BTS – IV– 04.14 – 0067

A

B.Tech. Degree IV Semester Examination April 2014

IT/CS/CE/SE/ME/EE/EB/EC/EI/FT 1401 ENGINEERING MATHEMATICS III

(2012 Scheme)

Time : 3 Hours — Maximum Marks : 100

PART A

(Answer *ALL* questions)

(8 × 5 = 40)

I. (a) $f(z)=z^n, n$ any positive integer, show that $f(z)$ is analytic and hence find its derivative.

(b) Show that the transformation $w=z^2$ maps the circle $|z-1|=1$ into the cardioid $\rho=2(1+\cos\phi)$.

(c) Find the poles and residues of $f(z)=\dfrac{z^2+z+1}{(z-1)^2(z+2)}$.

(d) Evaluate $\int_c \tan z\,dz$, c is the circle $|z|=2$.

(e) Form the partial differential equation by eliminating the function $f\left(x+y+z, x^2+y^2+z^2\right)=0$.

(f) Solve the partial differential equation $y^2p-xyq=x(z-2y)$.

(g) Derive one dimensional heat equation.

(h) Using method of separation of variables solve $\dfrac{\partial u}{\partial x}=2\dfrac{\partial u}{\partial t}+u,\ u(x,0)=6e^{-3x}$.

PART B

(4 × 15 = 60)

II. (a) If $f(z)$ is analytic show that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)\log|f(z)|=0$ (7)

(b) Show that $V=e^x(x\cos y-y\sin y)$ is harmonic function. Find the analytic function $f(z)$ for which V is the imaginary part. (8)

OR

III. (a) Find the image of the region bounded by the lines $x-y<z$ and $x+y>z$ under the mapping $w=1/z$. (6)

(b) Find the bilinear transformation which maps the points $z=\infty, i, o$ onto $w=o, i, \infty$. (4)

(c) Explain the transformation $w=\sin z$. (5)

(P.T.O.)

IV. (a) Evaluate $\int_c \frac{z}{z^2-1}\,dz$, where c is (9)

(i) $|z| = 1/2$

(ii) $|z-1| = 1/2$

(iii) $|z| = 2$

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$ using contour integration. (6)

OR

V. (a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ as Laurent's series in (9)

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $0 < |z-1| < 1$

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ using contour integration. (6)

VI. Solve (3x5=15)

(i) $x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$

(ii) $p^2 + q^2 = x + y$

(iii) $z = \frac{1}{p} + \frac{1}{q}$

OR

VII. Solve (3x5=15)

(i) $(D^3 - 2D^2D')z = e^{2x+y}$

(ii) $\left(D^3 - 4D^2D' + 4DD'^2\right)z = \sin(3x+2y)$

(iii) $x^2p^2 + y^2q^2 = z^2$

VIII. (a) Derive D-Alembert's solution of one dimensional wave equation. (5)

(b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$ (10)

OR

IX. (a) Obtain the solution of 2 dimensional Laplace equation. (5)

(b) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t. (10)

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