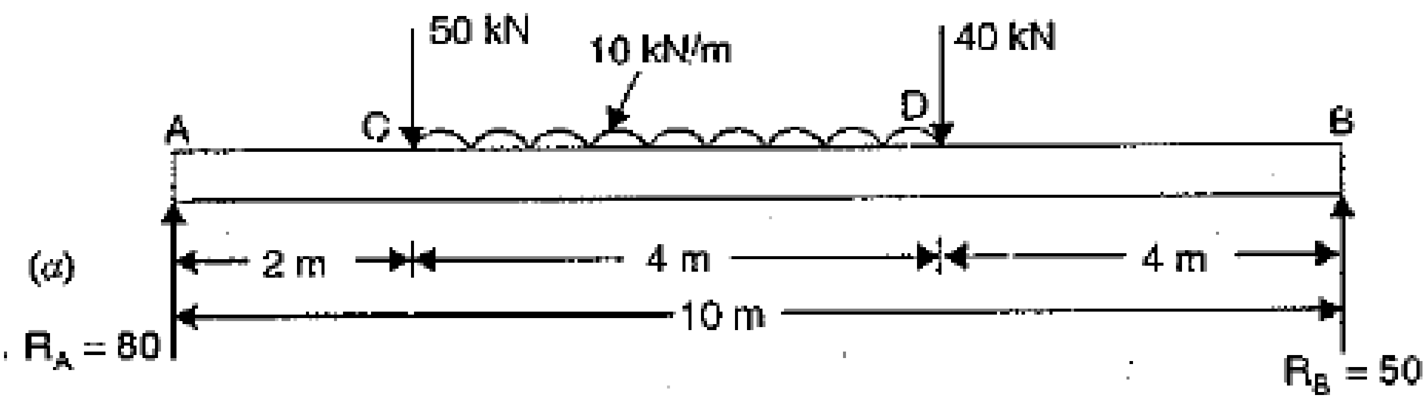


B.M. diagram


10 kN/m
5 kN/m
A
C
D
B
3 m
2 m
2 m
$R_A = 25$
$R_B = 15$
(a)
(b)
25
+
E
C
5
D
B
A
15
−
S.F. diagram
(c)
31.25
30
20
A
E
C
D
B
B.M. diagram

- Draw SFD and BMD. Also calculate maximum bending moment





Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40(2 + 4)$$

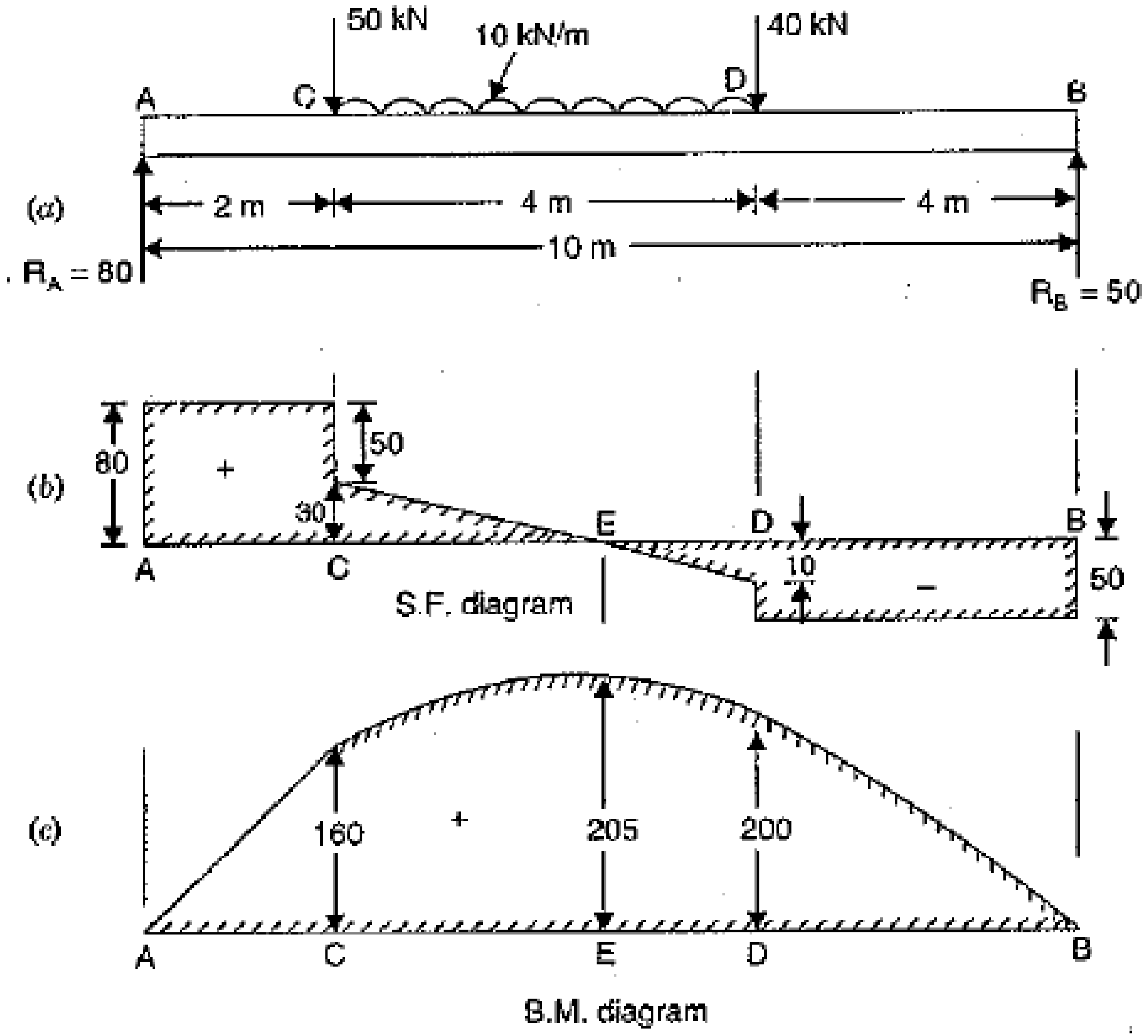
$$= 100 + 160 + 240 = 500$$

$$\therefore \quad R_B = \frac{500}{10} = 50 \text{ kN}$$

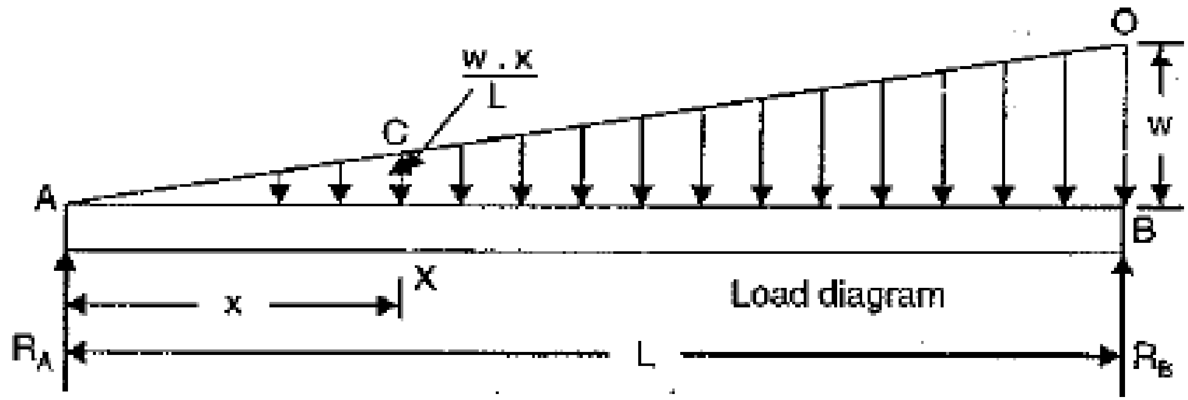
and

$$R_A = \text{Total load on beam} - R_B$$

$$= (50 + 10 \times 4 + 40) - 50 = 130 - 50 = 80 \text{ kN}$$


50 kN
10 kN/m
40 kN
A
C
D
B
(a)
2 m
4 m
4 m
10 m
$R_A = 80$
$R_B = 50$
(b)
80
+
50
30
E
D
B
10
−
50
A
C
S.F. diagram
(c)
160
+
205
200
A
C
E
D
B
B.M. diagram

SHEAR FORCE AND B.M. DIAGRAMS FOR A SIMPLY SUPPORTED BEAM CARRYING A UNIFORMLY VARYING LOAD FROM ZERO AT ONE END TO w PER UNIT LENGTH AT THE OTHER END



* Reaction forces:

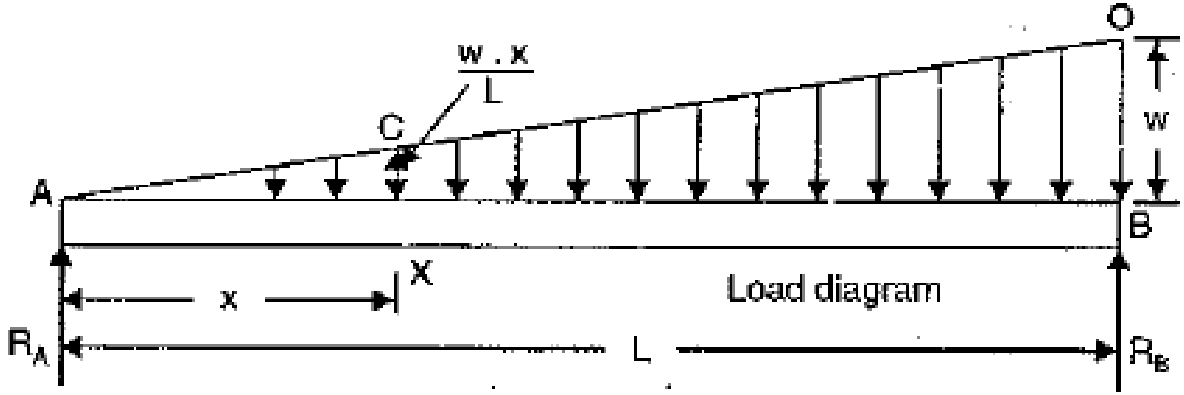


Fig. shows a beam AB of length L simply supported at the ends A and B and carrying a uniformly varying load from zero at end A to w per unit length at B. First calculate the reactions R_A and R_B.

Taking moments about A, we get

$$R_B \times L = \left(\frac{w.L}{2}\right).\frac{2}{3}L \qquad \left[\text{Total load}\left(=\frac{w.L}{2}\right)\text{is acting } \frac{2}{3}L \text{ from } A\right]$$

$\therefore$ $$R_B = \frac{w.L}{3}$$

and $$R_A = \text{Total load on beam} - R_B = \frac{w.L}{2} - \frac{w.L}{3} = \frac{w.L}{6}$$

- Calculation of Shear Force

Consider any section X at a distance x from end A. The shear force at X is given by,

$$F_x = R_A - \text{load on length } AX = \frac{w.L}{6} - \frac{w.x}{L}.\frac{x}{2}$$

$$\left(\text{Load on } AX = \frac{AX.CX}{2} = \frac{x.w.x}{2.L}\right)$$

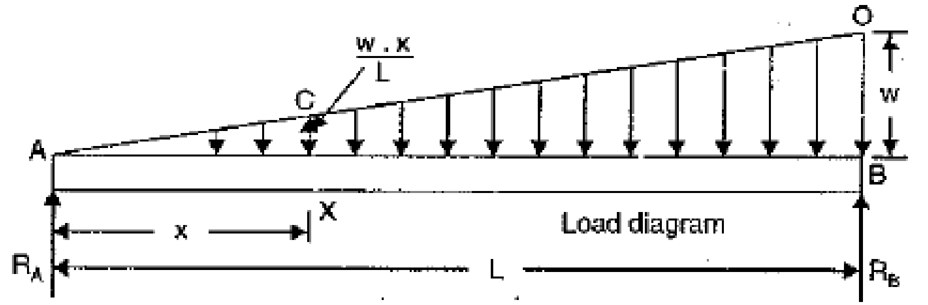
$$= \frac{wL}{6} - \frac{wx^2}{2L} \quad ...(i)$$

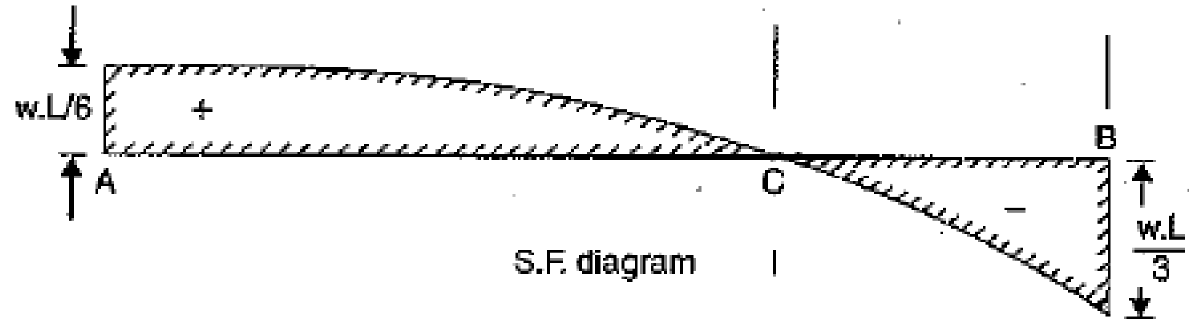
Equation (i) shows that S.F. varies according to parabolic law.

At A, $x = 0$ hence, $F_A = \frac{w.L}{6} - \frac{w}{2L} \times 0 = \frac{w.L}{6}$

At B, $x = L$, hence, $F_B = \frac{w.L}{6} - \frac{w.L^2}{2L} = \frac{w.L}{6} - \frac{w.L}{2} = \frac{w.L - 3w.L}{6} = -\frac{2w.L}{6} = -\frac{w.L}{3}$

The shear force is $+\frac{w.L}{6}$ at A and it decreases to $-\frac{w.L}{3}$ at B according to parabolic law.





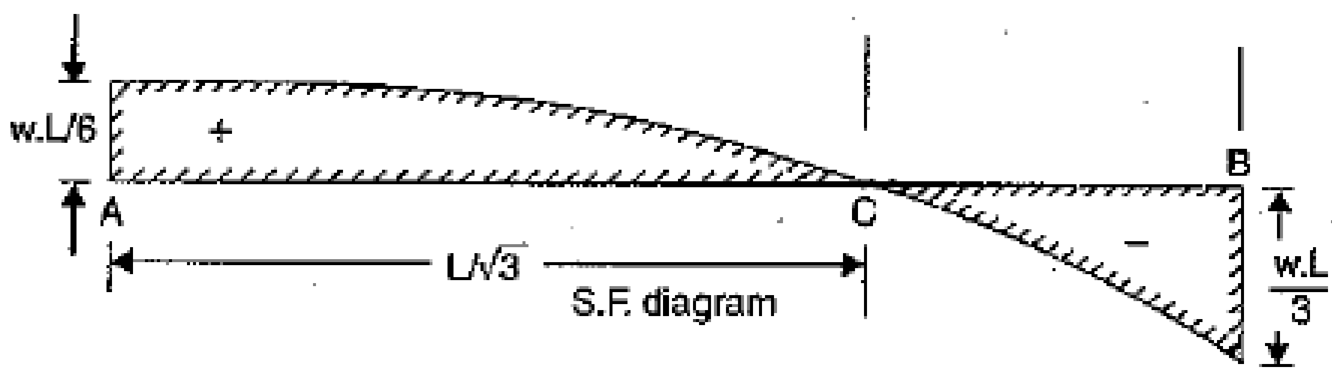
Somewhere between A and B, the S.F. must be zero. Let the S.F. be zero to a distance x from A. Equating the S.F. to zero in equation (i), we get

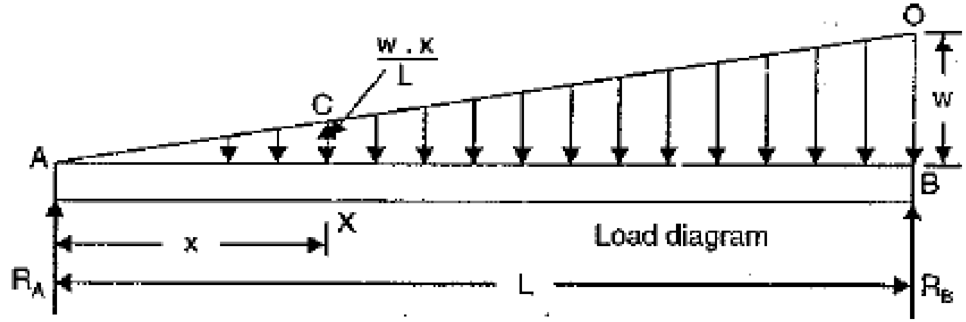
$$0 = \frac{wL}{6} - \frac{wx^2}{2L} \qquad \text{or} \qquad \frac{wx^2}{2L} = \frac{w.L}{6}$$

or

$$x^2 = \frac{w.L}{6} \times \frac{2L}{w} = \frac{L^2}{3}$$

$$\therefore \qquad x = \frac{L}{\sqrt{3}} = 0.577\,L$$





B.M. Diagram

The B.M. is zero at A and B.

The B.M. at the section X at a distance x from the end A is given by,

$$M_x = R_A.x - \text{Load on length } AX \cdot \frac{x}{3} \qquad \left(\because \text{ Load on } AX \text{ is acting at } \frac{x}{3} \text{ from } X\right)$$

$$= \frac{w.L}{6}.x - \frac{wx^2}{2L}.\frac{x}{3} = \frac{wL}{6}.x - \frac{wx^3}{6L}$$

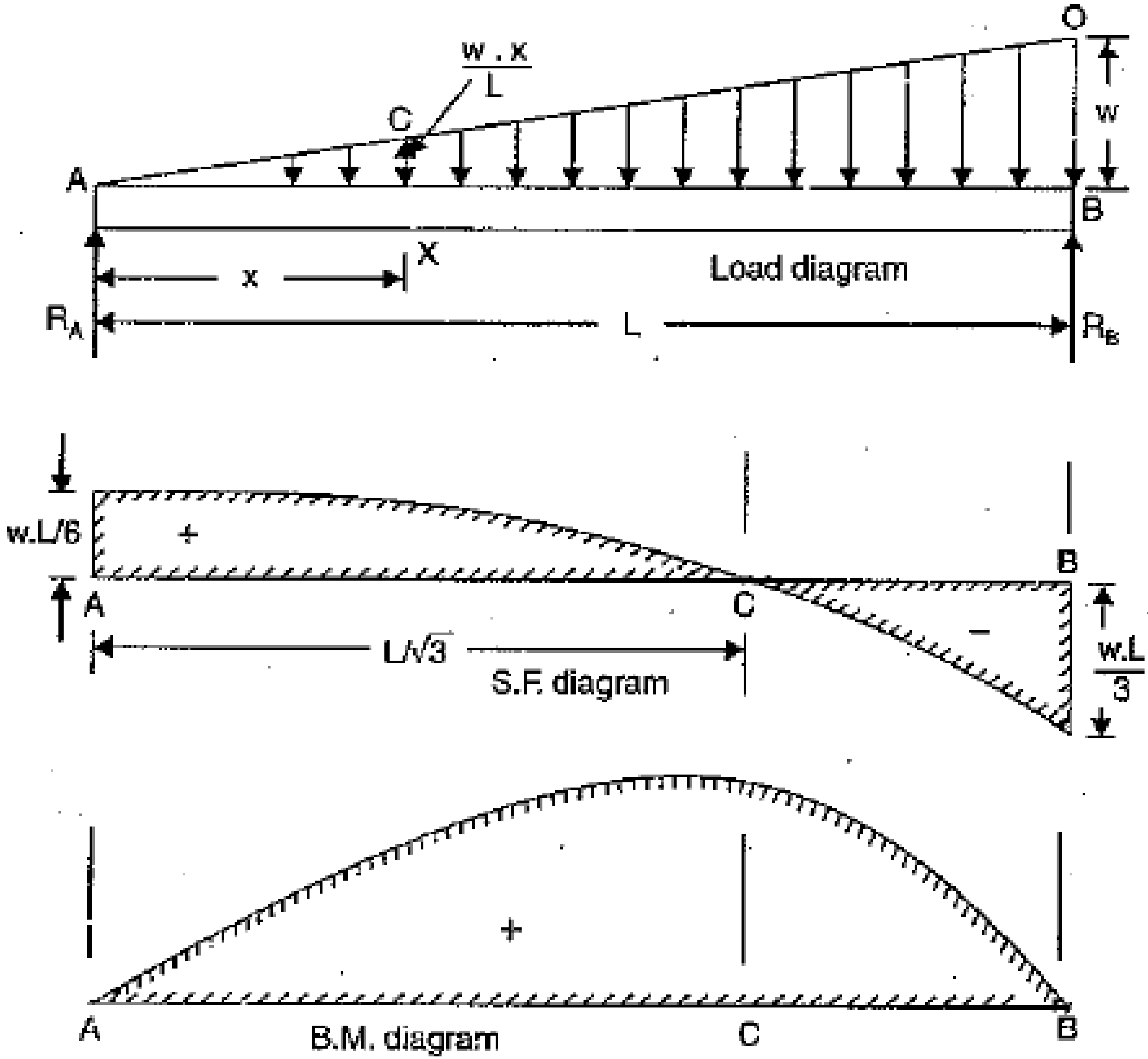
Equation (*ii*) shows the B.M. varies between A and B according to cubic law.

Max. B.M. occurs at a point where S.F. becomes zero after changing its sign.

That point is at a distance of $\frac{L}{\sqrt{3}}$ from A. Hence substituting $x = \frac{L}{\sqrt{3}}$ in equation (*ii*), we get maximum B.M.

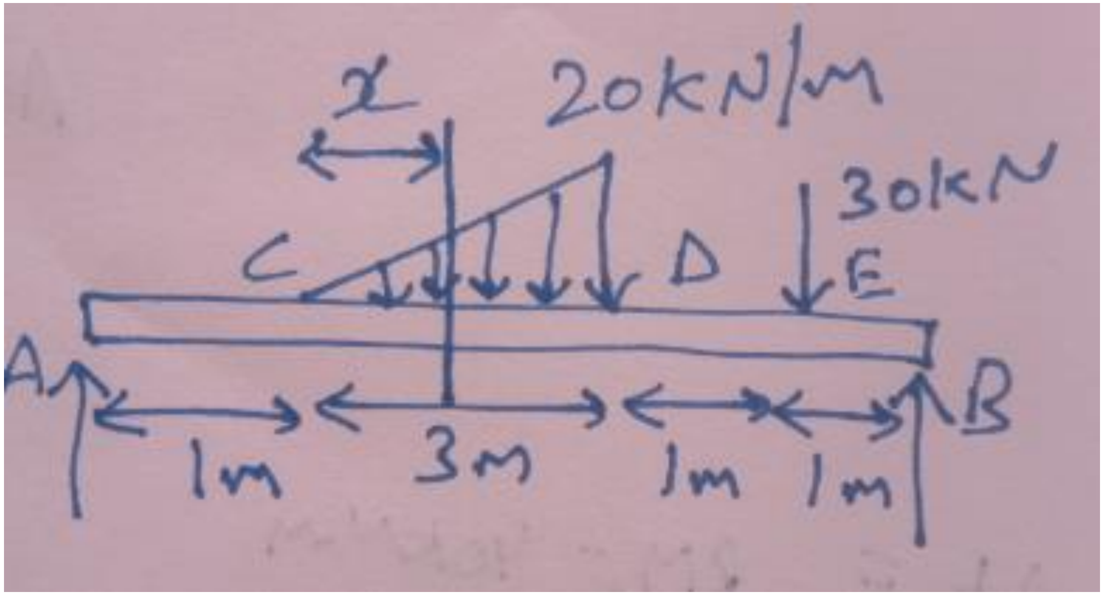
$$\therefore \qquad \text{Max. B.M.} = \frac{w.L}{6}.\frac{L}{\sqrt{3}} - \frac{w}{6L}.\left(\frac{L}{\sqrt{3}}\right)^3$$

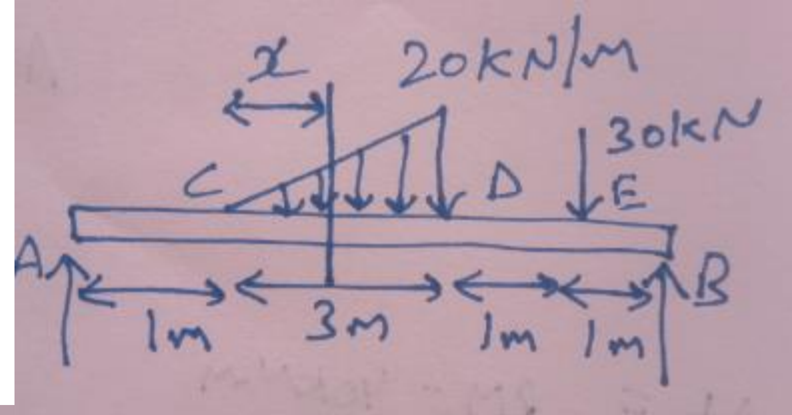
$$= \frac{wL^2}{6\sqrt{3}} - \frac{wL^2}{18\sqrt{3}} = \frac{3w.L^2 - wL^2}{18\sqrt{3}} = \frac{wL^2}{9\sqrt{3}}.$$



w . x
L
C
O
w
A
B
X
x
Load diagram
R_A
L
R_B
w.L/6
+
B
A
C
−
w.L
3
L/√3
S.F. diagram
+
A
B.M. diagram
C
B

- Draw SFD and BMD for the following loading conditions. Also calculate the maximum bending moment





$$R_A + R_B = \frac{1}{2} \times 20 \times 3 + 30$$

$$= 30 + 30 = 60\,kN$$

Take moments about A,

$$R_B \times 6 = 30 \times 5 + \frac{1}{2}(3 \times 20) \times \left(\frac{2}{3} \times 3 + 1\right)$$

$R_B = 40\,kN$ $\quad$ $R_A = 20\,kN$