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06CS34

Third Semester B.E. Degree Examination, Dec.09/Jan.10

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Let x be the set of all three digit integers that is x = { x is an integer / $100 \le x \le 999$}. If A_i is the set of numbers in x whose i^{th} digit is *i*, compute the cardinality of the set $A_1 \cup A_2 \cup A_3$. **(05 Marks)**

b. Using the laws of set theory, simplify each of the following :

i) $A \cap (B - A)$ ii) $\overline{\overline{(A \cup B)} \cap C \cup \overline{B}}$ **(05 Marks)**

c. State and prove Demorgan's laws set theory. **(05 Marks)**

d. Among the integers 1 – 200 find the numbers of integer's that are :

i) Divisible by 2 or 5 or 9 ii) Not divisible by 5

iii) Not divisible by 2 or 5 or 9 iv) Divisible by 5 or not by 2 or 9 **(05 Marks)**

2 a. Define tautology S.T

$[(p \vee q) \cap (p \to r) \wedge (q \to r)] \to r$ is a tautology by constructing truth table. **(05 Marks)**

b. Prove the following by using laws of logic :

i) $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$ ii) $[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(05 Marks)**

c. Verify the principles of duality for the logical equivalence :

i) $\sim (p \wedge q) \to \sim p \vee (\sim p \vee q) \Leftrightarrow \sim p \vee q$ ii) P.T. $[p \wedge (p \to q) \wedge r] \Rightarrow (p \vee q) \to r$ **(05 Marks)**

d. Check whether the following is a valid argument:

If I study, then I will not fail in the examination. If I do not watch T.V. in the evening I will study.

I failed in the examination

∴ I must have watch T.V. in the evening. **(05 Marks)**

3 a. Define an open statement. Write down the negation of following statements:

i) For all integer 'n' if n is not divisible by 2 then n is odd.

ii) If k, m, n are any integers when (k – m) and (m – n) are odd then (k – n) is even. **(05 Marks)**

b. For the universe of all integers, define the following open statements:

Let p(x) : x > 0, s(x) : x is divisible by 3, q(x) : x is even, r(x) : x is a perfect square, t(x) : x is divisible by 7. Write down the following quantified statements in symbolise form :

i) At least one integer is even ii) Some even integers are divisible by 3

iii) For every integer is either even or odd

iv) If x is even and a perfect square then x is not divisible by 3

v) If x is odd or is not divisible by 7 then x is divisible by 3. **(05 Marks)**

c. i) PT $\exists x q(x)$ follows logically from the premises $\forall x p(x) \to q(x)$ and $\exists x p(x)$

ii) PT the following argument is valid where 'C' is the specification element of universe

$\forall x[p(x) \to q(x)]$

$\forall x[q(x) \to r(x)]$ **(05 Marks)**

$\sim r(c) / \therefore \sim p(c)$

d. Give direct proof of the statement if n is odd integer and n^2 is an odd integer. **(05 Marks)**

4 a. Prove by mathematical induction that for all (+)ve integer $n \geq 1$, $1+2+3+\ldots\ldots+n = \frac{n(n+1)}{2}$ **(05 Marks)**

b. A sequence $\{a_n\}$ is defined successively by $a_1 = 4$, $a_n = a_{n-1}+n$, for $n \geq 2$ find an explicit form. **(05 Marks)**

c. The fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1}+F_{n-2}$ for $n \geq 2$. Evaluate F_2 to F_{10}. **(05 Marks)**

d. The Ackermann's number. $A_{m,n}$ are defined recursively for m, n $\in$N as follows:
$A_{o,n} = n + 1$, for $n \geq 0$
$A_{m,o} = A_{m-1,1}$ for $m \geq 0$
$A_{m,n} = A_{m-1,p}$ where $p = A_{m,n-1}$ form m, $n \geq 0$
P.T. $A_{1,n} = n + 2$ for all n $\in$N **(05 Marks)**

PART – B

5 a. Let A = {1 2 3 4}. Let 'R' be a relation on A defined by xRy iff x/y and y = 2x write down:
i) 'R' is a relation of set of ordered pairs
ii) Draw di-graph of R
iii) Determine in degrees and out-degrees of a diagraph **(05 Marks)**

b. Define the following terms and give an example for each:
i) Reflexive ii) Irreflexive iii) Anti symmetric
iv) Transitive v) Symmetry **(05 Marks)**

c. Let R be an equivalence relation on A and (ab)$\in$A then ST the following statement are:
i) $a \in [a]$ or $a \in R(a)$ ii) aRb iff, $[a] = [b]$ or $R(a) = R(b)$
iii) $[a] \cap [b] \neq \phi$ then $[a] = [b]$ **(05 Marks)**

d. Draw the hasse diagram respresented by the positive divisor of 36. **(05 Marks)**

6 a. Let A = B = {1 2 3 4}
i) f = (1 1) (2 3) (3 4) (4 2) ii) let A = {a b c}, B = {1 2 3 4} f = {(a 1) (b 1) (c 4)}
iii) A = {1 2 3 4}, b ={a b c d}, f = {(1 a) (2 a) (3 d) (4 c)}.
Determine 'f' is one-one or on to. **(05 Marks)**

b. If m and n are positive integers with $1 \leq n \leq m$, P.T. $s(m + 1, n) = s(m, n - 1) + n\, s(m, n)$ **(05 Marks)**

c. If $f : A \rightarrow B$ is invertible then it has unique inverse. **(05 Marks)**

d. State Pigeon-hole principle. If (n + 1) number are chosen from 1 to 2n at least one pair add to (2n + 1). **(05 Marks)**

7 a. Define Abelian group. Prove that a group G is abelian if and only if for all a, b $\in$ G, $(a, b)^{-1} = a^{-1}b^{-1}$ **(05 Marks)**

b. Define a cyclic group, show that every cyclic group is abelian but the converse is not true. **(05 Marks)**

c. State and prove Lagrange's theorem. **(05 Marks)**

d. Let $f : G \rightarrow H$ be a homomorphism from G into H. If G is abelian, prove that H is also abelian. **(05 Marks)**

8 a. Prove that every field is an integral domain. **(05 Marks)**

b. S is a subring of R if and only if :
i) For all a, b$\in$S, we have a + b $\in$S and ab$\in$S ii) For all a$\in$S, we have -a$\in$S **(05 Marks)**

c. Show that Z_5 is an integral domain. **(05 Marks)**

d. Prove that Zn is a field if and only if n is a prime. **(05 Marks)**

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