M 21394

Reg. No. : ....

Name : ....

Fourth Semester B.Tech. (Regular/Supplementary/Improvement – Including Part Time) Degree Examination, May 2012
(2007 Admn. Onwards)
PT 2 K6/2K6 CE/ME/EE/EC/CS/IT/AEI 401 : ENGINEERING MATHEMATICS – III

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　Max. Marks : 100

***Instruction** : Answer **all** questions.*

1. a) State the necessary and sufficient conditions for a function $f(z) = u+iv$ to be analytic. **5**

   b) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic and find v such that u+iv is analytic. **5**

   c) Find the poles of the function $f(z) = \dfrac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ and the residue at each pole. **5**

   d) Expand $f(z) = \dfrac{1}{z}$ in Taylor's series at $z = 1$. **5**

   e) Fit a straight-line $y = a + bx$ to the following data by the method of least squares.

x	0	1	3	6	8
y	1	3	2	5	4

5

   f) If X and Y are two discrete random variables with joint probability function given by $f(x, y) = \begin{cases} 1/3, & \text{for } (x, y) = (-1, 0), (0, 1), (1, 0) \\ 0 & \text{otherwise} \end{cases}$.

   Find cov (X. Y). Are X and Y independent ? **5**

   g) Classify the equation $\dfrac{\partial^2 u}{\partial x^2} + 4\dfrac{\partial^2 u}{\partial x \partial y} + 4\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial u}{\partial x} + 2\dfrac{\partial u}{\partial y} = 0$. **5**

   h) Using D'Alembert's method find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection, $f(x) = k(\sin x - \sin 2x)$. **5**

P.T.O.

2. a) i) If f(z) is an analytic function, then show that the transformation w = f(z) is conformal at all points where $f'(z) = 0$. **7**

ii) Construct the analytic function, f(z) = u + iv, where $(u - v) = e^x(\cos y - \sin y)$ using Milne Thompson Method. **8**

OR

b) i) Find the bilinear transformation which maps (–1, 0, 1) to (0, i, 3i). **7**

ii) Discuss the transformation $W = 1/z$. **8**

3. a) i) Using Cauchy's Residue theorem, evaluate $\int_C \frac{5z^2 - 3z + 2}{(z-1)^2} dz$ where C is $|z| = 3/2$. **8**

ii) State and prove Cauchy's integral formula. **7**

OR

b) i) Find the Laurent's series of $f(z) = \frac{z+2}{(z+1)(z+4)}$ in $1 < |z| < 4$. **7**

ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$. **8**

4. a) i) Find the correlation coefficient, equation of regression lines, angle between regression lines from the following data. **9**

x	1	2	3	4	5
y	2	5	3	8	7

ii) A fair coin is tossed 3 times. Let X denote the no. of heads and Y the no. of tails. Obtain the joint probability distribution of X and Y. Also find their marginal distributions. **6**

OR

b) i) An ordinary deck of 52 playing cards is randomly divided in to 4 piles of 13 cards each. Compute the probability that each pile has exactly one ace. **7**

ii) Consider the joint distribution of X and Y. **8**

↓X \ Y→	−2	−1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find (a) E(X), E(Y) (b) Cov (X, Y) and (c) σ_x, σ_y, ρ (X, Y).

5. a) i) Discuss the assumptions made and derive 1-dimensional wave equation. **7**

ii) The equation for heat conduction along a bar of length 'l' $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, neglecting the radiation. Find an expression for u(x,t), if the ends of the bar are maintained at zero temperature and if initially the temperature is T at the centre of the bar and falls uniformly to zero with time. **8**

OR

b) The points of trisection of a string are pulled aside through the same distance 'd' on opposite position of equilibrium and the string is released from rest. Find the expression for the displacement of the sting at any subsequent time and show that the mid point of the string is always at rest. **15**
