

(3 Hours)

[Total Marks :100

- N.B.** (1) Question No.1 is compulsory.
 (2) Attempt any **four** from the remaining six questions.
 (3) **Figures to the right** indicates full marks.

1. (a) A random variable x has the probability function : 5

$$\begin{array}{cccccc} X : & -2 & -1 & 0 & 1 & 2 & 3 \\ P(X=x): & 0.1 & k & 0.2 & 2k & 0.3 & 3k \end{array}$$
 Find (i) k (ii) $P(x \leq 1)$ (iii) $P(-2 < x < 1)$
 (iv) Obtain the distribution function of X
- (b) In the set of natural numbers, prove that the relation xRy if and only if $x^2 - 4xy + 3y^2 = 0$, is reflexive, but neither symmetric nor transitive. 5
- (c) Find the characteristic roots of $A^{30} - 9A^{28}$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ 5
- (d) Find Laurent's series for $f(z) = (z-3)\sin\left(\frac{1}{z+2}\right)$ about $z = -2$. 5
2. (a) If X, Y are independent Poisson variates such that $P(X=1) = P(X=2)$ and $P(Y=2) = P(Y=3)$ find the variance of $2X - 3Y$. 7
- (b) Find the residues of $f(z) = \frac{\sin \pi z}{(z-1)^2(z-2)}$ at its poles. 7
- (c) If $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$ find $\cos A$ 6
3. (a) Check whether $A = \{2, 4, 12, 16\}$ and $B = \{3, 4, 12, 24\}$ are lattices under divisibility? 7
 Draw their Hasse diagrams.
- (b) Nine items of a sample had the following values.
 45, 47, 50, 52, 48, 47, 49, 53, 51
 Does the mean of 9 items differ significantly from the assumed population mean 47.5 ?

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- (c) Find the characteristic equation of the matrix A and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ 6

$$\text{where } A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

4. (a) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 7

- (b) Let $S = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R} \right\}$ and + and \cdot be matrix addition and matrix multiplication. Is $(S, +, \cdot)$ an integral domain? Is it a field? 7

- (c) Show that $\int_C \frac{dz}{z+1} = 2\pi i$, where C is the circle $|z| = 2$. Hence deduce that 6

$$\int_C \frac{(x+1)dx + ydy}{(x+1)^2 + y^2} = 0 \quad \text{and} \quad \int_C \frac{(x+1)dy - ydx}{(x+1)^2 + y^2} = 2\pi$$

5. (a) The number of defects in printed circuit board is hypothesised to follow Poisson distribution. A random sample of 60 printed boards showed the following data. 7

Number of defects : 0 1 2 3

Observed frequency : 32 15 9 4

Does the hypothesis of Poisson distribution seem appropriate? 7

- (b) If f and g are defined as 7
 $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x - 3$
 $g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = 4 - 3x$
 (i) Verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.
 (ii) Solve $f \circ g(x) = g \circ f(1)$

- (c) For a distribution the mean is 10, variance is 16, γ_1 is 1 and β_2 is 4. Find the first four moments about the origin. Comment on the nature of the distribution. 6

6. (a) Prove that the set $A = \{0, 1, 2, 3, 4, 5\}$ is a finite Abelian group under addition modulo 6. 7

- (b) If $f(\xi) = \int_C \frac{3z^2 + 2z + 1}{z - \xi} dz$ where C is the circle $x^2 + y^2 = 4$ find the values of 7

(i) $f(3)$, (ii) $f'(1 - i)$, (iii) $f''(1 - i)$

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(C) A manufacturer known from his experience that the resistance of resistors he produces is normal with $\mu = 100$ ohms and standard deviation $\sigma = 2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms ? 6

7. (a) By using residue theorem evaluate $\oint_C \frac{\sin^6 z}{(z - \pi/6)^3} dz$ where C is $|z|=1$ 7

(b) The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is $1/4$. What is the probability of 4 successes in 6 independent trials ? 7

(c) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1/2 & 2 \end{bmatrix}$, prove that both A and B are not diagonalisable 6

but AB is diagonalisable.
