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06CS34

# Third Semester B.E. Degree Examination, June / July 08

# Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

**Note :** ***Answer any FIVE full questions, choosing at least TWO from each part.***

## PART - A

1 a. Define the following terms and give an example for each i) Set ii) Proper subset iii) Power set iv) Empty set v) Venn diagram. **(05 Marks)**

b. Using the laws of set theory, simplify each of the following

i) $A \cap (B\text{-}A)$ ii) $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}$ **(05 Marks)**

c. In a class of 30 students, 15 take arts, 8 take science, 6 take commerce, 3 take all the three courses. Show that 7 or more students take none of the course. **(05 Marks)**

d. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ respectively. Find the probability that the problem is solved. **(05 Marks)**

2 a. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following. i) $p \wedge q$ ii) $\neg p \vee q$ iii) $q \rightarrow p$ iv) $\neg q \rightarrow \neg p$. **(05 Marks)**

b. Verify that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$, for primitive statements p, q, and r. **(05 Marks)**

c. Prove the following logical equivalence using the laws of logic : $(\neg p \vee \neg q) \wedge (F_0 \vee P) \wedge P$. **(05 Marks)**

d. Establish the validity of the following argument using the rules of Inference.
$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$. **(05 Marks)**

3 a. What are the bound variables and free variables? Identify the same in each of the following expressions : i) $\forall_y \exists_z [\cos(x+y) = \sin(z-x)]$ ii) $\exists_x \exists_y [x^2 - y^2 = z]$. **(05 Marks)**

b. Let p(x) be the open statement "$x^2 = 2x$", where the universe comprises all integers. Determine whether each of the following statements is true or false. i) P(0) ii) P(1) iii) P(2) iv) P(-2) v) $\exists x$ P(x) . **(05 Marks)**

c. Provide the steps and reasons to establish the validity of the argument :

$$\begin{array}{l} \forall x[p(x) \rightarrow (q(x) \wedge r(x))] \\ \underline{\forall x[p(x) \wedge s(x)]} \\ \therefore \forall x[r(x) \wedge s(x)] \end{array}$$

**(05 Marks)**

d. Give a direct proof for each of the following

i) For all integers K and $l$, if k, $l$ are both even, then k+$l$ is even.

ii) For all integers k and $l$, if k, $l$ are even, then k.$l$ is even. **(05 Marks)**

4 a. Prove by Mathematical Induction that for every positive integer n, n 1, $2^{n-1}$. **(07 Marks)**

b. Apply backtracking technique to obtain an explicit formula for the sequence, defined by the recurrence relation $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$. **(06 Marks)**

c. Solve the linear recurrence relation $a_n = 4a_{n-1} + 5\,a_{n-2}$ with $a_1 = 2$, $a_2 = 6$. **(07 Marks)**

## PART - B

5 a. Define the following terms and give an example for each i) Reflexive ii) Irreflexive iii) antisymmetric iv) Transitive v) partition set. **(05 Marks)**

b. For A = {1, 2, 3, 4} and R = { (1, 2), (1, 3), (2, 4), (4,4)}. Compute i) $R^2$ ii) $R^3$ iii) $R^\infty$ iv) $M_R$ v) $(M_R)^t$. **(05 Marks)**

c. If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1,2\}$, $A_2 = \{2,3,4\}$ and $A_3 = \{5\}$, define relation R on A by xRy if x and y are in the same subset $A_i$, for $1 \le i \le 3$. Is R an equivalence relation? **(05 Marks)**

d. Draw the diagraph and Hasse diagram representing the positive divisors of 36. **(05 Marks)**

6 a. For each of the following function, determine whether it is one-to-one and determine its range.
i) $f : Z \to Z$, $f(x) = 2x+1$ ii) $f : Q \to Q$, $f(x) = 2x+1$ iii) $f : Z \to Z$, $f(x) = x^3 - x$
iv) $f : R \to R$, $f(x) = e^x$. v) f: $[0, \pi] \to R$, $f(x) = \sin x$. **(05 Marks)**

b. State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1. Show that if we select five points in the interior of this triangle, there must be atleast two whose distance apart is less than ½. **(05 Marks)**

c. Define function composition and let A = {1, 2, 3, 4}, B = {a, b, c} and C = {w, x, y, z} with $f : A \to B$ anf $g : B \to C$, given by f = {(1, a), (2, a), (3, b), (4, c) and g = {(a, x), (b, y), (c, z)}. For each of the element of A find gof. **(05 Marks)**

d. Let $f, g : z^+ \to z^+$ where for all $x \in z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x-1\}$, the maximum of 1 and x-1. i) What is the range of f? ii) Is f an onto function? iii) Is the function f one – to – one. iv) What is the range of g? v) Is g an onto function. **(05 Marks)**

7 a. Define i) Group ii) Subgroup iii) homomorphism iv) Cyclic group v) Coset. **(05 Marks)**

b. Prove that if G is a finite group of order n with H a subgroup of order m, then m divides n. **(05 Marks)**

c. Let G = $S_4$, the symmetric group on four symbols and let H be the subset of G where

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \right\}.$$

Construct a table to show that H is an abelian group of G. **(05 Marks)**

d. If G is a group, prove that for all a, b, $\in$ G, i) $(a^{-1})^{-1} = a$ ii) $(ab)^{-1} = b^{-1} a^{-1}$. **(05 Marks)**

8 a. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R)$, prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a unit of this ring if and only if $ad - bc \neq 0$. **(05 Marks)**

b. Let (R, +, . ) be a commutative ring and let z denote the zero element of R. for a fixed element $a \in R$, define $N(a) = \{r \in R \mid ra = Z\}$. Prove that N(a) is an ideal of R. **(05 Marks)**

c. Construct a decoding table (with syndromes) for the code given by the generator

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$ **(05 Marks)**

d. Prove that for all $n \in N$, $10^n \equiv (-1)^n \pmod{11}$. **(05 Marks)**

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