

UNIT-II: Beams:

Flexural Behavior of Beams which do not undergo lateral buckling:-

i) Elastic Behavior:- when the strains in the extreme fibres $\epsilon_{max} < \epsilon_y$ i.e. $f_{max} < f_y$ the behavior of the beam is elastic. The stress and strain vary linearly across the depth of the beam. Therefore the moment of resistance of the section is $M = f_{max} \cdot Z_e$

where f_{max} = Maximum stress,

Z_e = Elastic section modulus

$$Z_e = \frac{I}{y_{max}}$$

ii) Elastic/plastic Behavior:- when $\epsilon_{max} = \epsilon_y$ i.e. $f_{max} = f_y$ the yielding of the extreme fibres takes place, whereas the corner fibres of the beam remains elastic. The corresponding moment of resistance is called yield moment (M_y) and is given by

$$M_y = f_y \times Z_e$$

When $\epsilon_{max} > \epsilon_y$ there is no increase in the value of f_{max} which is limited to f_y . In this case the moment of resistance of resistance is obtained by considering both the yielded position and cross section becomes plastic.

At this stage further deformation is possible at a constant moment and it is called the formation of plastic hinge.

$$M_p = f_y (A_c y_c + A_t y_t)$$

$$= f_y \frac{A}{2} (y_c + y_t)$$

$$M_y = f_y Z_p$$

$$Z_p = \frac{M_p}{f_y}$$

1. Sectional properties of Rolled Steel sections:-

Rolled steel sections of different shapes and sizes are readily available in market for designing simple ~~beam~~ beams. Using such readily available rolled sections, their geometrical properties such as depth and width of section, thickness and height of web, thickness of flange, position of centroid, moment of inertia, plastic & elastic section Moduli values, etc., are required. IS 800 and its tables provided those values for all standard sections. Table 4.6 of IS 800-2007 provides few important geometrical and physical properties of all I & C sections.

Elastic section Modulus = Z_e

$$Z_e = \frac{M_y}{f_y}$$

plastic section Modulus (Z_p)

$$Z_p = \frac{M_p}{f_y}$$

Shear factor:- The ratio of plastic moment of resistance to elastic moment of resistance (or) the ratio of plastic section modulus to the elastic section modulus of the section is called shape factor.

$$\text{Shape factor} = \frac{I_p}{I_y} = \frac{Z_p}{Z_y}$$

Shape factor for different sections.

Rectangular section = 1.5

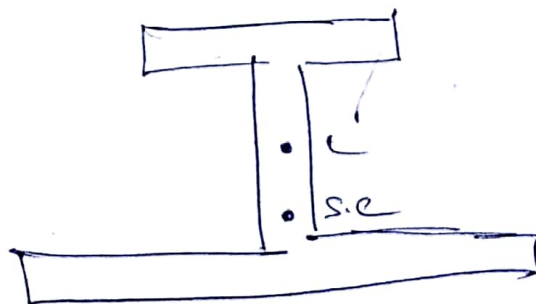
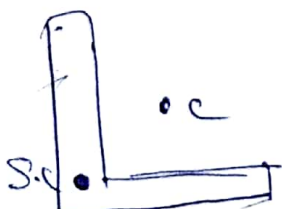
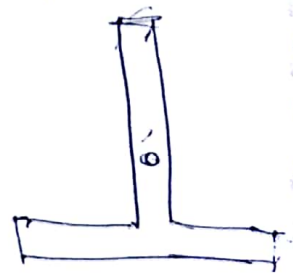
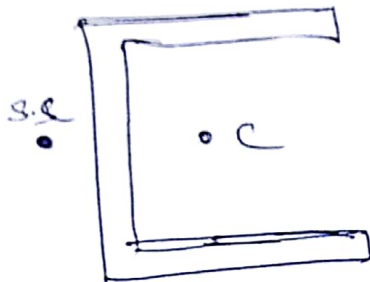
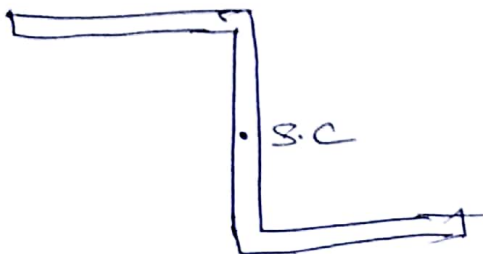
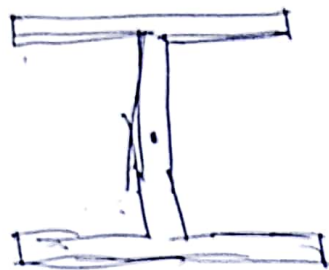
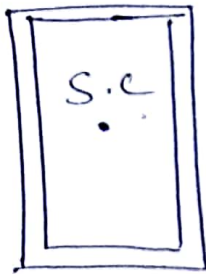
Circular section = 1.7

Shear - Centre and its significance

Shear centre is the point in the plane of cross section through which the resultant load has to pass so that the cross section is free from twisting.

Shear centre coincides with centroid of the cross section in certain sections.

Examples :-



For this case of laterally supported beams, load passing through the shear center may not matter because the compression flange is restrained.

For the case of laterally unsupported beams, if the load doesn't pass through the shear center it may destabilize (or) stabilize the lateral-torsional buckling.

Shear behaviour:-

$$\tau = \frac{VQ}{I \cdot tw}$$

τ = shear stress, V = shear force, tw = thickness of the web
 I = Moment of Inertia of the section about z-z axis
 Q = Moment of area above (or) below the depth where shear stress is calculated.

$$\tau_{av} = \frac{V}{dw \cdot tw}$$

$$\tau_y = \frac{f_y}{1.3} = 0.58 f_y$$

Plastic shear capacity of beam = V_p

$$V_p = 0.58 \cdot f_y \cdot dw \cdot tw$$

Design Strength of I in Bending as per IS-800-2007

Laterally supported beams -

i) When the factored design shear force does not exceed $0.6 V_d$, where V_d is the design shear strength of the cross section. The design bending strength M_d shall be taken as.

$$M_d = \beta_b \cdot \frac{Z_p f_y}{\gamma_{m0}}$$

$\beta_b = 1.0$ for plastic & compact sections

$\beta_b = \frac{Z_e}{Z_p}$ for semi compact sections

Z_p, Z_e = plastic & elastic section modulus

f_y = yield stress of the ~~factored~~ material.

γ_{m0} = P.S. Factor = 1.1

To avoid irreversible deformation under serviceability load, M_d shall be less than $1.2 \frac{Z_p f_y}{\gamma_{m0}}$ in case of simply supported & $1.5 \frac{Z_p f_y}{\gamma_{m0}}$ in cantilever beams.

Shear lag effect As per IS 800: 2007

In practice shear force influences the bending stress in the flanges and causes the section to warp. This results in non-uniform distribution of flexural stresses producing higher stresses near the junction of a web and lower

Stresses of points away from it. This effect is known as shear lag.

The shear lag effects in flanges may be disregarded provided,

a) For outstanding effects (supported along one edge)

$$b_o \leq \frac{l_o}{20} \text{ and } .$$

b) For internal elements (supported along two ~~by~~ edges)

$$b_i \leq \frac{l_o}{10}$$

Where l_o = length between points of zero moment in the span

b_o = width of the flange with outstand.

b_i = width of the flanges as an internal element.

Design strength in shear as per IS 800-2007

The factored design shear force, V in a beam due to external action shall satisfy,

$$V \leq V_d$$

where V_d = design strength

$$V_d = \frac{V_m}{\gamma_{m0}}$$

γ_{m0} = partial safety factor against shear failure

The nominal plastic shear resistance under pure shear

is given by $V_m = V_p$

$$\frac{A_m \cdot f_{yw}}{\sqrt{3}} = \frac{A_v \cdot f_{yw}}{\sqrt{3}}$$

A_v = shear area
 f_{yw} = yield strength of web

For rolled section.

$$A_m = htw$$

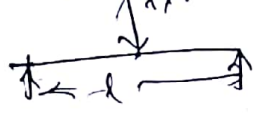
$$V_n = \frac{htw \cdot f_y}{\sqrt{3} \gamma_{m0}}$$

The design shear strength of beam

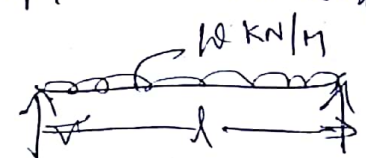
$$V_d = \frac{htw f_y}{\sqrt{3} \gamma_{m0}}$$

Maxi deflection in the Beams

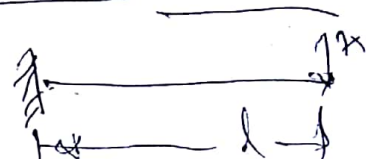
① Simply supported beam with central point load

$$S_{max} = \frac{Wl^3}{48EI}$$


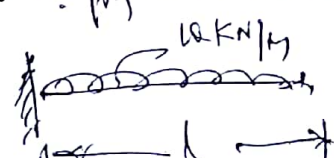
② S.S.B with u.d.l w kN/m over entire span

$$S_{max} = \frac{5wl^4}{384EI}$$


③ Cantilever with a point load at free end

$$S_{max} = \frac{Wl^3}{3EI}$$


④ Cantilever with u.d.l w kN/m over entire span

$$S_{max} = \frac{wl^4}{8EI}$$


Classification of Cross sections

- 1) Class - I (Plastic Cross-section)
- 2) class - II (Compact cross-section)
- 3) class - III (Semi Compact cross-section)
- 4) class - IV (Slender cross-section)

① For rolled steel section

$$\frac{b_f}{t_f} < 9.4 \epsilon \rightarrow \text{class I}$$

$$\frac{b}{t_f} < 10.5 \epsilon \rightarrow \text{class II}$$

where $b = \frac{b_f}{2}$

$$\frac{b}{t_f} < 15.7 \epsilon \rightarrow \text{class III}$$

$$\epsilon = \sqrt{\frac{250}{f_y}}$$

$$\epsilon = \sqrt{\frac{250}{250}}$$

$$\epsilon = \sqrt{1}$$

$$\epsilon = 1$$

②

$$\frac{d}{t_w} < 84 \epsilon \text{ class I}$$

$$\frac{d}{t_w} < 105 \epsilon \text{ class II}$$

$$\frac{d}{t_w} < 126 \epsilon \text{ class III}$$

where $\epsilon = \sqrt{\frac{250}{f_y}}$

b = clear distance b/w lateral supports

(if b is not taken as $\frac{b_f}{2}$ where b_f = width of flange)

t_f = thickness of flange

d = depth of web = $(h - 2t_f)$

t_w = thickness of web

Q An ISWB 300 @ 472 N/m acts as simply supported beam with an effective span of 6m. Calculate the design shear strength of beam. Assume Fe 250 grade steel and assume that the beam is laterally supported.

Solution. For ISWB 300 at 472 N/m.

$$h = 300 \text{ mm}$$

$$t_w = 7.4 \text{ mm}$$

$$f_y = 250 \text{ MPa}$$

$$\gamma_{m0} = 1.10$$

Design shear strength

$$V_d = \frac{V_m}{\gamma_{m0}}$$

$$V_d = \frac{A_v f_y}{\sqrt{3} \gamma_{m0}} = \frac{h \times t_w \cdot f_y}{\sqrt{3} \gamma_{m0}} \quad \left[\because A_v = h \times t_w \right]$$

$$V_d = \frac{300 \times 7.4 \times 250}{\sqrt{3} \times 1.10} = 291.299 \times 10^3 \text{ N}$$

$$V_d = 291.299 \text{ kN}$$

A simply supported beam ISMB 400 @ 604 N/m has an effective span of 5 mtrs. Find.

- 1) The design bending strength of beam
- 2) The " shear " " "
- 3) The intensity of UDL that the beam can carry under service conditions.

(a) The Maximum deflection:-

Assume that the beam is laterally supported and the grade of steel is E250

Solution:

Form steel tables For IS 400 @ 600 N/m

$$h = 600 \text{ mm}$$

$$I = 20458.4 \times 10^4 \text{ mm}^4$$

$$b_f = 140 \text{ mm}$$

$$t_f = 16 \text{ mm}$$

$$t_w = 8.9 \text{ mm}$$

$$Z_e = 1022.9 \times 10^3 \text{ mm}^3$$

$$Z_p = 1176.2 \times 10^3 \text{ mm}^3$$

① Section classification.

$$C = \sqrt{\frac{250}{t_f}} = \sqrt{\frac{250}{16}} = 1$$

$$\frac{b}{t_f} = \frac{140}{16} = 4.375 < 9.14 C$$

$$\frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{400 - 2(16)}{8.9} = 41.35 < 84 C$$

Hence the section is classified as class I (compact)

② Design bending strength of beam.

For plastic section $\beta_b = 1.0$

$$M_d = \frac{\beta_b \times Z_p \times f_y}{\gamma_{m0}} = \frac{1 \times 1176.2 \times 10^3 \times 250}{1.10} = 267.32 \text{ kN-m}$$

For S.S beam, M_d shall be less than $\frac{1.2 \times Z_e \times f_y}{\gamma_{m0}}$

$$= \frac{1.2 \times 1022.9 \times 10^3 \times 250}{1.10} = 278.97 \text{ kN-m}$$
$$= 278.97 \times 10^6 \text{ N-mm}$$

B.M for S.S beam = 278.87 > $M_d = 267.32 \text{ kN-m}$

③ Design Shear strength of beam

$$V_d = \frac{V_m}{\gamma_{mo}} = \frac{A_v \sigma_{ty}}{\sqrt{3} \cdot \gamma_{mo}}$$

$$V_d = \frac{n \cdot t_w \cdot \sigma_{ty}}{\sqrt{3} \cdot \gamma_{mo}} = \frac{400 \times 8.9 \times 250}{\sqrt{3} \times 1.1}$$

$$V_d = 467129 \text{ N}$$

or $467.129 \times 10^3 \text{ KN}$

④ Intensity of UDL = (M_d)

10, 13, 18, 24
31, 33, 46
1, 4, 6, 7, 11

$$\frac{w_u l^2}{8} = M_d = 267.32$$

$$\frac{w_u \times (5)^2}{8} = 267.32$$

$$w_u = \frac{267.32 \times 8}{25} = 57 \text{ KN/m}$$

⑤ Maximum deflection.

$$f_{\text{max}} = \frac{5}{384} \cdot \frac{w l^4}{EI} \quad (\because w = \text{KN/m})$$

$$= \frac{5}{384} \times \frac{57 \times (5)^4}{2 \times 10^5 \times 20458.4 \times 10^4}$$

$$= 11.34 \text{ mm}$$

⊗ An ISMB 500 @ 852 N/m has to carry an UDL of 36 kN/m inclusive of its self wt over a span of 10 mts. check the beam against limiting deflection if $f_u = 400 \text{ N/mm}^2$ and $E = 2 \times 10^6 \text{ N/mm}^2$. Assume the supported elements are not susceptible to cracking.

Span of the beam (L) = 10 mts or 10000 mm

From table 6 of IS 800-2007,

Limiting value of Maximum deflection,

$$\delta_{\text{limit}} = \frac{l}{300} = \frac{10000}{300} = 33.33 \text{ mm}$$

Max deflection at mid span

$$\delta_{\text{max}} = \frac{5}{384} \cdot \frac{W l^4}{E I} \quad (\text{where } W = \text{KN/m})$$

$$= \frac{5}{384} \times \frac{36 \times (10,000)^4}{0.2 \times 10^6 \times 45218 \times 10^4}$$

$$= 51.83 \text{ mm} \checkmark$$

From steel table for IS MB @ 852 N/m

$$I_{zz} = 45218.30 \times 10^4 \text{ mm}^4 \checkmark$$

$$\delta_{\text{max}} > \delta_{\text{limit}} \checkmark$$

\therefore The beam is considered as unsafe

Design procedure of laterally supported simple beam

For the given type of beam and given loading
Calculate the design bending moment and design stress

Assuming the section is plastic Calculate the plastic section modulus required,

$$Z_p \text{ required} = \frac{M_{\text{design}}}{f_y} \left[\because M_d = \frac{P_b \cdot Z_p f_y}{\gamma_{m0}} \right]$$

Choose a rolled steel section from steel tables having Z_p more than Z_p required. Write the sectional properties of chosen section.

Check for Bending:-

1) Decide the type of c/s - plastic (or) semi-compact or Compact

ii) Calculate the design bending strength

$$M_d = \frac{\phi_b Z_p f_y}{\gamma_{m0}}$$

This should be less than $\frac{1.2 Z_e f_y}{\gamma_{m0}}$ for S.S. Beams

& $\frac{1.5 Z_v f_y}{\gamma_{m0}}$ for Cantilever beams.

If M_d is greater than design bending moment (M) the beam is safe against bending

Check for shear

Calculate the design shear strength of beam.

$$V_d = \frac{h \times t_w \cdot f_y}{\gamma_{m0}}$$

If $V_d >$ the design shear force (V) then the beam is safe against shear.

Check for deflection

i) Calculate Max deflection in the beam (S_{max})

ii) This should not be greater than limiting deflection given in table 6 of IS 800:

2007.

iii) If $S_{max} < S_{limit}$ then the beam is safe against deflection.

When point load acting on central beam.

$$S_{limit} = \frac{l}{200}$$

$$M_d = S_{limit} = \frac{l}{200}$$

Design a simply supported beam of an effective span 6m with C-sections a UDL of 20 kN/m including self wt. If the compression flange of the beam is laterally restrained. Check the beam for shear and deflection. The grade of steel is Fe 250.

Solution:

① Load = 20 kN/m (If self wt not given assumed as 500N)
Factored load = $20 \times 1.5 = 30 \text{ kN/m}$

effective span = $l = 6 \text{ m}$

Design B.M = $M = \frac{W_u l^2}{8} = \frac{30 \times 6^2}{8} = 135 \text{ kN-m}$
or $135.00 \times 10^6 \text{ N-mm}$

Design shear force = $V = \frac{W_u l}{2} = \frac{30 \times 6}{2} = 90 \text{ kN}$
 $V = 90 \times 10^6 \text{ N-mm}$

② Assuming the section is plastic section

plastic section modulus = Z_p

$$Z_p = \frac{M \times \gamma_{mo}}{f_y} = \frac{135 \times 10^6 \times 1.1}{250}$$

$$Z_p = 594 \times 10^3 \text{ mm}^3 \text{ (from steel tables)}$$

③ Choose ISHB 300 @ 102 N/m

$h = 300 \text{ mm}$, $b_f = 140 \text{ mm}$

$r_f = 12.40 \text{ mm}$, $t_w = 7.5 \text{ mm}$

$$Z_e = 573.6 \times 10^3 \text{ mm}^3$$

$$Z_p = 641.28 \times 10^3 \text{ mm}^3$$

selection of section

④ $\lambda = \sqrt{\frac{b_f}{t_w}} = \sqrt{\frac{250}{250}} = 1$

$$\frac{b}{t_w} = \frac{b_f/2}{t_w} = \frac{140/2}{7.5} = 9.33 < 9.4 \text{ } \epsilon$$

$$\frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{300 - 2(12.4)}{7.5} = 36.69 < 84.6$$

Hence the section is classified as plastic section
 - plastic section $\beta_b = 1.0$

⑤ check for Bending

Design Bending Strength = M_d

$$M_d = \frac{\beta_b Z_p f_y}{\gamma_{mo}} = \frac{1.0 \times 641.278 \times 10^3 \times 250}{1.05} = 148.11 \times 10^6 \text{ N-mm}$$

$$M_d = 148.11 \text{ kN-m}$$

This should be less than $\frac{1.22 Z_p f_y}{\gamma_{mo}}$ for S.S. beam

$$= \frac{1.2 \times 573.60 \times 10^3 \times 250}{1.10} = 135.00 \times 10^6 \text{ N-mm}$$

$$= 135.00 \text{ kN-m}$$

$$M_d = 148.11 \text{ kN-m} > 135 \text{ kN-m}$$

$$M_d > M$$

∴ The beam is not safe against bending.

⑥ Check for Shear

$$\text{Design S.P (V)} = 90 \text{ kN}$$

Design shear strength of beam = V_d

$$V_d = \frac{h t_w f_y}{\gamma_{mo} \sqrt{3}} = \frac{300 \times 7.5 \times 250}{1.10 \times \sqrt{3}} = 295.235 \text{ kN}$$

$$V_d = 295.24 \text{ kN} > 90 \text{ kN}$$

Hence it is safe.

② check for deflection:-

$$\text{Max deflection in the beam} = \delta_{\text{max}} = \frac{5}{384} \times \frac{Wl^4}{EI}$$

$$\delta_{\text{max}} = \frac{5}{384} \times \frac{20 \times (6000)^4}{2 \times 10^5 \times 86036 \times 10^4}$$

$$\delta_{\text{max}} = 19.644 \text{ mm}$$

take from table 6 of IS 800-2007, ~~900~~

$$\text{Limiting deflection } \delta_{\text{limit}} = \frac{\text{span}}{240}$$

$$\delta_{\text{limit}} = \frac{l}{240} = \frac{6000}{240} = 25 \text{ mm}$$

$$\delta_{\text{max}} < \delta_{\text{limit}} \quad \text{Hence it's safe}$$

Design a laterally supported simply supported beam of 4m span; loaded for a concentrated load of 400 kN at mid span. The load is transferred through baseplates of 200mm length to the supports. Design a check for deflection using IS MB 400 section which is available.

Data

$$\text{Span} = 4 \text{ mts (or) } 4000 \text{ mm}$$

$$\text{Support width} = 200 \text{ mm}$$

$$\text{Load on beam} = P = 400 \text{ kN (at mid span)}$$

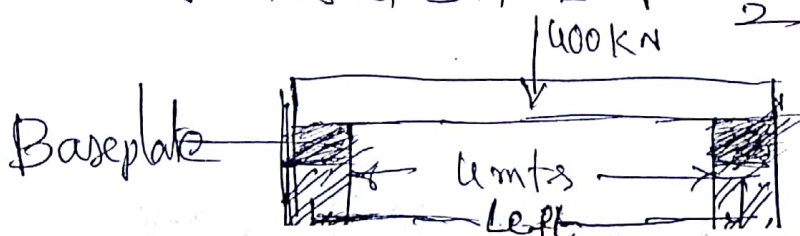
$$L_{\text{eff}} = 4 + 0.2 = 4.2 \text{ m}$$

$$\text{Factored load} = 1.5 \times 400 = 600 \text{ kN} = W$$

$$\text{Max/Factored bending moment} = \frac{Wl}{4} = \frac{600 \times 4.2}{4}$$

$$M = 630 \text{ kN-m}$$

$$\text{Factored S.F} = V = \frac{P}{2} = \frac{600}{2} = 300 \text{ kN}$$



② properties of ISMB 400

$$h = 400 \text{ mm}, b_f = 140 \text{ mm}, t_f = 16 \text{ mm}$$

$$t_w = 8.9 \text{ mm}; Z_e = 1102 \times 10^6 \text{ mm}^3 \quad Z_p = 1.176 \times 10^6 \text{ mm}^3$$

$$r_1 = 14 \text{ mm}; I_{zz} = 204.58 \times 10^5 \text{ mm}^4$$

Out standing flange = $b = \frac{b_f}{2} = \frac{140}{2} = 70 \text{ mm}$

Depth of web = $d = h - 2(t_f + r_1) = 400 - 2(16 + 14) = 340 \text{ mm}$

③ Section classification :-

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

$$\frac{b}{t_f} = \frac{70}{16} = 4.375 < 9.4 \epsilon$$

$$\frac{d}{t_w} = \frac{340}{8.9} = 38.2 < 84 \epsilon$$

∴ Section is plastic section.

$$\therefore \beta_b = 1$$

$$\frac{d}{t_w} = 38.2 < 67 \epsilon$$

∴ checks for web-buckling will not be required.

Pr. Check for shear capacity

$$V_d = \frac{f_y \times A_v}{\sqrt{3} \times \gamma_{mo}} = \frac{f_y \times h \times t_w}{\sqrt{3} \times \gamma_{mo}}$$

$$V_d = \frac{250 \times 400 \times 8.9}{\sqrt{3} \times (1.1)} = 467.128 \times 10^3 \text{ N}$$

$$V_d = 300 \times 10^3 \text{ N}$$

∴ $V_d > V_u$ Hence it is safe

⑤ Design bending strength

$$M_d \leq 1.2 \cdot Z_p \cdot f_y \cdot \frac{\gamma_{m0}}{\gamma_{m1}}$$

$$M_d = \beta_b \cdot Z_p \cdot \frac{f_y}{\gamma_{m0}} = 1.0 \times 1.176 \times 10^6 \times \frac{250}{1.10}$$

$$M_d = 267.27 \times 10^6 \text{ Nm} \leq 1.2 \times 1.02 \times 10^6 \times \frac{250}{1.10}$$

$$M_d = 267.27 \times 10^6 \leq 278.18 \times 10^6 \text{ Nm} \quad \checkmark$$

Hence it is adequate.

⑥

check for deflection.

$$S_{max} = \frac{w l^3}{48 E I} = \frac{400 \times 10^3 \times (4000)^3}{48 \times 2 \times 10^5 \times 204.58 \times 10^6}$$

$$S_{max} = 13.03 \text{ mm}$$

~~Permissible~~ permissible deflection (S)

As per IS 800-2007

$$\text{Permissible deflection} = S = \frac{l}{300} = \frac{4000}{300} = 13.33 \text{ mm}$$

$$S_{max} < S$$

∴ The beam is safe against deflection.

✓ A simply supported beam of span 4.5 mtrs consist of rolled steel section ISLB 450 @ 640 N/m. The compression flange is laterally unsupported. Determine the design bending strength of the beam.

Span of the beam = $l = 4.5 \text{ mtrs}$

Section of beam ISLB 450 @ 640 N/m

Find design bending strength = $M_d = ?$

② properties of IS LB 450...

$$h = 450 \text{ mm}, b_f = 170 \text{ mm}, t_f = 13.4 \text{ mm}$$

$$t_w = 8.6 \text{ mm}, I_{zz} = 275.36 \times 10^6 \text{ mm}^4, r_1 = 161 \text{ mm}$$

$$Z_e = 1.223 \times 10^6 \text{ mm}^3, Z_p = 1.401 \times 10^6 \text{ mm}^3$$

$$I_{yy} = 8.53 \times 10^6 \text{ mm}^4$$

$$\text{Outstanding flange} = b' = \frac{b_f}{2} = \frac{170}{2} = 85 \text{ mm}$$

$$\begin{aligned} \text{Height of the web} &= 450 - (2t_f + r_1) \\ &= 450 - 2(13.4 + 16) = 391.2 \text{ mm} \end{aligned}$$

This is a case of laterally unsupported beam. Let us determine the elastic lateral torsional buckling moment (Refer IS 800-2007 page 54)

$$M_{cr} = \sqrt{\frac{\pi^2 E I_y}{(L_{LT})^2} \left[G J_1 + \frac{\pi^2 E I_w}{(L_{LT})^2} \right]}$$

$$G = \frac{E}{2(1+\mu)} = \frac{2 \times 10^5}{2(1+0.3)} = 76.92 \times 10^3 \text{ N/mm}^2$$

$$I_t = \sum \frac{b t^3}{3} = 2 \left(b_f \cdot \frac{t_f^3}{3} \right) + (h - t_f) \cdot \frac{t_w^3}{3}$$

$$= 2 \left[170 \times \frac{(13.4)^3}{3} \right] + (450 - 13.4) \cdot \frac{(8.6)^3}{3}$$

$$I_t = 272.7 \times 10^3 + 92.50 \times 10^3 = 365.26 \times 10^3 \text{ mm}^4$$

$$L_{LT} = 4.5 \text{ m} = 4500 \text{ mm}$$

$$h_f = (h - t_f) = 450 - 13.4 = 436.6 \text{ mm}$$

$$I_w = (2 - \beta_f) \cdot \beta_f \cdot I_y \cdot h_f^2$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5 \text{ (assumed)} \quad (\because I_{fc} = I_{ft})$$

$$I_w = (1 - 0.15) \times 0.15 \times 8.53 \times 10^6 \times 436.67^2$$

$$I_w = 4.064 \times 10^{11} \text{ mm}^4$$

$$\frac{\pi^2 E I_y}{(L_T)^2} = \frac{\pi^2 \times 2 \times 10^5 \times 4.064 \times 10^{11}}{(4500)^2} = 3.961 \times 10^{10}$$

$$M_{cr} = \sqrt{831.48 \times 10^3 \left[76.92 \times 10^3 \times 365.26 \times 10^3 + 3.961 \times 10^{10} \right]}$$

$$M_{cr} = 237.25 \times 10^6 \text{ Nmm} = \underline{\underline{237.25 \text{ kNm}}}$$

non-dimensional slenderness ratio,

$$\lambda_{LT} = \sqrt{\frac{R_b \cdot Z_p \cdot f_y}{M_{cr}}} = \beta_b = 1 \text{ for plastic section}$$

$$\lambda_{LT} = \sqrt{\frac{1 \times 1.40 \times 10^6 \times 250}{237.25 \times 10^6}} = 1.21$$

$$\left(\lambda_{LT} = \frac{1}{1.338 + \left[1.338^2 - 1.21^2 \right]^{0.5}} \right)$$

$$\lambda_{LT} = 0.523 < 1$$

$$\lambda_{LT} = \frac{1}{\phi_{LT} + \left[\phi_{LT}^2 - \lambda_{LT}^2 \right]^{0.5}}$$

$$\phi_{LT} = 0.15 \left[1 + \alpha_{LT} (\lambda_{LT}^2 - 0.2) + \lambda_{LT}^2 \right]$$

Imperfection factor, $\alpha_{LT} = 0.21$ for rolled sections.

$$= 0.15 \left[1 + 0.21 (\lambda_{LT}^2 - 0.2) + \lambda_{LT}^2 \right]$$

$$\phi_{LT} = 1.338$$

$$\lambda_{LT} = \frac{1}{1.338 + \left(1.338^2 - 1.21^2 \right)^{0.5}}$$

$$\lambda_{LT} = 0.523 < 1$$

Design bending Compressive stress

$$f_{cd} = \frac{\gamma_{M1} \cdot f_y}{\gamma_{M0}} = \frac{0.523 \times 250}{1.10} = 119 \text{ N/mm}^2$$

∴ Design bending strength

$$M_d = \beta_b \times Z_p \times f_{cd}$$

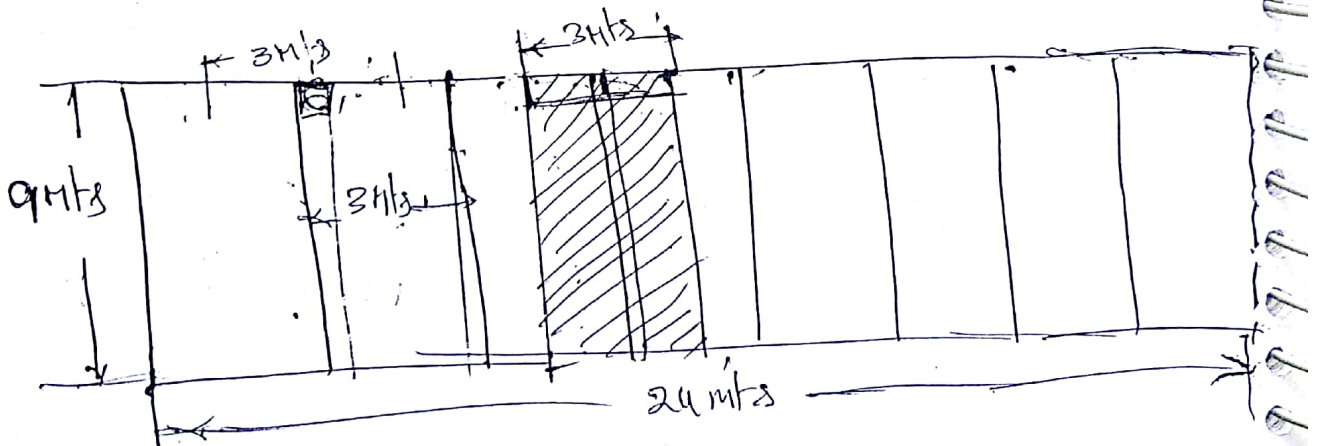
$$= 1 \times 1.401 \times 10^6 \times 119$$

$$M_d = 166.76 \times 10^6 \text{ Nmm}$$

$$M_d = \underline{\underline{166.76 \text{ kN-m}}}$$

Design a simply supported I-section to support the slab of hall 9m x 24m with beams spaced at 3m c/c. The thickness of the slab is 100mm. Consider a floor finish load of 0.5 kN/m² and a live load of 3 kN/m². The grade of the steel is E250. Assume that an adequate lateral support is provided to the compression flange.

Solution



1) Load parameter length of beam.

a) Slab = $\frac{0.1 \times 1 \times 25 \times 25}{1000} \times 3 \times 25 = 7.5 \text{ kN/m}$ (R.C.C.)

b) Floor finish = $3 \times 0.5 = 1.5 \text{ kN/m}$

c) Liveload = $3 \times 3 = 9 \text{ kN/m}$

d) Assume self wt = 0.5 kN/m

25 kN/m^2
 $2.5 \times 3 = 7.5 \text{ kN/m}$

Total load = $7.5 + 1.5 + 9 + 0.5 = 18.5 \text{ kN/m}$

Factored load = $w_u = 18.5 \times 1.5 = 27.75 \text{ kN/m}$

Effective span (l) = $9 + 0.3 = 9.3 \text{ m}$

$3 \text{ kN/m} \times 3 \text{ m}$
 9 kN/m

2) Design of B.M. = (M) & S.F. (V)

$M = \frac{w_u l^2}{8} = \frac{27.75 \times 9.3}{8} = 300 \text{ kN-m}$

$M = 300 \times 10^6 \text{ N-mm}$

Design S.F. = $V = \frac{w_l l}{2} = \frac{27.75 \times 9.3}{2} = 129 \text{ kN}$

$V = 129 \text{ kN} \text{ (or)} 129 \times 10^3 \text{ N}$

3) Assuming the section is plastic, plastic section modulus = Z_p

$Z_p = \frac{M_{\text{max}}}{f_y} = \frac{300 \times 10^6 \times 1.1}{250} = 1320 \times 10^3 \text{ mm}^3$

4) choose 450 @ 601 kN/m

$h = 450 \text{ mm}$, $b_f = 170 \text{ mm}$, $t_f = 13.4 \text{ mm}$

$t_w = 8.6 \text{ mm}$, $Z_e = 1222.8 \times 10^3 \text{ mm}^3$

$Z_p = 1404.4 \times 10^3 \text{ mm}^3$

$Z_{22} = 27536.1 \times 10^3 \text{ mm}^4$

5) classification of section

$$a) \epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$b) \frac{b_f}{t_f} = \frac{b_f/2}{t_f} = \frac{170/2}{13.4} = 6.34 < 9.4 \epsilon$$

$$c) \frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{450 - 2 \times 13.4}{8.6} = 49.2 < 81.4 \epsilon$$

Hence the section is plastic. For plastic section $\beta_b = 1$

6) check for Bending :-

$$M_d = \frac{\beta_b \cdot Z_p f_y}{\gamma_{mo}} = \frac{1 \times 1401.4 \times 10^3 \times 250}{1.1}$$
$$= 318.5 \times 10^6 \text{ N-mm}$$
$$\text{or } 318.50 \text{ kN-m}$$

This (M_d) should be less than $\frac{1.22 Z_p f_y}{\gamma_{mo}}$

$$= \frac{1.2 \times 1401.4 \times 10^3 \times 250}{1.1}$$
$$= 333.76 \times 10^6 \text{ N-mm}$$

$$M_d < \frac{1.22 Z_p f_y}{\gamma_{mo}} \text{ or } 318.5 \text{ kN-m}$$

$$M_d > M$$

$$318.5 \text{ kN-m} > 300 \text{ kN-m}$$

Hence the section is safe in Bending

⑦ check for deflection.

$$\delta_{\max} = \frac{5}{384} \frac{Wl^4}{EI}$$

$$= \frac{5}{384} \times \frac{18.41 \times (9300)^4}{2 \times 10^5 \times 22540.4 \times 10^4}$$

$$= 60.389111$$

$$\delta_{\text{limit}} = \frac{l}{240} = \frac{9300}{240} = 38.75 \text{ mm}$$

$$\delta_{\max} > \delta_{\text{limit}}$$

Unsafe

Redesign

| |
|---------------|
| 68858, 18, 75 |
| 1730792520 |

⑧ Choose another section ISHB 450 at 710 N/m.

$$h = 450 \text{ mm}; b_f = 150 \text{ mm}$$

$$t_f = 17.4 \text{ mm}; t_w = 9.4 \text{ mm}$$

$$t_w = 9.4 \text{ mm}$$

$$Z_e = 1350.70 \times 10^3 \text{ mm}^3$$

$$Z_p = 153.4 \times 10^3 \text{ mm}^3$$

$$I_{zz} = 30133 \times 10^4 \text{ mm}^4$$

Classification of section.

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\frac{b}{t_f} = \frac{150/2}{17.4} = 4.3 < 9.4 \epsilon$$

$$\frac{d}{t_w} = \frac{450 - 2 \times 174}{9.4} = 44.17 < 84 \epsilon$$

Hence the section is plastic

∴ for the plastic section $\beta_b = 1$

9) check for bending

$$\text{Design bending strength } (M_d) = \frac{\beta_b Z_p f_y}{\gamma_{mo}}$$

$$= \frac{1 \times 1553.4 \times 10^3 \times 250}{1.10}$$
$$= 353.045 \times 10^6 \text{ N-mm}$$

$$M_d = 353.05 \text{ kN-m}$$

$$\text{This should be less than } \frac{1.12 Z_e f_y}{\gamma_{mo}} = \frac{1.12 \times 1350.7 \times 10^3 \times 250}{1.1}$$

$$M = 368.37 \times 10^6 \text{ N-mm}$$
$$\text{or } 368.37 \text{ kN-m}$$

$$M_d < M \quad (\text{ie})$$

$$353.05 \text{ kNm} < 368.37 \text{ kNm}$$

Hence it is safe

check for shear

$$\text{Design shear strength } (V_d) = \frac{A_{tw} f_y}{\beta \gamma_{mo}}$$

$$V_d = \frac{450 \times 9.4 \times 250}{\beta \times 1.10} = 555.043 \text{ N}$$

$$V_d = 555.043 \text{ kN}$$

$$V_d \gg V$$

Design shear strength \gg Design shear force

$$555.05 \text{ kN} > 129 \text{ kN}$$

Hence the section is safe.

Check for deflection

$$\delta_{max} = \frac{5}{384} \times \frac{Wl^4}{EI} = \frac{5}{384} \times \frac{18.64 \times (9300)^4}{2 \times 10^5 \times 30390.18 \times 10^4}$$
$$= 29.87 \text{ mm}$$

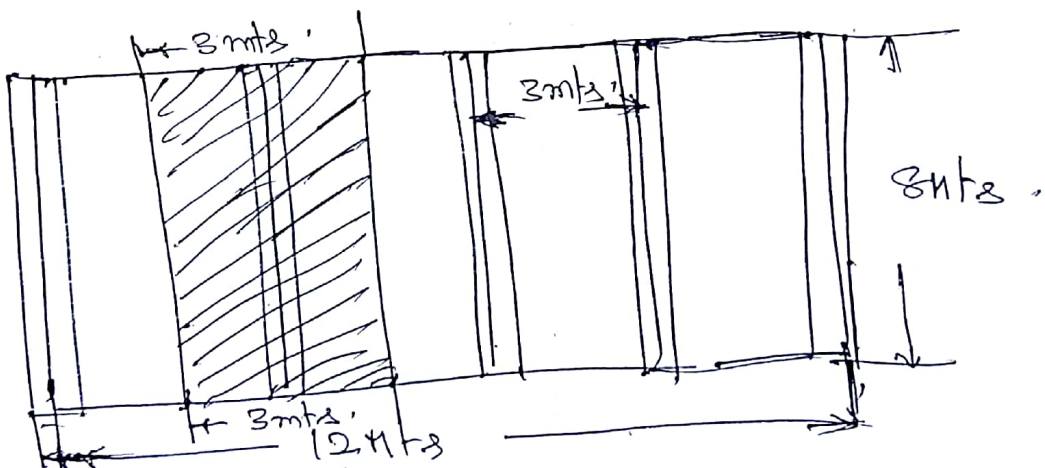
$$\delta_{limit} = \frac{l}{240} = \frac{9300}{240} = 38.75 \text{ mm}$$

$$\delta_{max} < \delta_{limit}$$

Hence the section is safe.

Choose the section is ISM 150 @ 710 mm

⊙ A roof of a hall measuring 8m x 12m consist of 100mm thick R.C.C slab supported on steel I beams spaced at 3m apart. The finished load may be taken as 0.15 kN/m² and live load as 2.15 kN/m². Design the steel Assume $f_y = 250 \text{ N/mm}^2$



⊙ Load per metre length of beam.

i) slab = $\frac{100}{1000} \times 3 \times 25 = 7.5 \text{ kN/m}$

ii) Floor finish = $3 \times 1.5 = 4.5 \text{ kN/m}$ ^{7.5}

iii) live load = $3 \times 1.5 = 4.5 \text{ kN/m}$ ^{1.5}

(ii) Assume the self wt of beam = 0.5 kN/m

$$\text{Total load} = 7.5 + 4.5 + 4.5 + 0.5 = 17 \text{ kN/m}$$

(2) Factored load = $17 \times 1.5 = 25.5 \text{ kN/m}$

(3) effective span = $8 + 0.3 = 8.3 \text{ m}$ (Support width is 300 mm)

(4) Design the B.M (M) = $\frac{WuL^2}{8}$ (Assume the roof supported as S.S.)

(5)
$$\frac{25.5 \times 8.3^2}{8} = 219.5865 \text{ kN} \text{ or } 219.5865 \times 10^6 \text{ N}$$

Design B.M = $219.586 \times 10^6 \text{ N-mm}$

Design S.F = $\frac{WuL}{2} = \frac{25.5 \times 8.3}{2} = 105.825 \text{ kN}$

or $105.825 \times 10^3 \text{ N}$

(5) Assuming the section is plastic section, plastic section modulus Z_p required = $\frac{M_{req}}{f_y}$

$$= \frac{219.586 \times 10^6}{1.10}$$

$$Z_p = 199.624 \times 10^3 \text{ mm}^3$$

(6) choose BS 460 @ 600 kN/m

$$h = 400 \text{ mm} ; b_f = 160 \text{ mm}$$

$$t_w = 8.9 \text{ mm} ; t_f = 16 \text{ mm}$$

$$Z_e = 1022.9 \times 10^3 \text{ mm}^3$$

$$Z_p = 1176.2 \times 10^3 \text{ mm}^3$$

$$I_{xx} = 20458.4 \times 10^4 \text{ mm}^4$$

$$\textcircled{7} \quad \varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\cdot \frac{b}{t_f} = \frac{100/2}{16} = 3.125 < 9.4 \varepsilon = 9.4 \varepsilon = 9.4$$

$$\frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{400 - 2 \times 100}{8.9} = 41.34 \leq 84 \varepsilon$$

Hence the section is plastic

⑧ Check for bending:

$$\text{Design bending strength} = (M_d) \cdot \frac{\beta_b \cdot Z_p \cdot f_y}{\gamma_{m0}}$$

$$= \frac{1 \times 1176.2 \times 10^3 \times 250}{1.10} = 267.318 \times 10^6 \text{ N-mm}$$

\therefore This should be less than $\frac{1.2 Z_e f_y}{\gamma_{m0}} = \frac{1.2 \times 1022.9 \times f_y}{1.10} = 106.1 \text{ kN}$

$$= 267.318 \times 10^6 \text{ kN-mm}$$

$$= 267.31 \times 10^6 \text{ N-mm}$$

$$M_d = 267.31 \text{ kN-m} > 219.58 \text{ kN-m}$$

$$M_d > M_1$$

⑨ check for shear

$$\text{Design shear strength} = (V_d) \frac{h \cdot t_w \cdot f_y}{\sqrt{3} \gamma_{m0}}$$

$$\frac{100 \times 3.9 \times 250}{\sqrt{3} \times 1.1} = 467128 \text{ N} \approx 467.12 \text{ kN}$$

9. check for deflection. $V_d > V$ Hence the section is safe

$$\delta_{max} = \frac{5}{384} \frac{W L^4}{EI} = \frac{5}{384} \times \frac{17604 \times 8300^4}{2 \times 10^5 \times 20458.4 \times 10^4}$$

$$\delta_{lim} = 1/240 = 8300/240 = 34.58; \delta_{max} < \delta_{lim} \text{ safe}$$

Q. A 3.5 m long cantilever beam has to carry a UDL of 12 kN/m including the self weight. The beam is laterally supported. Design the beam & check it safety against shear & deflection $f_y = 250 \text{ MPa}$.

Solution:

1) Total load (w) = 12 kN/m .

Factored load (w_u) = $12 \times 1.5 = 18 \text{ kN/m}$.

2) Effective span (l_e) = 3.5 m .

Design B.M (M) = $\frac{w_u l^2}{2} = \frac{18 \times 3.5^2}{2}$

= 110.25 kN-m

= $110.25 \times 10^6 \text{ N-mm}$

3) Design S.F (V) = $w_u l = 18 \times 3.5$
= $63 \text{ kN} = 63 \times 10^3 \text{ N}$

4) Assume the section is plastic

Plastic section modulus = Z_p

$Z_p = \frac{M_{\text{design}}}{f_y} = \frac{110.25 \times 10^6 \times 1.10}{250}$

$Z_p \approx 485 \times 10^3 \text{ mm}^3$

5) Choose ISLB 300 @ 370 N/m .

$h = 300 \text{ mm}$, $b_f = 150 \text{ mm}$

$t_f = 9.4 \text{ mm}$, $t_w = 6.7 \text{ mm}$

$Z_e = 488.9 \times 10^3 \text{ mm}^3$

$Z_p = 554.3 \times 10^3 \text{ mm}^3$

$A_{ef} = 7338.9 \times 10^4 \text{ mm}^2$

Classification of section.

$$e = \sqrt{\frac{250}{250}} = 1$$

$$\frac{b}{t_f} = \frac{150/2}{t_f} = \frac{150}{2 \times 9.4} = 7.98 < 9.4e$$

$$\frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{300 - 2 \times 9.4}{6.7} = 41.97 < 84e$$

Hence the section is plastic

∴ For plastic sections, $\beta_b = 1$

⑥ check for bending

$$M_d = \frac{\beta_b \times Z_p \times f_y}{\gamma_{mo}} = \frac{1 \times 554.3 \times 10^3 \times 250}{1.10}$$

$$= 125.97 \times 10^6 \text{ N-mm}$$

$$= 125.97 \text{ kN-m}$$

For cantilever this should be less than $\frac{1.5 Z_e f_y}{\gamma_{mo}}$

$$= \frac{1.5 \times 688.9 \times 10^3 \times 250}{1.10}$$

$$= 166.67 \times 10^6 \text{ N-mm}$$

$$= 166.67 \text{ kN-m}$$

$$\therefore M_d = 125.97 \text{ kN-m} > 110.25 \text{ kN-m}$$

$$M_d > m$$

Hence the section is safe in bending.

⑦ Checks for shear

$$V_d = \frac{h t_w f_y}{\sqrt{3} \cdot \gamma_{mo}} = \frac{300 \times 6.7 \times 250}{\sqrt{3} \times 1.10}$$

$$V_d = 263744 \text{ N}$$

$$V_d = 263.74 \text{ kN} > 63 \text{ kN}$$

Hence the section is safe in shear

8) check for deflection

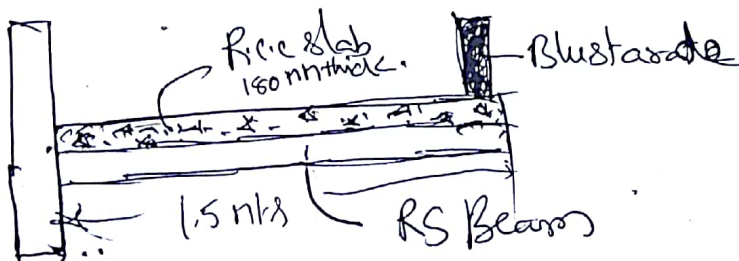
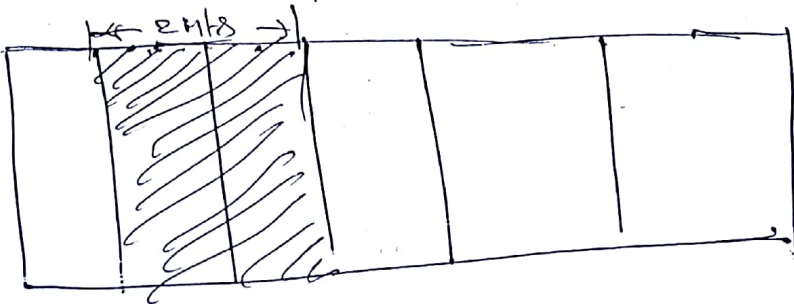
$$\begin{aligned} \text{Max deflection in beam} &= \delta_{\text{max}} = \frac{wL^4}{8EI} \\ &= \frac{12 \times (3000)^4}{8 \times 2 \times 10^5 \times 7332.9 \times 10^4} = 13.67 \text{ mm} \end{aligned}$$

$$\delta_{\text{limiting}} = \frac{l}{120} = \frac{3500}{120} = 29 \text{ mm}$$

$\delta_{\text{exp}} < \delta_{\text{limiting}}$ Hence it is safe

∴ provide RSLB 300 @ 370 N/m as a Cantilever beam.

Q A 1.5 m wide balcony ^{consist} ~~const~~ of 180 mm thick R.C.C slab supported on RS beam placed 2 m c/c of Cantilevering from a stone masonry work of 400 mm thickness. At the end of the balcony there is a R.C.C balustrade of 100 mm thickness and 800 mm height. If the balcony carries live load of 2 kN/m² design the rolled beam.



Solution:-

① Load per metre length (kN/m)

$$\begin{aligned} \text{Self wt of Rice slab} &= 1 \times \frac{180}{1000} \times 25 \times 2 = 9 \text{ kN/m} \\ &= 1.0 \times 0.18 \times 25 \times 2 = 9 \text{ kN/m} \end{aligned}$$

$$\text{Line load} = 2 \times 2 \text{ m} = 4 \text{ KN/m} \quad \text{Converted cm to KN/m}$$

$$\text{Assume the self wt} = 0.5 \text{ KN/m}$$

$$\text{Total load} = 9 + 4 + 0.5 = 13.5 \text{ KN/m}$$

$$\text{Factored load} = W_u = 13.5 \times 1.5 = 20.25 \text{ KN/m}$$

② Concentrated load at free end due to balustrade

$$W = \left(\frac{100}{1000} \times \frac{800}{1000} \times 25 \right) \times 2$$

$$= 4 \text{ KN} \quad (\text{Concentrated load})$$

$$\text{Factored load} = 1.5 \times 4 = 6 \text{ KN}$$

$$\text{effective span} = (l_e) = 1.5 \text{ m}$$

$$\text{③ Design B.M (M)} = \frac{W_u l^2}{8} + \frac{W l}{2}$$

$$= \frac{20.25 \times 1.5^2}{8} + \frac{6 \times 1.5}{2}$$

$$= 31.78 \text{ KN-m}$$

$$= 31.78 \times 10^6 \text{ N-mm}$$

$$\text{Design S.F} = V = W_u l + W$$

$$= 20.25 \times 1.5 + 6.00$$

$$V = 36.37 \text{ KN}$$

④ Assuming the section is plastic

$$\therefore \text{The section modulus required} = Z_p = \frac{M_{\text{max}}}{f_y}$$

$$Z_p = \frac{31.78 \times 10^6 \times 1.10}{250} = 139.83 \text{ mm}^3$$

$$Z_p = 139.83 \text{ mm}^3$$

⑤ Choose. I SLB 175 @ 164 N/m

$$h = 175 \text{ mm}, b_f = 90 \text{ mm}$$

$$t_f = 6.9 \text{ mm}; t_w = 5.1 \text{ mm}$$

$$Z_e = 125.5 \times 10^3 \text{ mm}^3$$

$$I_{22} = 1096.20 \times 10^4 \text{ mm}^4$$

Classification of section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\frac{b}{t_f} = \frac{b/2}{t_f} = \frac{90}{2 \times 6.9} = 6.2 < 9.1 \epsilon$$

$$\frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{175 - 2 \times 6.9}{5.1} = 31.64 < 84 \epsilon$$

Hence the section is safe.

For plastic sections $\beta_b = 1$

⑥ check for bending

$$M_d = \frac{\beta_b \cdot Z_p \cdot f_y}{\gamma_{m0}} = \frac{1 \times 139.83 \times 250}{1.10}$$

$$= 32.57 \times 10^6 \text{ N-mm or } \underline{32.57 \text{ kN-m}}$$

For cantilever this should be less than $1.5 Z_e f_y$

$$= \frac{1.5 \times 125.3 \times 10^3 \times 250}{1.10} = 42.75 \times 10^6 \text{ N-mm}$$

$$M_d = 42.75 \text{ kN-m} > 31.78 \text{ kN-m}$$

$$M_d > M$$

Hence the section is safe in bending.

⑦ check for shear

$$V_d = \frac{h^2 w \cdot d_y}{\sqrt{3} \cdot \gamma_{mo}} = \frac{175 \times 6.90 \times 250}{\sqrt{3} \times 1.11} = 117110 \text{ N}$$

$$V_d = 117.110 \text{ kN} > 36.37 \text{ kN} \quad \checkmark$$

$V_d > V$ Hence it's safe in shear.

⑧ check for deflection.

$$\delta_{\text{Maxi}} = \text{Maxi deflection} = \frac{Wl^4}{8EI} + \frac{Wl^3}{3EI}$$
$$= \frac{13.5 \times (1500)^4}{8 \times 2 \times 10^5 \times 1096.2 \times 10^4} + \frac{6 \times (1500)^3}{3 \times 2 \times 10^5 \times 1096.2 \times 10^4}$$
$$= 5.94 \text{ mm}$$

$$\delta_{\text{limit}} = \frac{l}{120} = \frac{1500}{120} = 12.5 \text{ mm}$$

$$\delta_{\text{Maxi}} < \delta_{\text{limit}}$$

Hence the section is safe in deflection.
provide ISLB 175 @ 164 N/m as Cantilever beam.

∅ Determine the design bending strengths of a laterally restrained beam ISMB 300 @ 462 N/m. The yield stress of steel is 250 MPa.

Solution:

From steel tables;

For ISMB 300 @ 462 N/m

$$h = 300 \text{ mm}, b_f = 140 \text{ mm}, t_f = 12.4 \text{ mm}, t_w = 7.5 \text{ mm}$$

$$Z_p = 573.6 \times 10^3 \text{ mm}^3 \quad Z_p = 651.7 \times 10^3 \text{ mm}^3$$

Section classification

$$e = \sqrt{\frac{250}{S_u}} = \sqrt{\frac{250}{250}} = 1$$

$$\frac{b}{t_f} = \frac{b_s/2}{t_f} = \frac{b_s}{2t_f} = \frac{1100}{2 \times 12.4} = 5.64 < 9.4 \text{ E}$$

$$\frac{d}{t_w} = \frac{h - 2t_f}{t_w} = \frac{300 - 2(12.4)}{7.5} = 36.69 < 84 \text{ E}$$

Hence the section is classified as class-1 plastic

② Design bending strength of beam

For plastic sections $\beta_{pc} = 1.0$

$$M_d \leq \frac{\beta_{pc} Z_p f_y}{\gamma_{mo}} = \frac{1 \times 651.7 \times 10^3 \times 250}{1.1}$$

$$= 148.11 \times 10^6 \text{ N-mm}$$

$$= 148.11 \text{ kN-m}$$

For simply supported beams M_d shall be less

than $\frac{1.2 Z_e f_y}{\gamma_{mo}}$

$$= \frac{1.2 \times 573.6 \times 10^3 \times 250}{1.1}$$

$$= 156.44 \times 10^6 \text{ N-mm}$$

$$= 156.44 \text{ kN-m}$$

Compare the values, $M_d = 148.11 \text{ kN-m}$

Design bending strength of beam = 148.11 kN-m

Laterally un supported Beams

Beams are normally used so as to bend about major axis (Z-Z) rather than to bend about minor axis (Y-Y) since they have higher value of moment of inertia about that axis. In such case when the compression flange is not supported, it has a tendency to bend in the lateral direction with twisting bending of compression flange with twisting reduces the load carrying capacity of the section, such beams are called laterally un supported beams.

Lateral torsional Buckling of beams:-

Lateral torsional buckling is a limit state of structural usefulness when the deformation of a beam changes from plane deflection to a combination of lateral deflection and twisting.

The factors affecting the lateral torsional buckling strength: —

- 1) Distance bet lateral support to compression flange
- 2) End restraint & at intermediate support locations.
- 3) Type and position of loads
- 4) Type of cross sections
- 5) Material properties.
- 6) Non prismatic nature of members.
- 7) Moment gradient along the length
- 8) Magnitude & distribution of residual stress.

Calculate the moment carrying capacity of laterally unbraced unrestrained beam made of ISMB 400 and length of member is equal to 4m. Assume necessary data.

Solution:

① Given Section ISMB 400

Support Condition: Laterally unbraced (~~of unbraced~~ beam)

Span of member = 4m.

② Properties of ISMB 400 from steel table.

$$h = 400 \text{ mm}, b_f = 140 \text{ mm}, t_f = 16.00 \text{ mm}$$

$$t_w = 8.9 \text{ mm}; I_{zz} = 204.58 \times 10^6 \text{ mm}^4, I_{yy} = 6.221 \times 10^6 \text{ mm}^4$$

$$r_z = 161.5 \text{ mm}, r_y = 28.2 \text{ mm}$$

$$Z_e = 1.02 \times 10^6 \text{ mm}^3, Z_p = 1.176 \times 10^6 \text{ mm}^3$$

$$r_1 = 14 \text{ mm}$$

③ Section classification

$$e = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\text{out stand of flange } b = \frac{b_f}{2} = \frac{140}{2} = 70 \text{ mm}$$

$$\frac{b}{t_f} = \frac{70}{16} = 4.375 < 9.4e$$

~~$$d = h - 2t_f$$~~
$$d = h - 2(t_f + r_1)$$

$$d = 400 - 2(16 + 14) = 340 \text{ mm}$$

$$\frac{d}{t_w} = \frac{340}{8.9} = 38.20 < 84e$$

∴ The section is plastic ∴ $\beta_b = 1$

∴ also $\frac{d}{t_w} = 38.2 < 67 \epsilon$

checks for web bulking is not required

⑥ Design moment capacity of section

$$M_d = \beta_b \times Z_p \times f_{cd}$$

$$\beta_b = 1, Z_p = 1.176 \times 10^6 \text{ mm}^3 \text{ } \therefore f_{cd} = ?$$

$$f_{cd} = \frac{\chi_{LT} \cdot f_y}{\gamma_{mo}}$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}} \leq 1.0$$

$$\lambda_{LT} = \sqrt{\frac{L_{LT} \cdot f_y}{J_{crb}}}$$

$$J_{crb} = \frac{1.1 \cdot \pi^2 \cdot E}{(L_{LT}/r_y)^2} \left[1 + \frac{1}{20} \left(\frac{L_{LT}}{r_y} \cdot \frac{t_f}{w_y} \right)^2 \right]^{0.5}$$

$$L_{LT} = 4 \text{ mts} = 4000 \text{ mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$r_y = 28.2 \text{ mm}, h = 400 \text{ mm}, t_f = 16 \text{ mm}$$

$$J_{crb} = \frac{1.1 \times \pi^2 \times 2 \times 10^5}{(4000/28.2)^2} \left[1 + \frac{1}{20} \frac{\left(\frac{4000}{28.2} \right)^2}{400/16} \right]^{0.5}$$

$$f_{cd} = 107.919 \times 1.615 = 174.30 \text{ N/mm}^2$$

$$\lambda_{LT} = \sqrt{\frac{S_y}{S_{cr,b}}} = \sqrt{\frac{250}{17430}} = 1.20$$

Imperfection factor

$$\alpha_{LT} = 0.21 \text{ (for rolled steel section)}$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\lambda_{LT} - 0.2) + \lambda_{LT}^2 \right]$$

$$= 0.5 \left(1 + 0.21 (1.20 - 0.2) + 1.20^2 \right)$$

$$\phi_{LT} = 1.325$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}^2 + \lambda_{LT}^2 \right)^{0.5}} \leq 1.0$$

$$= \frac{1}{1.325 + \left[1.325^2 + 1.20^2 \right]^{0.5}}$$

$$= \frac{1}{1.886} = 0.53 < 1.0$$

∴ Hence it is O.K

Design bending compressive stress, = f_{cd}

$$f_{cd} = \frac{\chi_{LT} \cdot S_y}{\gamma_{m0}} = \frac{0.53 \times 250}{1.1}$$

$$f_{cd} = 120.45 \text{ N/mm}^2$$

∴ Moment carrying capacity of ISMB 400

$$M_d = \beta_b \times Z_p \times f_{cd}$$

$$= 1 \times 1.176 \times 10^6 \times 120.45$$

$$M_d = 141.654 \times 10^6 \text{ Nmm} = \underline{141.654 \text{ kNm}}$$