BTS – III– 11.13 – 1169

# B.Tech. Degree III Semester Examination November 2013

## EC 1302 PROBABILITY AND RANDOM PROCESS

*(2012 Scheme)*

Time : 3 Hours Maximum Marks : 100

**PART A**

(Answer ***ALL*** questions)

(8 x 5 = 40)

I. (a) If X is a Poisson variate such that $P(X=2)=9P(X=4)+90P(X=6)$, find the standard deviation.

(b) The joint density function of X and Y is given by

$$f(x,y)=\begin{cases} xe^{-x(y+1)} & ,x>0,y>0 \\ 0 & ,\text{elsewhere} \end{cases}$$

Are X and Y independent?

(c) Define Random Process. What is a wide sense stationary (WSS) process?

(d) If $X(t)=A+Bt$ where A and B are independent random variables with $E(A)=p$ and $E(B)=q$, $V(A)=\sigma_A^2$ and $V(B)=\sigma_B^2$, find auto correlation $R(t_1,t_2)$.

(e) Define:

(i) Power spectral density function

(ii) Cross power spectral density

(iii) Time average of random process

(f) The transition probability matrix (TPM) of the Markov Chain $\{x_n\}$, with $n = 1, 2, 3, \ldots$ having 3 states 1, 2, 3 is given by

$$P=\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$. Also given that

$P(X_0=1)=0.7$, $P(X_0=2)=0.2$ and $P(X_0=3)=0.1$.

Find $P(X_3=2, X_2=3, X_1=3, X_0=2)$.

(g) Examine whether the function $Y(t)=x^3(t)$ is linear or not.

(h) Define (i) shot noise (ii) white noise

**PART B**

(4 x 15 = 60)

II. (a) Define Binomial distribution. Find its mean and variance. (7)

(b) The following table represents the bivariate distribution of $(X,Y)$. Calculate $E(X)$, $E(Y)$ (8)

Covariance and correlation between $X$ and $Y$.

Also find $V(X/Y=1)$

| Y \ X | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0.01 | 0.01 | 0.02 |
| 1 | 0.22 | 0.10 | 0.43 |
| 2 | 0.07 | 0.08 | 0.06 |

**OR**

III. (a) Derive the mean and variance of Poisson distribution. (7)

(b) Let X be a random variable with probability density function (8)

$$(pdf)\ f(x)=\begin{cases} 2/3 \text{ when } x=1 \\ 1/3 \text{ when } x=2 \\ 0 \quad \text{elsewhere} \end{cases}$$

Find the moment generating function (mgf) and the central moments $\mu_2$ and $\mu_3$

(P.T.O.)

IV. (a) If $X(t) = A\cos t + B\sin t$ where A and B are independent random variables each of which assumes values $-1$ and 2 with probabilities 2/3 and 1/3 respectively. Show that $X(t)$ is a WSS process. (7)

(b) Prove that the sum of two independent Poisson processes is also a Poisson process. (8)

**OR**

V. (a) If $X(t)$ is a random process in which $C(\tau) = q\,e^{-\alpha|\tau|}$, show that $X(t)$ is mean ergodic. (7)

(b) Show that the process of $X(t)$ such that (8)

$$P\{X(t)=n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}} & ;\ n = 1.2.3,.... \\ \dfrac{at}{1+at} & ;\ n = 0 \text{ is evolutionary} \end{cases}$$

VI. (a) If $R(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} ; |\tau| < T \\ 0 \quad ; \text{otherwise,} \end{cases}$ find power spectral density (7)

(b) Let $X_n$ be a Markov Chain with 3 states 0, 1, 2 and the transition probability matrix (8)

(7PM) is $P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$. Also given that

$P[X_0 = i] = \frac{1}{3}; i = 0,1,2.$ Find

(i) $P[X_2 = 2, X_1 = 1 / X_0 = 2]$ (ii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$

(iii) $P[X_2 = 2, X_1 = 1, X_0 = 2]$ (iv) $P[X_2 = 2 / X_1 = 1]$

**OR**

VII. (a) If the TPM of a Markov Chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the steady-state distribution of the chain. (6)

(b) A fair die is tossed repeatedly. If $X(n)$ denote the maximum of the numbers occurring in the first $n$ tosses, find the probability matrix $P$ of the Markov Chain $\{X_n\}$, $P(X_2 = 6)$ and $P^2$. (9)

VIII. (a) If a system is such that its input $X(t)$ and its output $Y(t)$ are related by a convolution integral $Y(t) = \int_{-\infty}^{\infty} h(u) \times (t-u)\,du$, $h(t)$ being a unit impulse function, show that it is a linear time invariant system. (8)

(b) A linear time invariant (LTI) system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{yy}(\tau)$ corresponding to an input $X(t)$. (7)

**OR**

IX. (a) Show that if the input to a time invariant stable linear system is a wide sense stationary (WSS) process, then the output *Y(t)* given by (7)

$y(t) = \int_{-\infty}^{\infty} h(u) \times (t-u)\,du$ where $h(t)$ is a unit impulse function is also a WSS process.

(b) If $\{N(t)\}$ is a band limit white noise such that the power spectral density $S_{NN}(w)$ is given by $S_{NN}(w) = \begin{cases} \dfrac{No}{2}\ for\ |w| < w_B, \\ 0 \text{ elsewhere} \end{cases}$ find the auto correlation function. (8)

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