Code No: NR220202 **NR** **Set No. 2**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Common to ME, MECT, AE, MMT, ETM, E.CONT.E, EIE, ECE, EEE
Time: 3 hours **Max Marks: 80**
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Prove that U = $\overset{-x}{e}$ $[(x^2 - y^2)\cos y + 2xy \sin y]$ is harmonic and find the analytic function whose real part is U.
   (b) Separate the real and imaginary parts of sin h z. [8+8]

2. (a) Evaluate $4\int\limits_0^\infty \frac{x^2dx}{1+x^4}$ using $\beta - \Gamma$ functions
   (b) Prove that $\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m,2^{4m-1}}$
   (c) Evaluate $\int\limits_0^2 (8-x^3)^{1/3}dx$ using $\beta - \Gamma$ functions [5+5+6]

3. (a) Prove that $J_0\,(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2} + \ldots\ldots\ldots\ldots$
   (b) $\int\limits_{-1}^{1} x^n P_n(x)dx = \frac{2^{n+1}.(n!)^2}{(2n+1)!}$ [8+8]

4. (a) Show that the transformation w=z^2 maps the circle |z-1|=1 into the cardioid r=2(1+cosθ) where w=re$^{i\theta}$ in the w-plane.
   (b) Find the bilinear transformation which maps the vertices (1+i, -i, 2-i) of the triangle T of the z-plane into the points (0, 1, i) of the w-plane. [8+8]

5. (a) Evaluate $\int\limits_C \frac{Cos\,z - \sin\,z\ \ dz}{(z+i)^3}$ with C: $|\,z\,| = 2$ using Cauchy's integral formula
   (b) Evaluate $\int\limits_{1-i}^{2+i} (2x+1+iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula [8+8]

6. (a) State and prove Laurent's theorem for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region
      i. 0 < | z − 1 | < 1
      ii. 1 < | z | < 2. [8+8]

7. (a) Use Rouche's theorem to show that the equation Z^5 + 15 Z +1=0 has one root in the disc | Z | < $\frac{3}{2}$ and four roots in the annulus $\frac{3}{2}$ < | Z | < 2.
   (b) Evaluate $\int\limits_0^\infty \frac{x^2dx}{(x^2+3b)}$ using residue theorem. [8+8]

8. (a) Find the poles of f(z) and the residues of the poles which lie on imaginary axis if f(z) = $\frac{(z^2+2z)}{(z+1)^2(z^2+4)}$

(b) Evaluate $\int\limits_C \frac{e^{2z}}{(z+1)^3}$ using residue theorem, C: $|z| = 2$. [8+8]

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Code No: NR220202 **NR** **Set No. 4**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Common to ME, MECT, AE, MMT, ETM, E.CONT.E, EIE, ECE, EEE
Time: 3 hours **Max Marks: 80**
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Prove that $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2} + ...$
   (b) $\int\limits_{-1}^{1} x^n P_n(x)dx = \frac{2^{n+1}.(n!)^2}{(2n+1)!}$ [8+8]

2. (a) Find the poles of f(z) and the residues of the poles which lie on imaginary axis if f(z) $= \frac{(z^2+2z)}{(z+1)^2(z^2+4)}$
   (b) Evaluate $\int\limits_C \frac{e^{2z}}{(z+1)^3}$ using residue theorem, C: $|z| = 2$. [8+8]

3. (a) State and prove Laurent's theorem for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region
      i. $0 < |\ z - 1\ | < 1$
      ii. $1 < |\ z\ | < 2$. [8+8]

4. (a) Evaluate $\int\limits_C \frac{Cos\, z - \sin\, z\ \ dz}{(z+i)^3}$ with C: $|\ z\ | = 2$ using Cauchy's integral formula
   (b) Evaluate $\int\limits_{1-i}^{2+i} (2x + 1 + iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula [8+8]

5. (a) Use Rouche's theorem to show that the equation $Z^5 + 15\ Z + 1 = 0$ has one root in the disc $|\ Z\ | < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |\ Z\ | < 2$.
   (b) Evaluate $\int\limits_0^\infty \frac{x^2 dx}{(x^2+3b)}$ using residue theorem. [8+8]

6. (a) Prove that U $= \overset{-x}{e}\ \ [(x^2 - y^2)\cos y + 2xy \sin y]$ is harmonic and find the analytic function whose real part is U.
   (b) Separate the real and imaginary parts of sin h z. [8+8]

7. (a) Evaluate $4\int\limits_0^\infty \frac{x^2 dx}{1+x^4}$ using $\beta - \Gamma$ functions
   (b) Prove that $\beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \frac{\pi}{m,2^{4m-1}}$
   (c) Evaluate $\int\limits_0^2 (8 - x^3)^{1/3} dx$ using $\beta - \Gamma$ functions [5+5+6]

8. (a) Show that the transformation w=z^2 maps the circle |z-1|=1 into the cardioid r=2(1+cosθ) where w=r$e^{i\theta}$ in the w-plane.

(b) Find the bilinear transformation which maps the vertices (1+i, -i, 2-i) of the triangle T of the z-plane into the points (0, 1, i) of the w-plane. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Use Rouche's theorem to show that the equation $Z^5 + 15\ Z + 1 = 0$ has one root in the disc $|\ Z\ | < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |\ Z\ | < 2$.
   (b) Evaluate $\int\limits_0^\infty \frac{x^2 dx}{(x^2+3b)}$ using residue theorem. [8+8]

2. (a) Evaluate $4\int\limits_0^\infty \frac{x^2 dx}{1+x^4}$ using $\beta - \Gamma$ functions
   (b) Prove that $\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m,2^{4m-1}}$
   (c) Evaluate $\int\limits_0^2 (8-x^3)^{1/3} dx$ using $\beta - \Gamma$ functions [5+5+6]

3. (a) Prove that $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2} + ...$
   (b) $\int\limits_{-1}^1 x^n P_n(x) dx = \frac{2^{n+1}.(n!)^2}{(2n+1)!}$ [8+8]

4. (a) Prove that $U = e^{-x}\ [(x^2-y^2)\cos y + 2xy\sin y]$ is harmonic and find the analytic function whose real part is U.
   (b) Separate the real and imaginary parts of sin h z. [8+8]

5. (a) Find the poles of f(z) and the residues of the poles which lie on imaginary axis if $f(z) = \frac{(z^2+2z)}{(z+1)^2(z^2+4)}$
   (b) Evaluate $\int\limits_C \frac{e^{2z}}{(z+1)^3}$ using residue theorem, C: $|z| = 2$. [8+8]

6. (a) State and prove Laurent's theorem for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region
      i. $0 < |\ z - 1\ | < 1$
      ii. $1 < |\ z\ | < 2$. [8+8]

7. (a) Show that the transformation $w=z^2$ maps the circle $|z-1|=1$ into the cardioid $r=2(1+\cos\theta)$ where $w=re^{i\theta}$ in the w-plane.
   (b) Find the bilinear transformation which maps the vertices (1+i, -i, 2-i) of the triangle T of the z-plane into the points (0, 1, i) of the w-plane. [8+8]

8. (a) Evaluate $\int\limits_C \frac{Cos\,z - \sin\,z\ \ dz}{(z+i)^3}$ with C: $|\ z\ | = 2$ using Cauchy's integral formula

(b) Evaluate $\int_{1-i}^{2+i} (2x + 1 + iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula [8+8]

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Code No: NR220202 **NR** **Set No. 3**

II B.Tech II Semester Examinations,APRIL 2011
MATHEMATICS - III
Common to ME, MECT, AE, MMT, ETM, E.CONT.E, EIE, ECE, EEE

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int\limits_C \frac{Cos\, z - \sin\, z\;\; dz}{(z+i)^3}$ with C: $|\, z\,| = 2$ using Cauchy's integral formula

   (b) Evaluate $\int\limits_{1-i}^{2+i} (2x + 1 + iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula [8+8]

2. (a) Find the poles of f(z) and the residues of the poles which lie on imaginary axis if f(z) $= \frac{(z^2+2z)}{(z+1)^2(z^2+4)}$

   (b) Evaluate $\int\limits_C \frac{e^{2z}}{(z+1)^3}$ using residue theorem, C: $|z| = 2$. [8+8]

3. (a) Prove that $J_0\,(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2}+$...........

   (b) $\int\limits_{-1}^{1} x^n P_n(x)dx = \frac{2^{n+1}.(n!)^2}{(2n+1)!}$ [8+8]

4. (a) Prove that U $= \overset{-x}{e}\;\; [(x^2 - y^2)\cos y + 2xy \sin y]$ is harmonic and find the analytic function whose real part is U.

   (b) Separate the real and imaginary parts of sin h z. [8+8]

5. (a) Evaluate $4\int\limits_0^{\infty} \frac{x^2dx}{1+x^4}$ using $\beta - \Gamma$ functions

   (b) Prove that $\beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \frac{\pi}{m,2^{4m-1}}$

   (c) Evaluate $\int\limits_0^2 (8 - x^3)^{1/3}dx$ using $\beta - \Gamma$ functions [5+5+6]

6. (a) State and prove Laurent's theorem for an analytic function f (z).

   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region

      i. $0 <$ | z – 1 | < 1
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   (b) Find the bilinear transformation which maps the vertices (1+i, -i, 2-i) of the triangle T of the z-plane into the points (0, 1, i) of the w-plane. [8+8]

8. (a) Use Rouche's theorem to show that the equation $Z^5 + 15\ Z + 1 = 0$ has one root in the disc $\mid Z \mid < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < \mid Z \mid < 2$.

   (b) Evaluate $\int\limits_0^\infty \frac{x^2 dx}{(x^2+3b)}$ using residue theorem. [8+8]

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