

FACULTY OF INFORMATICS

B.E. 2/4 (IT) II-Semester (Supplementary) Examination, January 2011

PROBABILITY AND RANDOM PROCESS

Time : Three Hours]

[Maximum Marks : 75

*Answer ALL questions from Part A. Answer any FIVE questions from Part B.***PART—A** (Marks : 25)

1. If X and Y are two random variables; prove that $E(X + Y) = E(X) + E(Y)$. 2
2. Two dice are rolled, find the probability that the first die should contain even number or a total of '8'. 3
3. State the properties of Moment Generating Function (MGF). 2
4. State Cauchy Schwartz Inequality. 2
5. State properties of joint probability function (F(x, y)). 3
6. Explain ergodicity. 2
7. If X and Y are independent random variables (IRV) prove that :

$$V(aX - bY) = a^2V(X) + b^2 V(Y) \quad (\because a, b \text{ constants}).$$
 3
8. If $X(t) = P + Qt$ where P and Q are independent r.v.s. with $E(P) = P$; $E(Q) = \phi V$; $\text{Var}(P) = \pi^2$; $\text{Var}(q) = \sigma_2^2$ find $R(t_1, t_2)$. 3
9. Define Noise. 2
10. State the properties of Cross Correlation. 3

PART—B (Marks : 50)

11. (a) State and prove Baye's theorem. 5
- (b) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4 find the probability that '1' is received. 5
12. (a) Find Mean and Variance of Poisson random variable. 5
- (b) A r.v. X has mean 12 and variance 9 and an unknown probability distribution. Find $P(6 < X < 18)$ by using Tchebycheff's theorem. 5

13. (a) Find the power spectral density of WSS process with autocorrelation function :

$$R(\tau) = e^{-\alpha\tau^2}$$

5

(b) State the properties of Cross-correlation.

5

14. (a) If the power spectral density of WSS process is given by :

$$s(w) = \begin{cases} \frac{b}{a}(a - |w|) & |w| \leq a \\ 0 & |w| > a \end{cases}$$

find autocorrelation function of the process.

5

(b) Prove that $S_{yy}(w) = |H(w)|^2 S_{xx}(w)$.

5

15. Two random processes $X(t)$; $Y(t)$ are defined by $X(t) = A \cos w_0 t + B \sin w_0 t$ and $Y(t) = B \cos w_0 t - A \sin w_0 t$, show that $X(t)$ and $Y(t)$ are jointly wide sense stationary if A and B are uncorrelated r.v's with zero means and the same variances and w_0 is a const.

10

16. The two dimensional r.v's has a joint density $f(x, y) = 8xy$, $0 < x < y < 1$. Find :

(i) Mdf's of $f(x)$; $g(y)$.

(ii) Are X and Y independent ?

(iii) $V(Y)$

(iv) $Cov(X, Y)$

(v) $P(X < 1/2 \cap Y < 1/4)$.

10

17. (a) $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage $\{X(t)\}$ in a stationary random process with mean zero and $R_{XX}(\tau) = e^{-\alpha|\tau|}$ and $H(w) = \frac{R}{R + iLW}$. Find $S_{YY}(w)$ and $R_{YY}(\tau)$.

(b) Explain the terms :

(i) WSS

(ii) Markov Process

(iii) Central Limit Theorem.

5+5