Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The degree of the polynomial $(x^2 + x - 2) / (x - 1)$ is

a) 0 b) 1

c) 2 d) 3.

ii) If G be a group and $a, b \in G$. Then $\left(a^{-1} b \right)^{-1}$ is equal to

a) ab^{-1} b) $b^{-1}a$

c) $a^{-1} b^{-1}$ d) $b^{-1}a^{-1}$.

iii) $\frac{\partial}{\partial x}\left(x^{y}\right) =$

a) 1   b) yx^{y}

c) $x^{y} \log x$ d) yx^{y-1}.

iv) If $P = \{2, 4, 6, 7, 8, 9\}$, $Q = \{1, 2, 6, 9\}$ then $P \cap Q$ is

a) $\{1, 2, 6\}$ b) $\{2, 6, 9\}$

c) $\{1, 6, 9\}$ d) $\{4, 6, 9\}$.

v) The value of $\underset{x \to 3}{Lt} \frac{x^{3} - 3^{3}}{x - 3}$ is

a) -12 b) 12

c) 27   d) -27.

vi) If A be a matrix whose inverse exists then which of the following is not true ?

a) $\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$

b) $A^{-1} = (\det(A))^{-1}$

c) $\left(A^{2}\right)^{-1} = \left(A^{-1}\right)^{2}$

d) none of these.

vii) The equation $x^4 + 2x^2 - 7x - 5 = 0$ has

a) one real roots and three complex roots

b) one complex roots and three real roots

c) two real roots and two complex roots

d) four real roots.

viii) Cardan's method is used for solving equation of degree

a) 2 b) 3

c) 4 d) none of these.

ix) If α, β, γ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha\beta$ is

a) 3 b) 6

c) 2 d) none of these.

x) $f(x, y) = \sqrt{x} + \sqrt{y}$ is a function of degree

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) 0 d) $\frac{1}{4}$.

xi) The equation $r = 3 \sin \theta + 4 \cos \theta$ represents

a) a parabola
b) an ellipse
c) a straight line
d) a circle.

xii) The inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 2 \\ -\frac{3}{2} & 3 \end{bmatrix}$
c) $\begin{bmatrix} -2 & 4 \\ -3 & 6 \end{bmatrix}$
d) does not exist.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

3. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$, two of whose roots are in the ratio 3 : 2.

4. Prove that, any square matrix can be expressed assume of a symmetric matrix and a skew-symmetric matrix.

5. If $u = \tan^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u.$$

6. A function $f(x)$ is defined as follows

$$f(x) = 1 + x \text{ when } x \le 2,$$

$$= 5 - x \text{ when } x > 2.$$

Show that $f(x)$ is continuous at $x = 2$ but $f'(2)$ does not exist.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) State Descart's rule of sign. Using this rule find the nature of the roots of the equation

$$x^4 - 7x^3 + 21x^2 - 9x + 21 = 0.$$

b) Solve the following system of linear equations by Cramer's rule

$$x - y + 2z = 1$$

$$x + y + z = 2$$

$$2x - y + z = 5.$$

c) If by a transformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, Prove that $a^2 + b^2 = a'^2 + b'^2$.

8. a) Show that the equation $20x^2 + 15xy + 9x + 3y + 1 = 0$ represents a pair of intersecting straight lines which are equidistant from the origin.

b) Show that $\cos x > 1 - \frac{x^2}{2}$ if $0 < x < \frac{\pi}{2}$.

c) If α, β, γ be the roots of the equation

$x^3 - px^2 + qx - r = 0$, then find the value of $\sum \frac{1}{\alpha}$.

9. a) If $A = \{ a, b, c, d, e \}$, $B = \{ c, a, e, g \}$ and $C = \{ b, e, f, g \}$,

then show that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

b) Reduce the following equation to the canonical form and determine the nature of the conic represented by it

$x^2 - 4xy + 4y^2 - 12x - 6y - 39 = 0$.

c) Evaluate $\lim_{x \to 1} \left(\frac{x}{x - 1} - \frac{1}{\log x} \right)$.

10. a) Evaluate $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$.

b) If PSQ be a focal chord of a conic with focus S and semi-latus rectum l, then prove that $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$.

c) If $A - 2B = \begin{bmatrix} 0 & 6 & 26 \\ 6 & -9 & 12 \\ 2 & 9 & -10 \end{bmatrix}$ and

$2A + B = \begin{bmatrix} 10 & -3 & 4 \\ 12 & -3 & 4 \\ 4 & 3 & 0 \end{bmatrix}$, find A and B.

11. a) If G be a group such that $(ab)^2 = a^2b^2 \;\forall\; a, b \in G$, show that the group G is abelian.

b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx = \frac{\pi}{8} \log 2$.

c) If $y = e^{-x} \sin x$, then show that $y_4 + 4y = 0$.