3E1416

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B. Tech. (Sem. III) (Main & Back) Examination, January - 2013
3AI6 Advanced Engg. Mathematics
(Common for Mech., AE & PI)

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

*Attempt any **five** questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*
Units of quantities used/ calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. NIL

UNIT - I

1 (a) Find the Fourier series for $f(x)=x+x^2, -\pi<x<\pi$.

Hence show that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ 8

(b) Obtain the expansion for y from the following table upto the first harmonic : 8

x	0	1	2	3	4	5
y	9	18	24	28	26	20

OR

1 (a) Find the Fourier sine and cosine transform of $f(x)$,

where $f(x)=1,\ for\ 0<x<a$
$=0\ for\ x>a$

8

(b) Solve $\frac{du}{dt}=\frac{d^2u}{dx^2}$, given that $u_x(0,t)=0$ and

$$u(x,0)=\begin{cases} x, & 0\le x\le 1 \\ 0, & x>1 \end{cases}$$

$u(x,t)$ is bounded and $x>0,\ t>0$. 8

UNIT - II

2 (a) An inulated rod of length l has i ends A and B kept at 0^oC and 100^oC respectively until steady state conditions prevail. If the temperature of B is then suddenly reduced to 0^oC and kept so, while that of end A is maintained, find the temperature $u(x,t)$ at distance x from A at time t. 8

(b) Find the laplace transform of $\frac{1}{t}\sin at$. 8

OR

2 (a) Find the Inverse Laplace transform of $\frac{11s^2-2s+5}{(s-2)(2s-1)(s+1)}$. 8

(b) Solve the following equation :

$(D^2+3D+2)x(t)=1,\ x(0)=0,\ D(x)=0$ at $t=0$. 8

UNIT - III

3 (a) If $f(z)=\begin{cases} \frac{x^3y(y-ix)}{x^6+y^2}, & z\ne 0 \\ 0, & z=0 \end{cases}$

prove that $\frac{f(z)-f(0)}{z-0}\to 0$ as $z\to 0$ along any radius vector but not as $z\to 0$ along the curve $y=ax^3$. Is this function differentiable at $z=0$? 8

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. 8

OR

3 (a) Evaluate the integral $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where c is the circle $|z| = 3$, by using Cauchy's integral formula. 8

(b) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ at all its poles in the finite plane. 8

UNIT - IV

4 (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by

$y = y_0 \sin^3 \pi x / l$.

It is released from rest from this position. Find the displacement $y(x,t)$. 8

(b) Prove $\left(1 - 2xt + t^2\right)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) t^n$.

8

OR

4 (a) Prove that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x), n \geq 0$.

8

(b) Prove that $\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x)\right]$

8

UNIT - V

5 (a) The ordinates of the normal curve are given by the following table :

x	0.0	0.2	0.4	0.6	0.8
y	0.3989	0.3910	0.3683	0.3332	0.2897

Evaluate :

(i) $y(0.25)$

(ii) $y(0.62)$

(iii) $y(0.43)$

8

(b) Use Lagrange's interpolation formula to find y when $x=2$, given that

$x=$	0	1	3	4
$y=$	5	6	50	105

8

OR

5 (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

(i) Trapezoidal rule

(ii) Simpson's $\frac{1}{3}$ rule

(iii) Simpson's $\frac{3}{8}$ rule.

8

(b) Define the operators δ and μ and prove that

$$\delta[f(x)g(x)]=\mu[f(x)]\delta[g(x)]+\mu[g(x)]\delta[f(x)]$$

8
