

21 : 1st half.12-AM(w)

Con. 3577-12.

GN-5543

(3 Hours)

[Total Marks : 100

- N.B. :** (1) Question No. 1 is compulsory.
 (2) Attempt any four questions out of remaining questions.

1. (a) Construct dual of the following LPP : 5

$$\text{Max } z = 8x_1 + 3x_2$$

$$\text{Subject to } x_1 - 6x_2 \geq 2$$

$$5x_1 + 7x_2 = -4, x_1, x_2 \geq 0$$
- (b) Find the orthogonal trajectory of the family of curves given by : 5

$$2x - x^3 + 3xy^2 = a$$
- (c) Evaluate $\int_c \frac{1}{z} dz$ where c is the upper half of $|z|=1$. 5
- (d) Show that every Skew-Hermitian matrix can be expressed in the form $P + iQ$ 5
 Where P is real Skew-Symmetric and Q is real Symmetric matrix.

2. (a) Determine the analytic function $f(z)$ where $u = -r^3 \sin 3\theta$ 6
- (b) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory 6
- (c) Use simplex method to solve the LPP $\text{Max } z = 3x_1 + 5x_2 + 4x_3$ subject to the 8
 constraints

$$2x_1 + 3x_2 \leq 8, \quad 2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15, \quad x_1, x_2, x_3 \geq 0$$

3. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$ using contour integration. 6
- (b) Find Eigen values and Eigen vectors of A^3 where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ 6
- (c) Use Penalty method (Big M) to solve Mini $z = 4x + y$ subject to $3x + y = 3,$ 8
 $4x + 3y \geq 6, \quad x + 2y \leq 4 \quad x, y \geq 0$

4. (a) If $w = f(z)$ then prove that $\frac{dw}{dz} = (\cos \theta - i \sin \theta) \frac{\partial w}{\partial r}$ 6

(b) If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 3a \\ 2 & 1 & 3b \\ 2 & -2 & 3c \end{bmatrix}$ is orthogonal find a, b, c . Also find A^{-1} 6

(c) Use Dual Simplex method to solve the LPP 8

Minimise $z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

subject to $5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$

$$x_2 + 5x_3 - 6x_4 \geq 10$$

$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

[TURN OVER

5. (a) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \cdot \tan A = A \cdot \tan 3A$ 6

(b) Find the image of the line $y - x + 1 = 0$ under the transformation $w = \frac{1}{z}$. Also find the image of the line $y = x$ under the same transformation. Draw rough sketches. 6

(c) Solve the NLPP using the method of Lagrangian Multipliers. 8

$$\text{Minimize } z = x_1^2 + x_2^2 + x_3^2$$

$$\text{subject to } x_1 + x_2 + 3x_3 = 2, \quad 5x_1 + 2x_2 + x_3 = 5,$$

$$x_1, x_2, x_3 \geq 0$$

6. (a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and hence find A^{-1} , $A^3 - 5A^2$ 6

(b) Find the bilinear transformation that maps the points $\infty, i, 0$ onto the points $0, i, \infty$. Find the fixed points. 6

(c) State and prove Cauchy's integral formula and hence evaluate $\int_c \cot z \, dz$ where 8

$$c \text{ is } \left| z + \frac{1}{z} \right| = \frac{1}{3}$$

7. (a) Obtain two Laurents series for : 6

$$\frac{(z-2)(z+2)}{(z+1)(z+4)}$$

(b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a \cos \theta + a^2} d\theta$ using residues. 6

(c) Solve the following N. L. P. P. using Kuhn-Tucker conditions 8

$$\text{Minimise } z = 7x_1^2 + 5x_2^2 - 6x_1$$

$$\text{Subject to } x_1 + 2x_2 \leq 10$$

$$x_1 + 3x_2 \leq 9$$

$$x_1, x_2 \geq 0$$