Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH/NEW/APM(NEW)/TT(NEW)/AUE(NEW)/CHE(NEW)/ME(NEW)/PE(NEW)/CE(NEW)/SEM-4/M-402/2013

2013

MATHEMATICS – III

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Short Answer Type Questions)

1. Answer any *ten* of the following : 10 × 2 = 20

a) The probability density function of a random variable X is

$$f(x) = \begin{cases} k(2x-1), 0 \le x \le 2 \\ 0, \quad \text{elsewhere} \end{cases}$$

Find the value of k.

b) If A and B are two events with $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, find $P(A/B)$.

c) If X follows a Binomial distribution with parameters 4 and $\frac{1}{3}$ then find $P(X=1)$.

d) If X is normally distributed with mean = 0 and variance = 1, then find $E(X^2)$.

e) If $f(x) = x \sin x$, $-\pi \le x \le \pi$ be presented as a Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

then find the value of a_0.

f) Define an even function and an odd function. Determine whether the function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$ is even or odd.

g) State Perseval's Identity for a function which can be expanded as a Fourier series.

h) If $F(k)$ is the Fourier transform of $f(x)$, prove that the Fourier transform of $f(x-a)$ is $e^{iak}F(k)$.

i) Find the poles of the function $f(z) = \dfrac{4-3z}{(z-1)z(z-3)}$.

j) Evaluate $\oint_C \dfrac{3z^2-2}{z-1}\,dz$ where C is the circle $|z| = \dfrac{1}{2}$.

k) Show that $u(x,y) = 2x - x^3 + 3xy^2$ is a harmonic function.

l) Prove that $f(z) = |z|$ is nowhere differentiable.

m) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

n) Show that $x = 2$ is a regular singular point and $x = 0$ is an irregular singular point of the equation

$$x^2(x-2)^2\frac{d^2y}{dx^2} + 2(x-2)\frac{dy}{dx} + (x+1)y = 0$$

o) Using separation of variable method convert the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ into two second order ODEs where u is a function of x and y.

GROUP – B

Answer any *five* questions taking at least *one* question from each module. $5 \times 10 = 50$

Module I : Fourier Series and Fourier Transform

2. a) Find the Fourier series of the function which is 2π-periodic :

$$f(x) = x - x^2, \ -\pi \le x \le \pi$$

Hence find the sum of the series

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \infty$$ 4 + 3

b) Find the Fourier transform of $f(t) = e^{-a|t|}$, where $a > 0$ and $-\infty < t < \infty$. 3

3. a) Find the Fourier transform of the function

$$f(t) = \begin{cases} 1 \text{ for } |t| \le 1 \\ 0 \text{ for } |t| > 1 \end{cases}$$ 5

b) Find the Fourier sine transform of the function $f(t) = e^{-t}, \ t > 0$.

Using Parseval's identity evaluate $\int_0^\infty \frac{x^2}{(x^2+1)^2} \mathrm{d}x$. 5

4. a) Find the Fourier cosine series for the function $f(x) = \pi - x$ on $[0, \pi]$. Hence deduce the value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \infty$. 4 + 1

b) State convolution theorem for Fourier transform. Find the Fourier cosine transform of e^{-ax} and e^{-bx}. Hence using Parseval's relation show that

$$\int_0^\infty \frac{dk}{(k^2+a^2)(k^2+b^2)} = \frac{\pi}{2ab(a+b)}, \quad a>0,\ b>0.$$

1 + 2 + 2

Module II : Calculus of Complex Variable

5. a) Find the analytic function $f(z) = u(x,y) + iv(x,y)$ where $u(x,y) = 4xy - x^3 + 3xy^2$. 5

b) Expand the following function in a Laurent's series valid in the region $2 < |z| < 3 : f(z) = \frac{(z^2-1)}{(z+2)(z+3)}$. 5

6. a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the origin. 5

b) Evaluate by Cauchy's integral formula $\int_C \frac{z}{z^2-1}\, dz$, where C is the circle $|z| = 2$. 5

7. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$, by residue theorem. 5

b) Expand the function $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$. 5

Module III : Probability

8. a) A box contains 5 red balls and 10 white balls. Two balls are drawn at random without replacement. What is the probability that (i) the second ball is white, (ii) the first ball is red, given that the second ball is white ? 3 + 2

b) Show that for the exponential distribution

$$f(x) = \lambda e^{-\lambda x}, x > 0$$

$$= 0, \quad \text{elsewhere}$$

the mean and the standard deviation are both $\frac{1}{\lambda}$. 5

9. a) The mean weight of 500 male students of a college is 150 lbs and the standard deviation is 15 lbs. Assuming that the weights are normally distributed, find how many students weigh between 120 and 155 lbs. Given $\Phi(2) = 0 \cdot 9772$ and $\Phi(0 \cdot 33) = 0 \cdot 6293$. 5

b) Find the standard deviation of a Poisson distribution with parameter λ. 5

10. a) If X is a Poisson variate such that $P(X = 1) = 0 \cdot 2$ and $P(X = 2) = 0 \cdot 2$, then find $P(X = 0)$. 5

b) The probability that Ashok can solve a problem is $\frac{4}{5}$, that Amal can solve it is $\frac{2}{3}$ and that Abul can solve it is $\frac{3}{7}$. If all of them try independently, find the probability that the problem is solved. 5

Module IV : PDE and Series Solution of ODE

11. a) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$. 5

b) Deduce Rodrigue's formula :

$$P_n(x) = \frac{1}{2^n . n!} . \frac{d^n}{dx^n} (x^2 - 1)^n$$ 5

12. Using the method of separation of variables, solve the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$

with Boundary conditions : $u(0, t) = 0 = u(1, t)$ and Initial conditions : $u(x, 0) = f(x), u_t(x, 0) = g(x)$. 10

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