[4161] – 101

Seat No.	

F.E. Semester – I Examination, 2012
ENGINEERING MATHEMATICS – I
(2008 Pattern)

Time : 3 Hours Max. Marks : 100

Instructions : *I) Attempt Q. **1** or Q. **2**, Q. **3** or Q. **4**, Q. **5** or Q. **6** from Section **I** and Q. **7** or Q. **8**, Q. **9** or Q. **10**, Q. **11** or Q. **12** from Section **II**.*
*II) Answers to the **two** Sections should be written in **separate** books.*
III) Neat diagrams must be drawn wherever necessary.
IV) Black figures to the right indicate full marks.
V) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.
VI) Assume suitable data, if necessary.

SECTION – I

1. A) Define Rank of the matrix. Find the rank of matrix A by reducing it to its normal form. **6**

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

B) Show that the system **5**

$3x + 4y + 5z = \alpha$

$4x + 5y + 6z = \beta$

$5x + 6y + 7z = \gamma$

is consistent only when α, β, γ are in geometric progression.

P.T.O.

C) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$ and hence find A^{-1}. **7**

OR

2. A) Find Eigen values and Eigen vectors for the matrix **7**

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B) Examine whether the following vectors are linearly dependent. If so find the relation between them $x_1 = (1, 1, 1, 3)$, $x_2 = (1, 2, 3, 4)$, $x_3 = (2, 3, 4, 7)$. **6**

C) Given the transformation

$$Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the co-ordinates (x_1, x_2, x_3) corresponding to (2, 9, 5) in Y. **5**

3. A) Prove that

$$\frac{1+\cos\alpha+i\sin\alpha}{1-\cos\alpha+i\sin\alpha} = \left(\cot\frac{\alpha}{2}\right)\left[e^{i\left(\alpha-\frac{\pi}{2}\right)}\right].$$ **5**

B) Solve the equation $x^7 - x^4 + x^3 - 1 = 0$ by using De Moivre's theorem. **5**

C) Prove that

$$\log\left[\frac{1}{1-e^{i\theta}}\right]=\log\left(\frac{1}{2}\operatorname{cosec}\frac{\theta}{2}\right)+i\left(\frac{\pi}{2}-\frac{\theta}{2}\right).$$ **6**

OR

4. A) If z_1, z_2 and origin represent vertices of an equilateral triangle on the Argand diagram, show that **6**

$$\frac{1}{z_1^2}+\frac{1}{z_2^2}=\frac{1}{z_1 z_2}$$

B) If $\operatorname{cosec}\left(\frac{\pi}{4}+ix\right) = u + iv$ then prove that

$$(u^2 + v^2)^2 = 2(u^2 - v^2).$$ **5**

C) If $a=e^{i\alpha}, b=e^{i\beta}, c=e^{i\gamma}$ then prove that

$$\frac{(a+b)(b+c)(c+a)}{abc}=8\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\beta-\gamma}{2}\right)\cos\left(\frac{\gamma-\alpha}{2}\right).$$ **5**

5. A) If $y=\frac{1}{(x-2)(x-1)^2}$

then find n^{th} order differential coefficient of y w.r.t. x. **5**

B) If x = tan (log y) prove that,

$$(1 + x^2)y_{n+2} + [2(n + 1) x - 1]y_{n+1} + [n^2 - n - 2]y_n = 0.$$ **5**

C) Test convergence of the series (**any one**) : **6**

1) $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{28}}+\frac{1}{\sqrt{65}}+.....$

2) $\frac{1}{2}x+\frac{1}{2}.\frac{3}{4}x^2+\frac{1.3.5}{2.4.6}x^3+.....$

OR

6. A) Find n^{th} order differential coefficient of y w.r.t. x **5**

$$y=\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$

B) If $y = (\sin^{-1}x)^2$ then prove that

$(1 - x^2)\, y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$ **5**

C) Test convergence of the series (**any one**) : **6**

1) $\frac{1}{2}-\frac{2}{5}+\frac{3}{10}-\frac{4}{17}+.....$

2) $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$

SECTION – II

7. A) Using Taylor's theorem express

$(x - 2)^4 - 3(x - 2)^3 + 4(x - 2)^2 + 5$ in powers of x. **5**

B) Expand $(1+\sin x)^{1/2}$ upto x^6. **5**

C) Solve **any one** : **6**

1) Evaluate $\lim_{x\to 0}\left(\frac{\sinh x}{x}\right)^{1/x^2}$

2) Evaluate $\lim_{x\to \frac{1}{2}}\frac{\cos^2 \pi x}{e^{2x}-2xe}$

OR

8. A) Show that $\frac{x}{e^x-1}=1-\frac{x}{2}+\frac{x^2}{12}-\frac{x^4}{720}+...$ **5**

B) Expand $\tan^{-1} x$ in powers of (x – 1) as far as the term in $(x-1)^3$. **5**

C) Solve **any one** : **6**

1) Evaluate $\lim_{x\to 0}\frac{\sin x.\sin^{-1}x-x^2}{x^6}$

2) Find a and b if $\lim_{x\to 0}\frac{a\cos x-a+bx^2}{x^4}=\frac{1}{12}$

9. Attempt **any two** : **16**

A) If f(x, y) $=\frac{1}{x^2}+\frac{1}{2xy}+\frac{\log x-\log y}{x^2+y^2}$ then prove that

$xf_x+yf_y+2f=0$.

B) If $x=r\cosh\theta$, $y=r\sinh\theta$ then show that

$(x-y)(z_x-z_y)=rz_r-z_\theta$

C) If u = log $(x^3 + y^3 - x^2y - xy^2)$ prove that

$$u_{xx} + 2u_{xy} + u_{yy} = \frac{-4}{(x+y)^2}$$

OR

10. Solve **any two** : **16**

A) If x = u tan v, y = u sec v prove that

$(u_x)_y \ (v_x)_y = (u_y)_x \ (v_y)_x$.

B) If u = $f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$ show that

$$\frac{1}{x}u_x + \frac{1}{y}u_y + \frac{1}{z}u_z = 0.$$

C) If $x^m + y^m = b^m$ then prove that

$$\frac{d^2y}{dx^2} = -(m-1)b^m \frac{x^{m-2}}{y^{2m-1}}$$

11. A) If u = x + y + z, uv = y + z, uvw = z, show that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$$ **6**

B) Discuss the maxima and minima of $f(x, y) = x^3 + y^3 - 3axy$. **6**

C) Examine for functional dependence $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$. Find the relation between them if functionally dependent. **6**

OR

12. A) For the transformation x = u(1 – v), y = uv prove that

$$\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = 1.$$ **6**

B) Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum. **6**

C) Find the percentage error in the area of an ellipse when the errors of 2% and 3% are made in measuring its major and minor axis respectively. **6**
