BTS – I & II – 06.14 – 0405 Reg. No. | | | | | | | | |

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B.Tech Degree I & II Semester (Combined) Examination June 2014

IT/CS/CE/SE/ME/EE/EB/EC/EI/FT 1101 ENGINEERING MATHEMATICS I

(2012 Scheme)

Time: 3 Hours Maximum Marks: 100

PART – A

(Answer ***ALL*** questions)

(8 x 5 = 40)

I (a) Solve. $\frac{dy}{dx} = \cos(x+y+1)$.

(b) Solve $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$

(c) Test the convergence of the series $\sum \frac{n!2^n}{n^n}$.

(d) Define conditional convergence and absolute convergence of series with examples.

(e) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

(f) The torsional rigidity of a length of a wire is obtained from the formula $N = \frac{8\pi I l}{t^2 r^4}$. If l decreased by 2%, r is increased by 2%, t is increased by 1.5%, show that the value of N is diminished by 13% approximately.

(g) Find the entire length of the astroid $x = a\cos^3\theta, y = a\sin^3\theta$.

(h) Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy\, dx$.

PART – B

(4 x 15 = 60)

II. (a) Solve $(\sec x \tan x \tan y - e^x) + \sec x \sec^2 y\, dy = 0$. (5)

(b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$. (5)

(c) Solve $x^2 \frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$ (5)

OR

III (a) Solve $\frac{dx}{dt} + 2x - 3y = 5t, \frac{dy}{dt} - 3x + 2y = 2e^{2t}$. (8)

(b) An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and current i at any time t, given that at t = 0, $q = 0.05$ coulomb, $i = \frac{dq}{dt} = 0$ *when* $t = 0$. (7)

(P.T.O)

IV. (a) If $y = (x^2-1)^n$ Prove that $(x^2-1)Y_{n+2} + 2xY_{n+1} - n(n+1)Y_n = 0$ (8)

(b) State Leibnitz's test and test for convergence the series (7)

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \ldots\ldots..(0 < x < 1)$$

OR

V. (a) Examine the convergence of the series (5)

$$\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \ldots\ldots$$

(b) Find the interval of convergence of the series (5)

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$$

(c) Prove that log (secx) $= \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \ldots..$ (5)

VI. (a) State and prove Euler's theorem (5)

(b) If $X^x Y^y Z^z = c$ show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ (5)

(c) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighbourhood of $(1,1)$. (5)

OR

VII. (a) If $u = x + y + z$, $uv = y + z$ and $uvw = z$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$ (7)

(b) A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction. (8)

VIII. (a) Evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dy\, dx$ by changing the order of integration. (8)

(b) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (7)

OR

IX. (a) Find the surface area of the solid formed by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. (7)

(b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\cos\theta} \int_0^{\sqrt{a^2 - r^2}} r\, dz\, dr\, d\theta$. (8)

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