SE- SEM III (Rev.) Comp. 15/Nov-Dec 2012
Discrete Structure & Graph Theory

V-A4-II-Hf-Ex-12-D-12

Con. 7396-12.

KR-3533

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Solve any **four** questions out of remaining **six** questions.
(3) Assumptions made should be **clearly** stated.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Show that :— **6**

$1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = (4n^3 - n)/3$.

(b) Show that if any five numbers from 1 to 8 are choosen, then two of them will add to 9. **6**

(c) Out of 250 candidates who failed in an examination, it was revealed that 128 failed in mathematics, 87 in physics and 134 in aggregate. 31 failed in mathematics and in Physics, 54 failed in the aggregate and in mathematics, 30 failed in the aggregate and in physics. Find how many candidates failed. **8**

(i) in all the three subjects.
(ii) in mathematics but not in physics.
(iii) in the aggregate but not in mathematics.
(iv) in physics but not in aggregate or in mathematics.

2. (a) Determine whether the following relation are symmetric, asymmetric and antisymmetric. **6**

(i) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) Construct truth table to determine whether the given statement is a tautology, contradiction or neither :— **6**

(i) $(q \wedge p) \vee (q \wedge \sim p)$
(ii) $(p \vee \sim q) \wedge p$

(c) If R be a relation in the set of integers z defined by— **8**

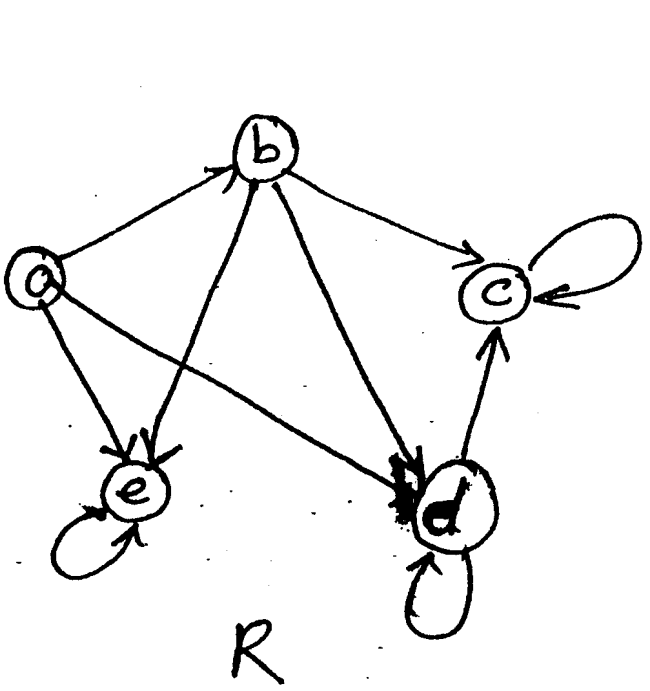
$R = \{(x, y) : x \in z, y \in z, x - y \text{ is divisible by } 3\}$

Show that the relation R is an equivalence relation and describe the equivalence classes.

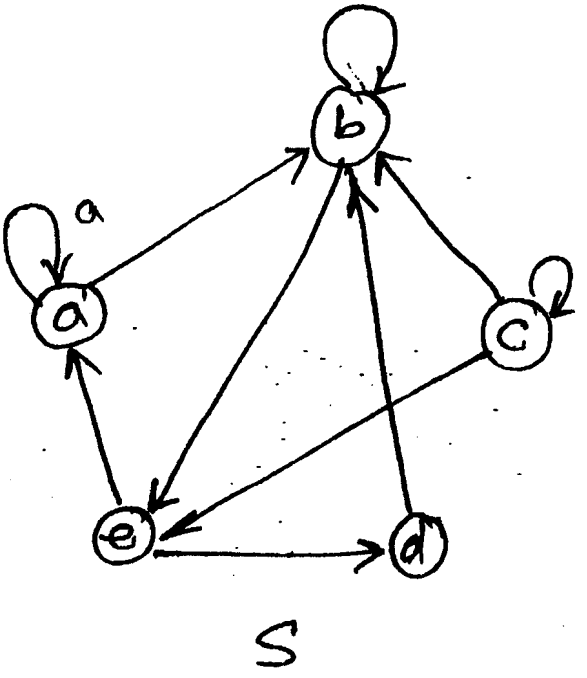
[TURN OVER

3. (a) Define with example injective, surjective and bijective function. **6**

   (b) Let A = { a, b, c, d, e } and Let R and S be two relations on A whose corresponding diagraph are shown below. Find $\overline{R}$, R^{-1}, $R \cap S$ and $R \cup S$. **8**



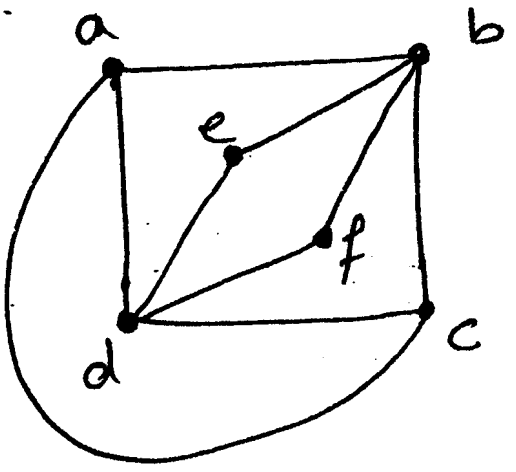
R

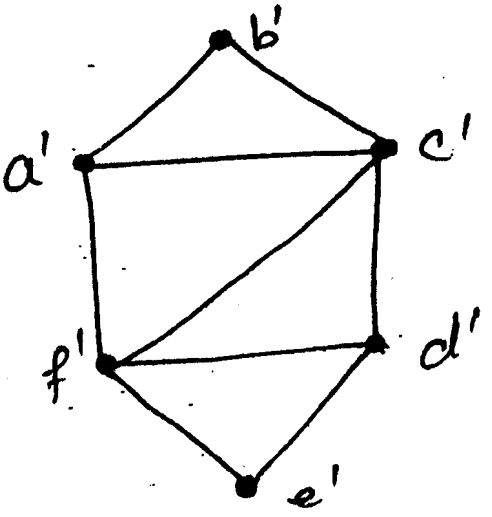


S

   (c) A connected planar graph has 10 vertices each of degree 3. Into how many regions does a representation of this planar graph split the plane ? **6**

4. (a) Determine whether the following pair of graphs are isomorphic or not. **6**





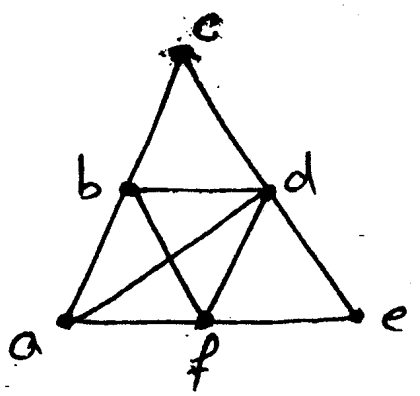
   (b) Let $f : R \to R$, $f(x) = x^2 - 1$, $g(x) = 4x^2 + 2$ find (i) $f \circ (g \circ f)$ (ii) $g \circ (f \circ g)$ **6**

   (c) Draw hasse diagram of the poset D60 and identify whether it is linearly ordered or not ? **8**

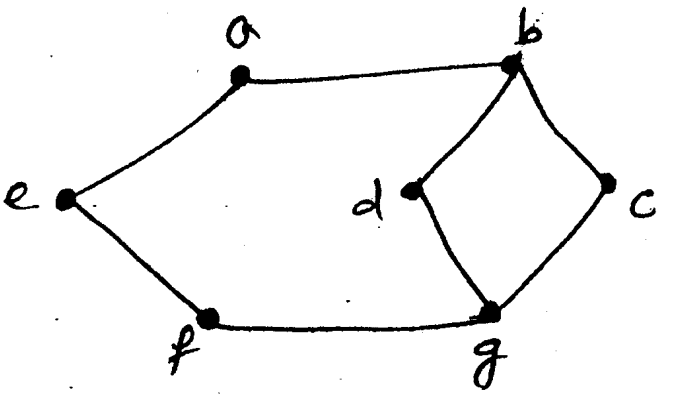
5. (a) Let A = { 1, 2, 3, 4 } and R = { (1, 2), (2, 1), (2, 2), (4, 3), (3, 1) }. Find the transitive closure of relation R by Warshall's algorithm. 6

(b) Define a ring and field. Let R = { 0, 1, 2, 3 }. Show that the modulo 4 system is a ring. 8

(c) Determine which of the following graph contain an Eulerian or Hamiltonian circuit. 6



(i)



(ii)

6. (a) Consider the (2, 6) group encoding function e : B2 → B6 defined by :— 8

e(00) = 0 0 0 0 0 0
e(01) = 0 1 1 1 1 0
e(10) = 1 0 1 1 0 1
e(11) = 1 1 0 0 1 1

Decode the following relative to maximum likelihood decoding function—

(i) 0 0 1 1 1 0 (ii) 1 1 1 1 0 1 (iii) 1 1 0 0 1 0

(b) Solve the recurrence relation $a_n = 4 (a_{n-1} - a_{n-2})$ where $a_0 = 1$, $a_1 = 1$. 6

(c) Show that { 1, 5, 7, 11 } is a abelian group under multiplication modulo 12. 6

7. (a) Define with example :— 8

(i) Normal subgroup.
(ii) Spanning tree.
(iii) Planar graph.
(iv) Quantifiers.

(b) Consider chains of divisors of 4 and 9 i.e. L_1 = { 1, 2, 4 } and L_2 = { 1, 3, 9 } and partial ordering relation of division on L_1 and L_2. Draw the lattice $L_1 \times L_2$. 6

(c) Prove that every field is an integral domain. 6