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06MAT21

Second Semester B.E. Degree Examination, June/July 08

Engineering Mathematics II

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, choosing at least two full questions from each part.***

Part - A

1 a. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **(06 Marks)**

b. State and prove Lagrange's mean value theorem. **(07 Marks)**

c. Expand $e^{\tan^{-1} x}$ by Maclaurin's series upto the term containing x^5 **(07 Marks)**

2 a. Evaluate :

i) $\lim\limits_{x \to \pi/2} (2x \tan x - \pi \sec x)$

ii) $\lim\limits_{x \to a} \left[2 - \left(\frac{x}{a}\right)\right]^{\tan\left(\frac{\pi x}{2a}\right)}$ **(06 Marks)**

b. Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using the Taylor's theorem. **(07 Marks)**

c. Find the maximum and minimum values of $x^2 + y^2$ subject to the condition $5x^2 + 6xy + 5y^2 = 8$ **(07 Marks)**

3 a. Evaluate the integral $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ by changing the order of integration. **(07 Marks)**

b. Evaluate the integral $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz \, dy \, dx}{(1+x+y+z)^3}$ **(07 Marks)**

c. With the usual notation, show that

$$\beta(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$$ **(06 Marks)**

4 a. Verify Green's theorem for $\oint_c \left[(xy + y^2)dx + x^2 dy\right]$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(07 Marks)**

b. Using the divergence theorem evaluate

$\iint_s \vec{f} \cdot \hat{n} \, ds$ where $\vec{f} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ **(07 Marks)**

c. Prove that cylindrical coordinate system is orthogonal. **(06 Marks)**

Part - B

5 a. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$ **(07 Marks)**

b. Solve: $\frac{d^3y}{dx^3} + y = 5e^x x^2$ **(07 Marks)**

c. Using the method of undetermined coefficients

Solve: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$ **(06 Marks)**

6 a. Solve: $\frac{d^2y}{dx^2}+4y=\tan 2x$ by using the method of variation of parameters. (07 Marks)

b. Solve: $x^3\frac{d^3y}{dx^3}+3y^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\ \cos(\log x)$ (07 Marks)

c. Solve the initial value problem

$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=0,$ given that $x(0)=0, \frac{dx(0)}{dt}=15.$ (06 Marks)

7 a. Find the Laplace transforms of

i) t^2e^{2t}

ii) $(\cos at-\cos bt)/t$ (07 Marks)

b. Find Laplace transform of the periodic function of period 2 a, which is defined by

$$f(t)=\begin{cases} t & \text{if } o\le t\le a \\ 2a-t & \text{if } a\le t\le 2a \end{cases}$$ (07 Marks)

c. Express $f(t)=\begin{cases} \cos t, & 0<t<\pi \\ \cos 2t, & \pi<t<2\pi \\ \cos 3t, & t>2\pi \end{cases}$ (06 Marks)

In terms of Heaviside unit step function and hence find $L\{f(t)\}$

8 a. Find i) $L^{-1}\left\{\frac{s-2}{s^2+7s+12}\right\}$

ii) $L^{-1}=\left\{\frac{e^{-6s}}{(s-4)^2}\right\}$ (06 Marks)

b. Using convolution theorem obtain the inverse Laplace transform of $\frac{s}{(s+2)(s^2+9)}$. (07 Marks)

c. Solve the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{-x}$ with $y(0)=1=y'(0)$ using Laplace transforms. (07 Marks)

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