



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.Tech(New)/CSE/IT/SEM-4/M-401/2013**

**2013**

**MATHEMATICS-III**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP - A**

**( Multiple Choice Type Questions )**

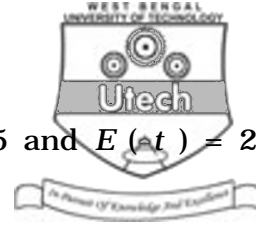
1. Choose the correct alternatives for any *ten* of the following :  
 $10 \times 1 = 10$

i) The number of generations of an infinite cyclic group is

- a) 1
- b) 2
- c) infinite
- d) none of these.

ii) The standard deviation of a sample mean for SRSWR is

- a)  $\sigma^2/n$
- b)  $\sigma/\sqrt{n}$
- c)  $\sigma/n$
- d)  $n$ .



iii) If  $t$  is a statistic such as  $E(t^2) = 5$  and  $E(t) = 2$ , then the standard error of  $t$  is

- a) 0
- b) 1
- c) 2
- d) none of these.

iv) If the exponential distribution is given by the probability density function  $f(x) = e^{-x}$ ,  $0 < x < \infty$ , then the mean of the distribution is

- a) 1
- b) 3
- c)  $\frac{1}{3}$
- d) none of these.

v) The probability of an event  $A$  is  $\frac{1}{3}$ , that of  $A + B$  is  $\frac{1}{2}$  and that of  $AB$  is  $\frac{1}{4}$ . Then the probability of  $B$  is

- a)  $\frac{1}{12}$
- b)  $\frac{5}{12}$
- c)  $\frac{1}{6}$
- d) none of these.

vi) Which one of the following sets forms a group under usual multiplication of complex numbers ?

- a)  $\{1, i\}$
- b)  $\{1, \omega, \omega^2\}$
- c)  $\{1, \omega^2\}$
- d)  $\{1, \omega\}$ .



vii) The distribution for which the mean and variance are equal is

- a) Poisson                      b) normal  
c) binomial                      d) exponential.

viii) In a Binomial (  $n, p$  ) distribution, if its mean and variance are 2 and  $\frac{4}{3}$  respectively, then the values of  $n$  and  $p$  are

- a)  $8, \frac{1}{4}$                           b)  $6, \frac{1}{3}$   
c)  $4, \frac{1}{2}$                           d) none of these.

ix) If  $G$  is a connected planar graph with  $n$  vertices,  $e$  edges and  $f$  faces, then  $n - e + f = 2$ . This statement is

- a) True                              b) False.

x) The mean of Binomial variate is

- a)  $np$                               b)  $np(1 - p)$   
c)  $\sqrt{np}$                             d) none of these.



xi) Kuratowski's graph is a

- a) planar graph
- b) regular graph
- c) tree
- d) none of these.

xii) The order of the dihedral group  $D_4$  is

- a) 4
- b) 6
- c) 8
- d) 64.

xiii) Every finite integral domain is a field. This statement is

- a) True
- b) False.

xiv) If  $A$  and  $B$  are two subgroups of a group  $G$ , then which of the following is always a subgroup of  $G$ ?

- a)  $A \cup B$
- b)  $G - A$
- c)  $G - B$
- d)  $A \cap B$ .

xv) The symmetric group  $S_3$  has

- a) 6 elements
- b) 8 elements
- c) 9 elements
- d) none of these.



**GROUP - B**  
**( Short Answer Type Questions )**

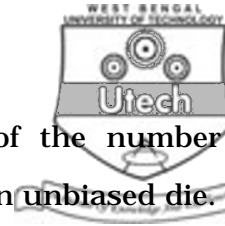
Answer any *three* of the following.  $3 \times 5 = 15$

2. Let  $( Q, + )$  be the additive group of rational numbers and  $( Q^+, \cdot )$  be the multiplicative group of positive rational numbers. Are these two groups isomorphic ? Justify your answer.
3. Prove Baye's theorem for repeated trials.
4. Examine whether function  $|x|$  in  $( - 1, 1 )$  and zero elsewhere is a density function.
5. Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.
6. Show that a field does not contain any zero divisor.

**GROUP - C**  
**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

7. a) Prove that a planar graph with  $n$  vertices,  $e$  number of edges and  $k$  number of components determines  $f$  number of regions, where  $f = e - n + k + 1$ . 8
- b) Let  $\bar{X}$  be the sample mean of samples of size  $n$  drawn at random from a population which is normally distributed with mean  $\mu$  and variance  $\sigma^2$ . Find the standard error of the statistic  $\bar{X}$ . 7



8. a) Find the mathematical expectation of the number of points obtained in a single throw of an unbiased die. 5
- b) Define Poisson distribution and find its mean and variance. 5
- c) Let  $f$  be a ring homomorphism from the ring  $Z$  of integers into itself such that  $f(1) = 1$ . Determine the homomorphism  $f$ . 5
9. a) Show that any simple connected planar graph with  $n$  vertices ( $n \geq 3$ ) has at most  $(3n - 6)$  edges. 7
- b) Prove that every nontrivial subgroup of the additive group  $Z$  of integers is cyclic. 4
- c) Let  $R$  and  $S$  be two rings and  $f : R \rightarrow S$  be a ring homomorphism. Show that kernel of  $f$  is a subring of  $R$ . 4
10. a) Determine the mean and variance of exponential distribution. 6
- b) Show that every cyclic group is commutative. 4
- c) Let  $H$  be a normal subgroup of a group  $G$  and  $G/H$  be the set of all cosets of  $H$  in  $G$ . Show that  $G/H$  forms a group under the composition

$$(aH) \cdot (bH) = (ab)H \text{ for all } a, b \in G. \quad 5$$



11. a) The probability density function of a random variable  $X$  is assumed to be of the form  $f(x) = cx^\alpha$ ,  $0 \leq x \leq 1$  for some number and constant  $c$ . If  $\{X_1, X_2, \dots, X_n\}$  is a random sample of size  $n$ , find the maximum likelihood estimate of  $\alpha$ . 5
- b) Let  $S'$  be the set defined by  $S' = \{z \in C : |z| = 1\}$ , where  $C$  is the set of all complex numbers. Show that  $S'$  forms a commutative group under usual multiplication of complex numbers. 5
- c) Let  $R$  be the additive group of real numbers and  $C^*$  be the multiplicative group of nonzero complex numbers. If  $f = R \rightarrow C^*$  is a group homomorphism defined by  $f(x) = e^{2\pi ix}$  for all  $x \in R$ , find the kernel of  $f$ . 5
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