

26.11.13

(3 Hours)

[Total Marks : 100]

N.B. : (1) Question no. 1 is compulsory.

(2) Answer any four questions out of remaining six questions.

1. (a) Find fourier expansion of  $f(x) = 4 - x^2$  in the interval  $(0, 2)$  5
- (b) Find the probability that atmost 5 defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective. 5
- (c) Given  $6y = 5x + 90$ ,  $15x = 18y + 130$ ,  $6x^2 = 16$ . Find 5
- (i)  $\bar{x}$  and  $\bar{y}$  (ii)  $r$  and (iii)  $6y^2$
- (d) Solve the two dimensional heat equation  $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$  which satisfies the conditions 5
- $u(0,y) = u(\ell,y) = u(x,0) = 0$  and  $u(x,a) = \sin \frac{n\pi x}{\ell}$ .
2. (a) Obtain fourier series for  $f(x) = x - x^2$ ,  $-\pi < x < \pi$  Hence deduce that 7
- $$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$
- (b) Seven dice are thrown 729 times. How many times do you expect atleast 4 dice to show three or five? 7
- (c) A continuous random variable X has p.d.f.  $f(x) = kx^2 e^{-x}$ ,  $x \geq 0$ . Find K, mean and variance. 6
3. (a) Using normal distribution find the probability that in a group of 100 persons there will be 55 males assuming that the probability of a person being male is  $\frac{1}{2}$ . 7
- (b) Derive wave equation for vibration of string 7
- (c) Obtain fourier expansion of  $f(x) = \sin ax$  in the interval  $(-\ell, \ell)$  where  $a$  is not an integer. 6
4. (a) Calculate correlation coefficient from the foll : data 7
- |   |    |    |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|----|
| x | 23 | 27 | 28 | 29 | 30 | 31 | 33 | 35 | 36 | 39 |
| y | 18 | 22 | 23 | 24 | 25 | 26 | 28 | 29 | 30 | 32 |
- (b) A die was thrown 132 times and the foll : frequencies were observed : 7
- |              |    |    |    |    |    |    |
|--------------|----|----|----|----|----|----|
| No. obtained | 1  | 2  | 3  | 4  | 5  | 6  |
| Frequency    | 15 | 20 | 25 | 15 | 29 | 28 |
- Test the hypothesis that the die is unbiased.

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- (c) Obtain complex form of fourier series for  $f(x) = \cosh 3x + \sinh 3x$  in  $(-3, 3)$ . 6

5. (a) A homogeneous rod of conducting material of length  $\ell$  has ends kept at zero temperature and the temperature at the centre is  $T$  and falls uniformly to zero at the two ends. Find the temperature  $u(x, t)$  at any time. 7

- (b) Obtain half range sine series for  $f(x)$  when 7

$$f(x) = x, \quad 0 < x < \frac{\pi}{2}$$

$$= \pi - x, \quad \frac{\pi}{2} < x < \pi$$

$$\text{Hence deduce } \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

- (c) Two independent samples of sizes 8 and 7 gave the foll : results 6

Sample 1 : 19 17 15 21 16 18 16 14

Sample 2 : 15 14 15 19 15 18 16

is the difference between the sample means significant ?

6. (a) Find the expansion of  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$  as a half range cosine series. 7

Hence S. T

$$(i) \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$(ii) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \frac{\pi}{12}$$

- (b) The diameter of a semicircular plate of radius  $a$  is kept at  $0^\circ\text{C}$  and the temperature at the semicircular boundary is  $T^\circ\text{C}$ . Find the steady state temperature  $u(r, \theta)$ . 7

- (c) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance. Whether the boys perform better than the girls? 6

7. (a) S. T the functions  $f_1(x) = 1$ ,  $f_2(x) = x$  are orthogonal on  $(-1, 1)$ . Determine the constants  $a$  and  $b$  such that the function  $f_3(x) = -1 + ax + bx^2$  is orthogonal to both  $f_1$  and  $f_2$  on that interval. 7

- (b) Find fourier integral representation of 7

$$f(x) = x, \quad 0 < x < a$$

$$= 0, \quad x > a, \quad f(-x) = f(x)$$

- (c) If  $u = x - y$ ,  $v = x + y$  and if  $x, y$  are uncorrelated, 6

$$\text{P.T r uv} = \frac{6x^2 - 6y^2}{6x^2 + 6y^2}$$